AGRICULTURAL RESEARCH INSTITUTE
PUSA

PROCEEDINGS

OF THE

ROYAL SOCIETY OF LONDON

SERIES A

CONTAINING PAPERS OF A MATHEMATICAL AND PHYSICAL CHARACTER.

VOL. LXXXII

LONDON:

PRINTED FOR THE ROYAL SOCIETY AND SOLD BY
HARRISON AND SONS, ST MARTIN'S LANE,
PRINTERS IN ORDINARY TO HIS MAJESTY.

SEPTEMBER, 1909.

LONDON
HARRISON AND SONS, PRINTERS IN ORDINARY TO HIS MAJESTY,
ST MARTIN'S LANE

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PROCEEDINGS OF

THE ROYAL SOCIETY.

Section A —Mathematical and Physical Sciences.

Address of the President, Lord Rayleigh, O.M., D.C.L., at the Anniversary Meeting on November 30, 1908.

Since the last Anniversary the Society has sustained the loss of eighteen Fellows and four Foreign Members

The deceased Fellows are —

The Right Hon Lord Kelvin, died December 17, 1907
Sir Alfred Baring Garrod, died December 28, 1907.
Robert Lewis John Ellery, died January 14, 1908.
Prof. James Bell Pettigrew, died January 31, 1908
William Ashwell Shenstone, died February 3, 1908
Sir John Denis Macdonald, died February 7, 1908
Lieutenant-General Sir Richard Strachey, died February 12, 1908
Dr William Edward Wilson, died March 6, 1908
Dr. Henry Clifton Sorby, died March 9, 1908
Sir John Eliot, died March 17, 1908.
The Duke of Devonshire, died March 24, 1908
Dr. James Bell, died March 31, 1908.
Colonel Andrew Wilson Baird, died April 2, 1908
Sir John Evans, died May 31, 1908.
Lord Blythswood, died July 8, 1908.
Arthur Lister, died July 20, 1908.
The Earl of Rosse, died August 29, 1908.
Prof. William Edward Ayrton, died November 8, 1908.

The deceased Foreign Members are:—

Pierre Jules César Janssen, died December 23, 1907.
Franz von Leydig, died April, 1908.
Henri Becquerel, died August 25, 1908
Éleuthère Élie Nicolas Mascart, died August 26, 1908

The list of deaths this year is exceptionally heavy, and includes the name of one of the most eminent scientific men of our generation, who occupied the Presidency of this Society from 1890 to 1895—I refer, of course, to Lord Kelvin.

We are fortunate in having secured for our 'Proceedings' a review of Kelvin's life and work, written by one who is especially well qualified for the difficult task I do not doubt that Professor Larmor is right in placing in the forefront of that work those fundamental advances in Thermodynamics which date from the middle of the last century. It was Kelvin who first grasped the full scope of the principle known as the Second Law, a law which may indeed well be considered to stand first in order of importance, regarded from the point of view of man's needs and opportunities It would be futile to attempt here a re-survey of the ground covered by Professor Larmor

My acquaintance with Kelvin was limited, until about 1880, a time when I was occupied with measurements relating to the electrical units, and received much appreciated encouragement From then onwards until his death I enjoyed the privilege of intimacy and, needless to say, profited continually from his conversation, as I had done before from his writings Our discussions did not always end in agreement, and I remember his admitting that a certain amount of opposition was good for him Such discussions often invaded the officers' meetings during the time that we were colleagues, not always to the furtherance of the Society's business. But I must not linger over these reminiscences, interesting as they are to me We shall never see his like

By the death of Sir Richard Strachey we have lost a man well known to the senior Fellows, who served repeatedly upon the Council and whose advice was always valued He was a born administrator, and by his work in India and afterwards at the Meteorological Office he rendered splendid service.

Dr. Sorby's researches extended over many fields, and in several of them he was a pioneer I suppose that his greatest achievement was the introduction of the method in which thin slices of rock are examined under the microscope. Among his many interesting observations are those upon the retardation of freezing in capillary tubes. It appears that the walls exercise

an influence at distances much greater than those usually regarded as molecular—evidence apparently of structure upon an extended scale Dr Sorby belonged to a class on whom England has special reason to congratulate herself, men who pursue science unprofessionally. The names of Cavendish, Young, Joule, and Darwin at once suggest themselves It is to be feared that specialisation and the increasing cost and complication of experimental appliances are having a prejudicial effect in this regard. On the other hand, the amateur is not without advantages which compensate to some extent Certainly, no one who has the root of the matter in him should be deterred by fears of such difficulties, and the example of Sorby suffices to show how much is open to ingenuity unaided by elaborate appliances

The name of Sir John Evans must not pass without special notice There are few in recent years to whom the Society has been more indebted Many of our Fellows hardly realise how important and laborious are the services rendered in the office of Treasurer Evans' scientific attainments, his knowledge of the world and of business, and his personal characteristics specially qualified him for office An appreciation, signed by well-known initials, has recently appeared in our 'Proceedings'

On the Foreign List, also the losses are heavy. We have especially to condole with our colleagues in France upon the havoc caused by death within the last year or two Janssen, and Mascart, who was much missed at the recent Electrical Conference, had reached a full age But Becquerel was in the full tide of life, and we had hoped to learn much more from him, as the discoverer of radio-activity, he had opened up inquiries whose significance seems ever on the increase Science has lost a leader, his friends and the world a charming personality

During the time that I was Secretary, and so concerned with the passing of mathematical papers through the Press, I was much struck with the carelessness of authors in the arrangement of their manuscript It is frequently forgotten that a line of print in the 'Transactions' and in the new form of the 'Proceedings' will hold much more than a line of ordinary manuscript, unless, indeed, the handwriting is exceptionally small. Unless the authors' indications were supplemented, it frequently occurred that several lines of print were occupied by what might equally well, and in my judgment much better, be contained in one line. Even practised writers would do well, when they regard their manuscript as complete so far as regards matter and phrasing, to go over it again entirely from the point of view of the printing. In this way much expense and space would be spared, and the appearance of the printed page improved. Professor Larmor has

drawn up a paper which has received the sanction of the Council and is appended to this Address, and will, it is hoped, be of service at once to authors and to the Society

Apart from questions of printing, the choice of symbols for representing mathematical and physical quantities is of some importance, and is embarrassed by varying usages, especially in different countries A Committee now sitting is concerned with the selection of symbols for electrical and magnetic quantities, but the question is really much wider. One hesitates to suggest another international conference, and perhaps something could be done by discussion in scientific newspapers. Obviously some give and take would be necessary When the arguments from convenience are about balanced, appeal might be made to the authority of distinguished men, especially of those who were pioneers in the definition and use of the quantity to be represented As an example of the difficulties to be faced, I may instance the important case of a symbol for refractive index In English writings the symbol is usually μ, and on the Continent n By the early optical writers it would seem that no particular symbol was appropriated In 1815* Brewster has m The earliest use of μ that I have come across is by Sir John Herschel,† and the same symbol was used by Coddington (1829) and by Hamilton (1830), both distinguished workers in optics On the other hand, n was employed by Fraunhofer (1815), and his authority must be reckoned very high As regards convenience, I should suppose that the balance of advantage would incline to μ, since n is wanted so frequently in other senses Another case in which there may be difficulties in obtaining a much to be desired uniformity is the symbol for electrical resistance

On a former occasion I indulged in comment upon the tendency of some recent mathematics, which were doubtless understood as the mild grumbling of an elderly man who does not like to see himself left too far behind In the same spirit I am inclined to complain of what seem unnecessary changes in mathematical nomenclature In my youth, by a natural extension of a long established usage relative to equations, we spoke of the *roots* of a function, meaning thereby those values of the argument which cause the *function* to vanish In many modern writings I read of the *zeroes* of a function in the same sense There may be reasons for this change; but the new expression seems to need precaution in its use, otherwise we are led to such flowers of speech as "zeroes with real part positive," which I recently came across.‡

* 'Phil Trans.,' 1815

† 'Phil. Trans,' 1821, p 230

‡ 'Proc Math. Soc.,' vol 31, p. 266.

But though I may use a little my privilege of grumbling over details, I hope I shall not be misunderstood as undervaluing the progress made in recent years, which, indeed, seems to me to be very remarkable and satisfactory, regarded from the scientific point of view On the other hand I cannot help feeling misgivings as to the suitability of the highly specialised mathematics of the present day for a general intellectual training, and I hope that a careful watch may be maintained to check, in good time, any evil tendencies that may become apparent

Among the notable advances of the present year is the liquefaction of helium by Professor Onnes of Leiden It is but a few years since Sir J Dewar opened up a new field of temperature by his liquefaction of hydrogen, and now a further extension is made which, if reckoned merely in difference of temperature, may appear inconsiderable, but seen from the proper thermodynamical standpoint is recognised to be far-reaching The exploration of this new field can hardly fail to afford valuable guidance for our ideas concerning the general properties and constitution of matter. Professor Onnes' success is the reward of labours well directed and protracted over many years

The discovery and application by Rutherford and Geiger of an electrical method of counting the number of α-particles from radio-active substances constitutes an important step, and one that appears to afford better determinations than hitherto of various fundamental quantities It would be of interest to learn what interpretation is put upon these results by those who still desire to regard matter as homogeneous

Another very interesting observation published during the year is that of Hale upon the Zeeman effect in sun-spots, tending to show that the spots are fields of intense magnetic force Anything which promises a clue as to the nature of these mysterious peculiarities of the solar surface is especially welcome Until we understand better than we do these solar processes, on which our very existence depends, we may do well to cultivate a humbler frame of mind than that indulged in by some of our colleagues

A theoretical question of importance is raised by the observations of Nordmann and Tikhoff showing a small chromatic displacement of the phase of minimum brightness in the case of certain variable stars. The absence of such an effect has been hitherto the principal argument on the experimental side for assuming a velocity of propagation in vacuum independent of frequency or wave-length. The tendency of the observations would be to suggest a dispersion in the same direction as in ordinary matter, but of almost infinitesimal amount, in view of the immense distances over which the propagation takes place. Lebedew has pointed out that

this conclusion may be evaded by assuming an asymmetry involving colour in the process by which the variability is brought about, and he remarks that although the dispersions indicated by Nordmann and Tikhoff are in the same direction, the amounts calculated from the best available values of the parallaxes differ in the ratio of 30 to 1 In view of this discrepancy and of the extreme minuteness of the dispersion that would be indicated, the probabilities seem at the moment to lie on the side of Lebedew's explanation, doubtless further facts will be available in the near future

I cannot abstain from including in the achievements of the year the remarkable successes in mechanical flight attained by the brothers Wright, although the interest is rather social and practical than purely scientific For many years, in fact ever since I became acquainted with the work of Penaud and Wenham, I have leaned to the opinion that flight was possible as a *feat* This question is now settled, and the tendency may perhaps be to jump too quickly to the conclusion that what can be done as a feat will soon be possible for the purposes of daily life But there is a very large gap to be bridged over, and the argument urged by Professor Newcomb and based on the principle of dynamical similarity, that the difficulties must increase with the scale of the machines, goes far to preclude the idea that regular ocean service will be conducted by flying machines rather than by ships But, as the history of science and invention abundantly proves, it is rash to set limits For special purposes, such as exploration, we may expect to see flying machines in use before many years have passed

The Report of the National Physical Laboratory for the year again indicates remarkable growth The various new buildings, which have been erected and equipped during recent years at a cost of about £33,000, are now occupied, and the result is that both researches and test work can be carried out with much greater ease and efficiency than previously The Executive Committee in charge of the Laboratory is indebted in the first instance to H M Government, and then to the numerous friends whose assistance has made this possible. At the same time, the needs for buildings are not nearly satisfied There has been during the year a very marked and important growth in the demand by manufacturers and others for assistance in metallurgical enquiries, which require investigations, frequently of a very complex character; and with the present accommodation for much of the Metallurgical Department this demand is difficult to satisfy. Thanks in great measure to the Goldsmiths' Company, the chemical side of this department is well provided for, but new buildings for the other branches of metallurgy are an urgent want.

The Report of the Treasury Committee of Inquiry referred to in the address of last year was communicated by the Treasury to the Royal Society, with the intimation that Their Lordships accept the recommendations of the Committee, and trust that the Royal Society may see their way to do the same. In their reply the President and Council, with the concurrence and advice of the Executive Committee of the Laboratory, expressed their readiness to use their best endeavours to carry the Report into effect. The Report has since been presented to Parliament.

The buildings of the Magnetic Observatory at Eskdalemuir are now occupied, but, unfortunately, difficulty has arisen in making the magnetograph rooms which are underground completely watertight, and the recording apparatus is not yet properly installed.

The third and fourth volumes of 'Collected Researches' of the Laboratory have been published during the year, and testify to the vigorous scientific activities of the staff The third volume is occupied chiefly with the account of the prolonged series of experiments on electric units carried out at the Laboratory by Prof Ayrton, Mr Mather, Dr Lowry, and Mr. Smith These researches proved of great value in the discussions at the International Conference on Electric Units, for which recently the Society provided accommodation and entertainment at the request of the Government.

The progress of the 'Royal Society Catalogue of Scientific Papers' has advanced a definite stage during the year, through the publication by the Cambridge University Press of the Index Volume of Pure Mathematics for the Nineteenth Century Owing to the magnitude of the material to be indexed in the several sciences, it has been necessary to adopt drastic measures of compression, and the 40,000 entries involved in the present section have thus been condensed into one royal octavo volume of some 700 pages An essential element in this saving of bulk has been the grouping of titles within each heading so as to avoid reprinting the leading words. It was, perhaps, inevitable that this device would occasionally be mistaken for an attempt at organic classification within the limits of the main headings, which are substantially those of the yearly 'International Catalogue of Scientific Literature' This had, indeed, been foreseen in the preface of the volume. As regards new actual sub-headings which have been introduced occasionally, the Committee remark that "These minor classifications, being often made mechanically on the basis of the explicit mention of the sub-heading, are not to be taken as exhaustive, cognate entries may be found elsewhere under the same main heading. The unit of classification is thus the complete numbered heading."

The Committee of the Catalogue have indeed been fully conscious

throughout of the difficulties of the task which they supervise, and it must be gratifying to the Director of the Catalogue and his staff to have the support of high authorities, not confined to this country, in their decision that in so extensive an undertaking practical feasibility must be the aim rather than an elusive theoretical perfection One advantage, at any rate, will accrue from bringing out a single volume well in advance, in that the Committee will be able to profit in the future work from the experience they have acquired

Through the kindness of Dr Schuster I had the opportunity of submitting to the Council, before the expiry of my term of office, a generous proposal which he makes for instituting a fund of £1500, the interest of which is to be applied to pay the travelling expenses of delegates of the Society to the International Association of Academies Dr Schuster felt that the absence of such a provision laid a burden upon delegates, and might operate to limit the choice of the Society. I was empowered by the Council to convey their cordial thanks to Dr Schuster, and I have now the pleasure of making his benefaction known to the Society at large

In taking leave of the honourable office which I have occupied for three years, I desire to thank the Society and especially my colleagues, the officers, for the consideration which they have uniformly shown me All the omens indicate that the Society will be represented by one well versed in its affairs, and whose scientific distinction and wide experience justify the highest hopes for his tenure of the chair

MEDALLISTS, 1908

Copley Medal

The Copley Medal is awarded to Dr Alfred Russel Wallace, F.R.S.

It is now sixty years since this distinguished naturalist began his scientific career During this long period he has been unceasingly active in the prosecution of natural history studies As far back as 1848 he accompanied the late Henry Walter Bates to the region of the Amazon, and remained four years there, greatly enriching zoology and botany, and laying at the same time the basis of that wide range of biological acquirement by which all his writings have been characterised From South America he passed to the Malay Archipelago and spent there some eight fruitful years. It was during his stay in that region that he matured those broad views regarding the geographical distribution of plants and animals which on his return to this country he was able to elaborate in his well-known classic volumes on

that subject. It was there, too, amid the problems presented by the infinite variety of tropical life, that he independently conceived the idea of the theory of the origin of species by natural selection which Charles Darwin had already been working out for years before His claims to the admiration of all men of science were recognised by the Royal Society forty years ago, when, in 1868, a Royal Medal was awarded to him Again, when in 1890, the Darwin Medal was founded, he was chosen as its first recipient He is still full of mental activity and continues to enrich our literature with contributions from his wide store of experience and reflection in the domain of Natural History As a crowning mark of the high estimation in which the Royal Society holds his services to science, the Copley Medal is now fittingly bestowed on him

RUMFORD MEDAL.

The Rumford Medal is awarded to Prof H A Lorentz, For Mem. R S

Prof Hendrik Antoon Lorentz, of Leiden, has been distinguished during the last quarter of a century by his fundamental investigations in the principles of the theory of radiation, especially in its electric aspect. His earliest memoirs were concerned with the molecular equivalents which obtain in the refractive (and dispersive) powers of different substances, in them he arrived at formulæ that still remain the accepted mode of theoretical formulation of these phenomena. The main result, that $(\mu^2-1)/(\mu^2+2)$ is proportional jointly to the density of distribution of the molecules, and to a function of the molecular free periods and the period of the radiation in question, rests essentially only on the idea of propagation in some type of elastic medium, and thus it was reached simultaneously, along different special lines, by H A Lorentz originally from Helmholtz's form of Maxwell's electric theory, and by L Lorenz, of Copenhagen, from a general idea of propagation after the manner of elastic solids.

The other advance in physical science with which Prof. Lorentz's name is most closely associated is one of greater precision, the molecular development of Maxwell's theory of electro-dynamics This subject was never entered upon by Maxwell himself, on the ground, probably, that the general relations of the æther, and in particular their dynamical bearings, offered a definite field which must be fully probed and explored before the uncertainties connected with molecular complexity became ripe for effective detailed treatment. But the theoretical difficulties connected with the simple law of the astronomical aberration of light, and particularly with the entire absence of any effect of the Earth's uniform motion in space on

terrestrial phenomena involving radiation, had more recently rendered this problem urgent Following on various purely optical papers on the phenomena of moving bodies, Prof Lorentz, in 1892, elaborated a general molecular treatment in the memoir "La Théorie Electro-magnétique de Maxwell, et son Application aux Corps Mouvants," which appeared in the 'Archives Néerlandaises,' and contains substantially the main root ideas of the subject. In 1905 it was re-expounded with further development in a tract entitled "Versuch einer Theorie der Electrischen und Optischen Erscheinungen in Bewegten Korpern," the main feature being the elimination of the dynamical element in the previous discussion in favour of a formulation by a system of abstract equations, after the way first set out by Maxwell himself as a summary of his final definite results as distinct from the formative ideas underlying them, and afterwards brought into prominence by the expositions of Heaviside and Hertz

By these writings Prof Lorentz has taken a predominant place in the modern evolution of electric and optical theory He has since been active in special applications, of which the best known has been his theoretical prediction of the physical features of the alteration of the lines of the spectrum in a magnetic field, which had been discovered and has since been developed by his colleague Zeeman

Royal Medals

The assent of His Majesty the King, our Patron, has been graciously signified to the following awards of the Medals presented annually by him to the Society

A Royal Medal to Prof John Milne, F R S., for his work on Seismology. In 1875, Dr. Milne accepted the position of Professor at Tokyo, which was offered to him by the Imperial Government of Japan His attention was almost immediately attracted to the study of earthquakes, and he was led to design new forms of construction for buildings and engineering structures with a view to resisting the destructive effects of shocks. His suggestions have been largely adopted, and his designs have been very successful for the end in view. Incidentally he studied the vibrations of locomotives, and showed how to obtain a more exact balancing of the moving parts, and thus to secure smoother running and a saving of fuel. Here again his suggestions were accepted, and his work was recognised by the Institution of Civil Engineers

He next devoted himself to the study of artificial shocks produced by the explosion of dynamite in borings. He then studied actual shocks as observed at nine stations connected by telegraph wires. A seismic study of Tokyo, and subsequently of the whole of northern Japan, followed. In this

latter work he relied on reports from 50 stations. The Government then took up the matter, increased his 50 stations to nearly 1000, and founded a Chair of Seismology for Mr Milne It is due to his energy, skill, and knowledge that the Japanese School of Seismology stands as the first in the world.

While still in Japan he attempted to obtain international co-operation through the representatives of 13 nationalities This first effort failed, but subsequently, on his return to England in 1895, he succeeded, and reports are now received by him from some 200 stations furnished with trustworthy instruments, and scattered all over the world On his return to England he at once established his own observatory at Shide, in the Isle of Wight, and the work has been carried on continuously from that time up to now, mainly by his own industry and resources

In Great Britain we owe everything in seismology to the British Association Their Committee was founded in 1880, and since that date Milne has been the moving spirit in the long career of its activity He has been the author of 29 annual reports, and these form in effect a history of the advance of seismology since it has been recognised as a definite branch of science

The knowledge which we have now acquired as to the internal constitution of the earth is more due to Milne than to any other man

The work of Dr Henry Head, F R S, on which is founded the award of the other Royal Medal, forms a connected series of researches on the Nervous System (made partly in conjunction with Campbell, Rivers, Sherren, and Thompson), published for the most part in 'Brain' at various times since 1893 up to the present date, and constituting one of the most original and important contributions to neurological science of recent times

His first paper (1893), founded on minute and laborious clinical investigation, established in a more precise manner than had hitherto been done the relations between the somatic and visceral systems of nerves. He confirmed from the clinical side the experimental researches of Sherrington on the distribution of the posterior roots of the spinal nerves

An inquiry into the pathology of Herpes Zoster (1900), which he proved abundantly to be due to inflammation of the posterior root ganglia, indicated that the areas of referred pain in visceral disease corresponded specially with the distribution of the fibres of the posterior roots subserving painful cutaneous sensibility

Continuing his investigations on the peripheral nerves, partly by experiments on himself, in conjunction with Rivers, and partly by examination of cases of accidental injuries to nerves, Head was led to formulate (1905) an

entirely novel conception and differentiation of the functions of the peripheral nerves, and of the paths for the respective forms of sensibility which they convey—epicritic, protopathic, and deep sensibility. This is generally regarded by neurologists as a research of quite exceptional originality and ability

Following the course of afferent impulses, Head next showed (1906) that the sensory paths of the peripheral nerves at their first synaptic junction with the spinal cord become re-arranged, and ascend in different relations in certain definite tracts

Davy Medal

The Davy Medal is awarded to Prof William Augustus Tilden, F R S

The researches of Prof Tilden extend into many domains His work on the specific heats of the elements in relation to their atomic weights described to the Society in the Bakerian Lecture for 1900 and in two later papers published also in the 'Philosophical Transactions,' was of high theoretical importance The employment of liquid oxygen as an ordinary laboratory reagent, rendered possible by the researches of Dewar and others, enabled Prof. Tilden to test the validity of Dulong and Petit's Law and of Neumann's Law over a much wider range of temperature than was possible before, and gave a truer estimate of the nature of their validity

In the region of organic chemistry, he has carried out important researches on the terpenes, such as that on the hydrocarbons from *Pinus sylvestris*, on terpin and terpinol, and on limettin

In inorganic chemistry, his investigation on *aqua regia* and on nitrosyl chloride are especially noteworthy. He has assisted much in clearing up many points with regard to *aqua regia* about which obscurity remained His introduction of nitrogen peroxide and especially of nitrosyl chloride as reagents has proved, in his own hands and in those of other workers, to be of very high value

Darwin Medal.

The Darwin Medal is awarded to Prof August Weismann for his contributions to the study of evolution. He was one of the early supporters of the doctrine of evolution by means of natural selection, and wrote in support of the Darwinian theory in 1868 His great series of publications from that date onward must always remain a monument of patient inquiry. In forming an estimate of his work it does not seem essential that we should

decide on the admissibility of his germ-plasm theory. It is in like manner unimportant that he was, in certain respects, forestalled by Galton, and that his own views have undergone changes. The fact remains that he has done more than any other man to focus scientific attention on the mechanism of inheritance. By denying the possibility of somatic inheritance, he has compelled the world to look at this question with a closeness of criticism that is wanting in all earlier inquiries In the opinion of what is perhaps the majority of naturalists, he has achieved much more than this—he has convinced them that the solution of the problem of evolution must be sought along the lines of his doctrine of germinal continuity Thus the preformist's point of view, for which he has done so much forms the basis on which Mendelians and Mutationists are at work

Weismann's work was highly estimated by Mr Darwin Thus he writes, in 1875 ('More Letters,' i, 356), of Weismann's paper on Seasonal Dimorphism "No one has done so much as you on this important subject, *i.e.* on the causes of variation" Again ('Life and Letters,' iii, 198)· "I have been profoundly interested by your essay on 'Amblystoma,' and think you have removed a great stumbling block in the way of evolution" And, once more, in January, 1877 ('Life and Letters,' iii, 231), Darwin wrote of Weismann's 'Studien zur Descendenzlehre' "They have excited my interest and admiration in the highest degree, and whichever I think of last seems to me the most valuable"

Hughes Medal.

The Hughes Medal is awarded to Prof Eugen Goldstein.

Prof Goldstein was one of the early workers on the modern detailed investigation of the electric discharge in rarefied gases, and by long continued researches has contributed substantially to the systematic analysis of the complex actions presenting themselves in that field Of these researches may be mentioned his observations of the effect of magnetic force on striations, of the phosphorescence produced by the cathode rays, and of the reflection of cathode rays

By his discovery of the so-called Kanal-Strahlen, or positive rays, he has detected an essential feature of the phenomenon, which, in his own hands and in those of other workers, has already thrown much needed light on the atomic transformations that are involved.

(APPENDIX.)

Practical Suggestions on Mathematical Notation and Printing.

It is a subject of common complaint that mathematical manuscripts are often prepared for press without due regard for the difficulties encountered in setting up the type, or for the appearance of the printed page

The Council of the Royal Society have had under consideration for some time the desirability of taking steps with a view to diminish the expense of printing and proof-corrections, and to avoid waste of space, and undue variety of notation in papers by different authors in the same volume They have approved of the reprinting, with modifications and additions, of the substance of a Report to the British Association on this subject, in the hope that greater uniformity and facility in mathematical typography may thereby be promoted The recommendations which follow are now offered, not in any authoritative way, but simply as a consensus of opinion, to this end it is understood that they were submitted in advance, for consideration and criticism, to the Council of the London Mathematical Society

Abstract of Report of British Association Committee *

With a view to the questions referred to them for consideration, the Committee appointed by the British Association made inquiries into the nature and processes of mathematical printing, and the difficulties attendant thereon, and it appeared to them that a statement of the results of these inquiries would form the best introduction to the suggestions which they had to make

The process of "composition" of ordinary matter consists in arranging types uniform in height and depth (or "body" as it is termed) in simple straight lines The complications peculiar to mathematical matter are mainly of two kinds

First, figures or letters of a smaller size than those to which they are appended have to be set as indices or suffixes, and consequently, except when the expressions are of such frequent occurrence as to make it worth while to have them cast upon type of the various bodies with which they are used, it becomes necessary to fit these smaller types in their proper positions by special methods This process, which is called "justification," consists in filling up the difference between the bodies of the larger and smaller types with suitable pieces of metal

* Report of the Committee, consisting of W Spottiswoode, F.R.S, Prof Stokes, F R.S, Prof Cayley, F R S, Prof. Clifford, F R.S, and J W L. Glaisher, F R S, appointed to report on Mathematical Notation and Printing, with the view of leading mathematicians to prefer in optional cases such forms as are more easily put into type, and of promoting uniformity of notation —'B A Report,' 1875, pp 337–339

The second difficulty arises from the use of lines or "rules" which occur between the numerator and denominator of fractions, and (in one mode of writing) over expressions contained under radical signs. In whatever part of a line such a rule is used, it is necessary to fill up, or compensate, the thickness of it throughout the entire line.

The complications above described may arise in combination or may be repeated more than once in a single expression, and in proportion as the pieces to be "justified" become smaller and more numerous, so do the difficulties of the workman, the time occupied on the work, and the chances of subsequent dislocation of parts augment.

The cost of "composing" mathematical matter may now (1908) in general be estimated at somewhat more than twice that of ordinary or plain matter, the recent adoption of the point system in the casting of types having greatly simplified mathematical justification.

There are many expressions occurring in mathematics which are capable of being written in more than one way, and of these some present much greater difficulties to the printer than others. This being so, the Committee were of opinion that instead of making any specific recommendations, the most useful course they could take would be to append a table of equivalent forms specifying those which do and those which do not involve justification, and also a list of mathematical signs which may fairly be expected to be found, in the usual sizes, ready to hand among a printer's materials.

In recommending in this qualified way some forms of notation in preference to others, the Committee wished it to be distinctly understood that they were speaking from the printing, and not from the scientific point of view, and they were quite aware that, even if some of the easier forms should be adopted in some cases, they may still not be of universal application, and that there may be passages, memoirs, or even whole treatises in which they would be inadmissible.

The Committee drew attention to the advantages which may incidentally accrue to mathematical science by even a partial adoption of the modifications suggested. Anything which tends towards uniformity in notation may be said to tend towards a common language in mathematics, and whatever contributes to cheapening the production of mathematical books must ultimately assist in disseminating a knowledge of the science of which they treat.

MATHEMATICAL SIGNS NOT INVOLVING "JUSTIFICATION"

$$\times \quad - \quad + \quad = \quad \surd \quad \pm \qquad \cdot \quad \cdot \qquad \} \quad < \quad > \quad -$$

$$(\quad [\quad \} \quad \int \quad \surd$$

$$a \quad a' \quad a_1 \quad a^2 \quad a_2 \quad a^1 \quad a_1$$

EQUIVALENT FORMS

Involving justification	Not involving justification
$\frac{x}{a}$	x/a or $x-a$ or $x:a$
$\sqrt{x}$	$\sqrt{x}$ or $x^{\frac{1}{2}}$
$\sqrt[n]{x}$	$\sqrt[n]{x}$ or $x^{\frac{1}{n}}$
$\sqrt{x-y}$	$\sqrt{(x-y)}$ or $(x-y)^{\frac{1}{2}}$
$\sqrt{-1}$	i or i
$x\sqrt{x+a}$	$x(x+a)$
$e^{\frac{\pi x}{a}}$	$e^{\pi x/a}$

This British Association List, which has been abbreviated and modified, is now incorporated in the following —

RECOMMENDATIONS REGARDING MATHEMATICAL NOTATION AND PRINTING.

Always—

instead of	$\frac{x}{3}$	$\frac{a+b}{2}$	$\frac{a+\frac{b}{2}}{\frac{c}{3}+\frac{d}{4}}$	$\frac{a}{b+\frac{i}{d}}$	$\sqrt{x}$	$\sqrt{-1}$	$\frac{1}{x}$	$\frac{1}{e^{\pi}}$
write	$\frac{1}{3}x$	$\frac{1}{2}(a+b)$	$\frac{a+\frac{1}{2}b}{\frac{1}{3}c+\frac{1}{4}d}$	$\frac{a}{b+c/d}$	$\sqrt{x}$ or $x^{\frac{1}{2}}$	i or i	x^{-1}	$e^{-\pi}$
instead of	$x\sqrt{x+a}$		$\sqrt{x-y}$		$e^{\frac{\pi x}{a}}$		$\int_0^{\frac{\pi}{2}}$	$\lfloor n$
write	$x(x+a)$		$\sqrt{(x-y)}$ or $(x-y)^{\frac{1}{2}}$		$e^{\pi x/a}$		$\int_0^{\frac{1}{2}\pi}$	$n!$

In current ordinary text—

instead of	$\frac{x}{a}$	$\frac{a+b}{c+d}$	$\frac{x}{y+\frac{i}{2}}$	$a/y+\frac{a}{b+c}$
write	x/a	$(a+b)/(c+d)$	$x/(y+\frac{1}{2}i)$	$\frac{x}{y}+\frac{a}{b+c}$

Excessive use of the slanting line, or solidus, is, however, undesirable, it may often be avoided by placing several short fractions or formulas, with the intervening words if any, on the same line, instead of setting out each one on a line by itself The last of the examples given above illustrates an improper use, in which symmetry is spoiled while nothing is gained, either both fractions should be written with the solidus, as $x/y+a/(b+c)$, or else neither as above.

The solidus should be of the same thickness as the horizontal line which it replaces, in some founts of type it is too thick and prominent.

Irregularities in the spacing of letters and symbols in the formulas as printed are often the cause of a general unsatisfactory appearance of the page.

For centimetres, millimetres, kilometres, grammes, kilogrammes, the abbreviations should be cm, mm, km, gm, kgm (not cms, etc), and so in similar cases. Present custom is against the use of the signs and '.'

Symbols which are not provided in the usual founts of type are, as a rule, to be avoided Compounded symbols such as a or $\bar{a}$ usually involve justification, and are thus liable to become deranged or broken. The two examples here given have, however, become so essential that separate founts should be provided for them

The use of a smaller fount for numerical fractions is now customary, thus always $\frac{1}{3}a$ instead of $a/3$ The use of negative exponents often avoids a complex fractional form, as also the use of the fractional exponents, such as $\frac{1}{2}$ and $\frac{1}{3}$ In the latter case $x^{\frac{1}{2}}$ is usually preferred to $x^{1/2}$, notwithstanding that the latter is more legible

Much is often gained in compactness and clearness by setting out two or more short formulæ on one line, instead of on consecutive lines; in that case they should be separated by spaces, indicated by the sign $\#$ on the MS This would apply with even greater force to expressions such as $x = a, = b, = c$

In the Preface to his 'Mathematical and Physical Papers,' vol 1, 1880, the late Sir George Stokes successfully introduced the limited use of the solidus notation, obtaining the assent and support of Lord Kelvin, Prof. Clerk Maxwell, Lord Rayleigh, the Editors of the 'Annalen der Physik,' and many other mathematicians He defined its use as restricted to the symbols immediately on the two sides of it, unless a brace or stop intervenes, thus $\sin n\pi x/a$ is to mean $\sin(n\pi x/a)$, but $\sin n\theta./r^n$, in case it is used, would mean $(\sin n\theta)/r^n$.

The Charges on Ions produced by Radium.

By C. E HASELFOOT, M.A., Hertford College, Oxford.

(Communicated by Prof J S Townsend, F R S Received November 3, - Read November 12, 1908)

In a paper* on "The Charges on Positive and Negative Ions in Gases," Prof Townsend has described a method for the direct determination of the quantity Ne, where N is the number of molecules in a cubic centimetre of a gas at standard pressure and temperature and e the charge on an ion His experiments were carried out on ions produced by the action of secondary Röntgen rays, and he showed that for negative ions the method led with great accuracy to the value $1{\cdot}23 \times 10^{10}$ for Ne, the same as that for NE, where E is the charge on a monovalent ion in a liquid electrolyte. For positive ions the value obtained in the first set of experiments was $2{\cdot}4 \times 10^{10}$, but subsequently with less penetrating secondary rays it was found to be as low as $1{\cdot}26 \times 10^{10}$ It would therefore appear that the positive ions have in some cases a single and in others a double atomic charge, whereas the charge on the negative ions is always the same

With a view of testing the theory for ions produced by radium, experiments have been made with an apparatus precisely similar to that used by Prof Townsend, and the results obtained confirm the reliability of the method After making due allowance for experimental and other known sources of error the positive ion appears to behave at all pressures and under all forces in accordance with the theory, but in the case of the negative ion some considerable deviations were observed, if the gas is very dry, but these disappear as soon as some water vapour is added

The method consists essentially in the production of ions in a field A, from which those of one sign are passed into a field B through a hole of definite size. Here they diffuse and the charge received by a disc placed immediately below and of the same size as the hole is compared with the total charge received by the disc and a ring which surrounds it. From the observed value of this ratio the quantity Ne is deduced.

Some modifications of the apparatus used for Rontgen rays were necessary, in order to deal with the ions produced by radium In these experiments the ions are generated in the space A by radium placed in shallow horizontal grooves f, covered with thin aluminium foil, in brass blocks F, standing on the ring E. The space B is bounded by a series of brass rings G, kept at

* 'Roy Soc. Proc.,' A, vol 81, 1908, p 464.

definite potentials in order to maintain the constancy of the field. C is a plate whose potential determines the field A The ions generated in A pass through the grating g and then through the hole h They diffuse, and the ratio of the charges received by the disc D and the ring R is measured

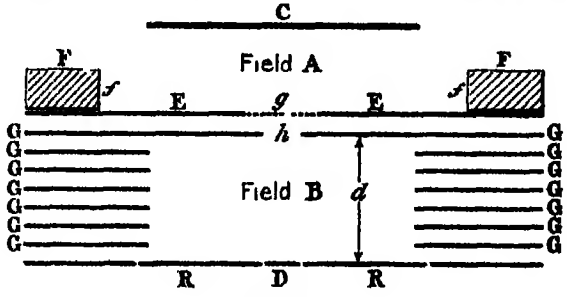


Fig 1

In conducting the experiments it is necessary to allow for the ions which are generated in the field B and also for the self-repulsion of the stream coming through the hole The charge acquired by the disc and ring due to ions produced in the field B is easily found by observing these charges when the field A is reversed Subtracting them from the charges received when the field A is the same as the field B, we obtain the charges due to ions coming through the aperture The special difficulty, however, is to avoid the large effects due to the emanation and to the induced radio-activity in the field B These become larger the longer the apparatus is left undisturbed, and in one experiment, made about two months after the apparatus was set up were found to be more than twice the effect due to the field A They were due to the emanation escaping through and round the aluminium foil, which was kept thin in order to obtain measurable effects, and the movements of this gas would probably account for the considerable irregularities observed during preliminary experiments Now it has been shown by Mme Curie that the activity induced on surfaces is greatest for the lower portions of the vessel containing the emanation, so that it was considered advisable to invert the apparatus (in the original form of which, shown in the figure, A was the upper field), and thus cause the more radio-active surfaces to be in the lower space A A means was also provided of drawing a current of air through the apparatus from B to A, and it was then found that the number of ions produced in the space B could be made small, thereby diminishing this correction. The other correction is for self-repulsion It can be shown that this effect varies inversely as the square of the electrical force Z in the field B, and approximately directly as the square of the pressure. It can thus be reduced at will by diminishing the pressure and increasing the force, but a

diminution of pressure, beyond a certain point, decreases too seriously the charge to be measured, whereas an increase of force concentrates such a large proportion of ions on the disc that the effect of the clearance between the disc and the ring may become comparatively large, and, further, any percentage error in the determination of the ratio leads to a far larger percentage error in the value of Ne. This will be clear from an inspection of the graph given in fig. 2.

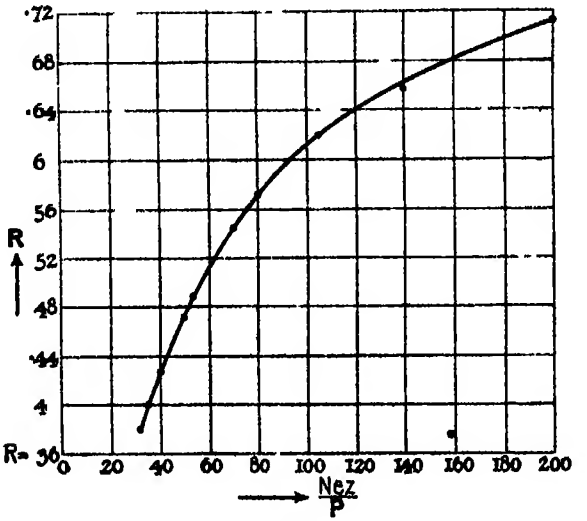


Fig. 2.

The method of deducing Ne from the observed value of R has been explained in the paper already referred to. The connection between these quantities is best shown by means of a curve, and the graph in fig. 2 gives R in terms of $(Ne\ Z)/P$, where Z is the electric force, and P atmospheric pressure. The curve corresponds to the case in which the aperture is 7 cm. from the disc, and the diameter of the aperture 1·5 cm. When $N\ e$ is $1{\cdot}23 \times 10^{10}$ and Z 1 volt per centimetre $(Ne.\ Z)/P$ is approximately 40.

Observations were made at pressures of 4·5, 9, and 14·5 mm. with forces of 1, 2, and 4 volts per centimetre both for positive and negative ions. Rejecting those subject to various sources of error, the following results remain:—For positive ions, three at 2 volts per centimetre giving $Ne = 1{\cdot}26 \times 10^{10}$, and five at 4 volts per centimetre giving $Ne = 1{\cdot}37 \times 10^{10}$, for negative ions, three at 2 volts per centimetre giving $Ne = 1{\cdot}24 \times 10^{10}$.

The observations at 1 volt per centimetre are almost all subject to too large a correction for self-repulsion, rising in some cases to 20 per cent., to make them reliable. Those for 4 volts per centimetre are subject to greater experi-

mental errors than those at 2 volts per centimetre, and this may possibly account for the large value of Ne deduced in the case of positive ions from observations at this force The most accurate results can be obtained at pressures of from 4 to 6 mm and forces of 1 5 to 2 volts per centimetre Further experiments are to be made under these conditions with air and other gases, and it is hoped by reducing the ionisation in the field B to obtain more accurate results.

As in the case of ions produced by Rontgen rays, it was found that the negative ions produced by radium did not obey the simple laws of diffusion when the air is very dry This appears from the fact that the ratio R does not vary with the force Z as given by the theory, and that it depends on the pressure As the drying proceeds, the departure of R from the value given by the curve (assuming $Ne = 1\,23 \times 10^{10}$) is most marked with large values of Z and small pressures Thus the mean of six observations at the force of 4 volts per centimetre, all giving very low values, is 0 584 (instead of 0 682) With a lower force this departure from theory occurs less frequently, and then as a rule at low pressures With a force of 1 volt per centimetre, and pressures varying from 8 to 2 5 mm, the numbers found on one day varied from 0 390 to 0 329, the theory gives 0 429 For a force of 2 volts per centimetre and a pressure of 2 5 mm the number obtained was 0 311, that given by the theory being 0 572 These experiments were then repeated after admitting moisture, and the values of the ratio obtained agreed with the theory within the limits of experimental error

A point of some interest is the fact that it was found easier to experiment with negative than with positive ions This is probably due to the action of the emanation By placing the field B, in which the diffusion takes place, at the top of the apparatus, the radio-activity caused by the emanation is confined as far as possible to the lower field A, and it is then found in the case of negative ions that the charge due to the ionisation occurring in field B seldom exceeds one-third of that coming from field A With positive ions, however, it is frequently as large as one-half In a strong electric field the excited radio-activity is confined entirely to the negative electrode, and Rutherford has found that this is the case down to a pressure of 10 mm When the disc D and ring R are receiving negative ions the force would tend to keep the emanation in the lower field, but for positive ions, when the disc and the ring round it form the negative electrode, the electric force tends to bring the emanation into the field B The correction for the ionisation in the field B is therefore large and subject to variations in the experiments with positive ions, and also from the same cause difficulties arise in making an accurate estimate of the effect of self-repulsion

So far the results are in satisfactory agreement with those obtained by Prof. Townsend, with Rontgen rays, and it is hoped to make further experiments and to see if it is possible to obtain positive ions with double the atomic charge by means of radium rays. Possibly the molecules of a gas are ionised in different ways by the α, β, and γ types of radiation, and it is not quite certain which kind of radiation had the predominating effect in the production of the ions in these experiments, though probably the greater numbers were due to the α-rays

I am greatly indebted to Prof Townsend for most valuable advice and assistance throughout.

A Comparison of the Radium Emanation Spectra obtained by different Observers

By T Royds, M.Sc., 1851 Exhibition Scholar

(Communicated by Prof E Rutherford, F R S Received November 26,—Read December 10, 1908)

In 1904 Sir William Ramsay and Prof Collie* gave a list of lines produced by the discharge in a vacuum tube containing radium emanation, but the uncertainty of these numbers made a redetermination desirable A later determination by Mr Cameron and Prof Ramsay† was communicated to the Royal Society on June 25, 1908, and was published on August 27, together with corrections, and a final compilation of verified emanation lines added on August 5 After Prof Rutherford had completed the measurements of the volume of the radium emanation, he and the writer were able to photograph the spectrum that had been observed in the course of this work, and we published in 'Nature,' July 9, 1908, the wave-lengths of the stronger lines observed by us in the emanation spectrum, and a more complete list, containing 73 lines, with an accuracy of 0 5 Å U, was given in the 'Philosophical Magazine' of August, 1908. Measurements which I have recently made to within 0 1 Å.U by means of a concave grating confirm the accuracy of our previous determinations

The complete purification of the radium emanation demands a lengthy and painstaking procedure, and is a matter of considerable difficulty, for the volume of pure emanation available in our experiments would occupy at

* 'Roy Soc. Proc.,' vol. 73, p. 470, 1904
† 'Roy. Soc. Proc.,' vol 81, p. 210, 1908

atmospheric pressure not more than one-tenth of a cubic millimetre The vacuum tube employed must therefore be of small dimensions, and all traces of foreign gases have to be removed from the walls and the electrodes of the tube In the experiments of Rutherford and Royds, using the method of purification recently developed by Prof Rutherford,* a complete day's work was taken up before the vacuum tube was filled with the pure emanation

We have observed the spectrum of the radium emanation at least eight times, and have obtained almost exactly the same spectrum on each occasion We have drawn attention to variations in the relative intensities due to the presence of foreign gas In the different experiments of Cameron and Ramsay, however, the spectrum is seen to vary considerably In their first experiment, the spectrum consisted chiefly of hydrogen, together with nitrogen, and also (presumably) mercury,† and of the lines remaining, which alone are given in their list, the strongest are quite absent from our spectrum, a few of the fainter lines are probably identical with some of the strongest emanation lines. In the second experiment of Cameron and Ramsay, allowing for a possible error of 4 or 5 Å U in their measurements, the strongest lines in the spectrum are, roughly speaking, those strongest in our spectrum At the end of their final list of August 5, Cameron and Ramsay state that our figures show a very close agreement with theirs A careful examination shows, however, that there are striking differences too marked to be explained as errors in measurement or variations of intensity due to impurities, for many of the strongest lines are absent altogether from our spectrum, and also from their own previous determinations

Investigating these differences several weeks ago, I noticed that after leaving out the well-marked emanation lines, the final spectrum of Cameron and Ramsay was almost identical with the xenon spectrum obtained by the discharge from a Leyden jar with spark gap‡ An examination of the accompanying table shows that the spectrum attributed by them to the radium emanation is mainly a compound of the xenon spectra obtained with and without Leyden jar, the jar spectrum having been brought out, probably,

* 'Phil Mag.,' May, 1908.

† In none of our photographs have the mercury lines been present, though the green line 5461 has been seen faintly in visual observations. Neither were the hydrogen lines, except H_β, seen when preliminary precautions had been taken for the removal of hydrogen from the electrodes

‡ The author is reminded that Prof. Liveing had remarked last summer the similarity of this emanation spectrum to that of xenon, Sir J. Dewar reported the circumstance at he British Association meeting in September The author is convinced that there is no real coincidence in the emanation spectrum with the spectra of any of the rare atmospheric gases.

by the heaviness of the discharge employed Practically the only strong xenon line not included in their list is the line 4862 69 (intensity 8), which was doubtless hidden by H_β 4861 49

After eliminating the lines due to xenon from their last photograph containing 54 lines, there remain 11 Of these, eight are seen to be some of the strongest lines of our spectrum. Taking into account the complete list of Ramsay and Cameron's lines, including those printed in italics, which were seen only in previous photographs, the number of emanation lines amounts to 16, there are, of course, numerous coincidences within the accuracy of their measurements, of xenon lines with possible emanation lines

Table

Cameron and Ramsay's emanation spectrum	Intensity	Xenon (Baly)	Intensity without jar	Intensity with jar	Rutherford and Royds's values	Intensity
7050	2					
6150	2					
6101	2	6097 80	—	7		
6055	2	6051 36	—	7		
5990 5	2	5976 67	—	7		
5679 5	2					
5586	2				5582 2	8
5446	1	5439 19	—	8		
5419	4	5419 40	—	10		
5370	2	5372 62	—	8		
5335	3	5339 56	—	9		
5289	6	5292 40	—	10		
5083	2	5080 88	—	7	Probably 5084 45	4
4979	2	4978 49	—	4	Possibly 4979 02	4
4936	2				Possibly H (C and R.)	
4920	3	4921 68	—	6		
		4919 85	—	4		
4883	2	4883 68	—	6		
4873	2	4876 68	—	7		
4843 5	10	4844 50	—	10		
4816	2	4818 15	—	4	Possibly 4817 33	4
4806 5	1	4807 19	6	1		
4768	2	4769 21	—	4		
4731 5	2	4734 30	8	1		
4724	4				4721 70	5
4695 5	2	4698 20	—	5		
		4697 17	7	—		
4681	5	4683 76	—	5	Possibly 4680 92	10
4672	7	4671 42	10	2		
4652 5	3	4652 15	—	6		
4645 5	6				4644 29	10
4626 5	10	4624 46	15	2	Possibly 4625 58	8
4616	2	4615 72	—	5		
4610	2				4609 40	7
4605	10	4608 21	—	10	Possibly 4604 46	4
4592	1	4592 22	—	6	Doubtful (C and R.)	
4585	4	4585 65	—	10		
4578 5	3	4577 36	—	6	Possibly 4577 77	7
4545 5	2	4545 34	—	8		
4541	1	4541 03	—	8		
4532 5	2	4532 67	—	5		

Cameron and Ramsay's emanation spectrum	Intensity	Xenon (Baly)	Intensity without jar	Intensity with jar	Rutherford and Royds's values	Intensity
4524	4	4524·83	6	—	Possibly H (C and R)	
		4524 38	—	5		
4509	4				4508·68	9
4505	2				4503 89	2
					Doubtful (C and R)	
4501	3	4501 13	10	2		
4481	4	4481 01	—	7		
4463 5	8	4462 38	—	20	Possibly 4460·0	10
4449	4	4448 28	—	10		
4441 5	<1	4441·08	—	3	Possibly 4439 88	2
4436 5	<1	4434 35	—	6	Possibly 4435·25	8
4416 5	4	4415·00	—	7		
4391	3	4395·91	—	10		
		4393 84	—	10		
4349	6				4349 81	15
4331	4	4330 63	—	15		
4307	2				4308 3	10
4246	4	4245 54	—	10		
4239	3	4238 37	—	10		
4204	5				4203 39	10
4189	3	4193 25	8	8	Probably 4188 2	5
		4180 20	—	10		
4167	6				4166 6	20
4114	3	4116 25	—	7	Probably 4114·71	7
4018 5	4				4017·90	10
3982	8				3981 83	12
3973	6				3971 71	9
3958 5	3				3957 30	7
3879 5	10	3880 60	—	6		
		3877·95	—	8		
3866 5	6				3867 6	4
3856 5	4					

It is not possible to explain the presence of so large a quantity of xenon as must have been in the spectrum tube as being due to a leakage of air into the apparatus Prof Rutherford and I have made special experiments to test whether xenon was present with our preparations of emanation, but could not detect any trace of this gas. It is probable that there has been, in the experiments of Cameron and Ramsay, an unsuspected contamination with xenon.

Cameron and Ramsay attribute a line at 4058 5 to the active deposit of the radium emanation, since its relative intensity increases with the length of time that the discharge has passed Mercury lines will, of course, behave similarly as the emanation is driven into the walls. It appears probable that they have overlooked a mercury line at 4057·9, which is present, according to Eder and Valenta, in the vacuum tube discharge at ordinary temperatures, with an intensity 4 compared with intensity 10 for the mercury line 4046 8

The Extension of Cracks in an Isotropic Material.

By A Mallock, F.R.S

(Received November 9,—Read December 10, 1908)

The formation and extension of cracks in solids is a matter of considerable practical importance, but, as far as I am aware, the strains in the material at the extremity of a spreading crack have not been considered in detail either by engineers or physicists

It is a matter of common observation that in some materials a crack will spread with great facility whilst in others the reverse is the case Glass and indiarubber may be mentioned as extreme cases Between these two, endless gradations of brittleness and toughness can be found

That the facility with which a crack spreads does not depend only on the breaking strain of the material or on the work required to cause rupture is apparent from the fact that (*a*) although the tension modulus of rupture for glass is comparable with (though less than) the same modulus for cast iron and brass, the facility with which a crack can be started and spread in it is immensely greater, and (*b*) though the modulus of rupture for indiarubber is comparatively small it requires more work to break a piece of this substance by tension than to break in the same way a piece of glass of equal area the reason of course being the very small extension which can be given to glass without rupture

Any specified strain in a solid can be represented as a combination of shear and volume extension or compression, and both for volume extension and shear there are limits which if exceeded either cause rupture or leave the material in an altered condition when the stress is removed

The ordinary tests which are applied to structural materials involve in general both volume alteration and shear, but the limits for the two forms of strain are distinct and perhaps independent of one another

In liquids the coefficient of distortion μ is evanescent and there is no limit as regards the magnitude of the shear, but κ the volume coefficient is finite and comparable with that for solids, and there are limiting values for the volume expansion of liquids which if exceeded cause discontinuity.

In many liquids, *e.g*, water and mercury, it is known that the breaking strain for volume extension is large although there are experimental difficulties

in measuring the exact amount There is, however, no known limit of rupture for the volume compression of either solids or liquids *

It would be a matter of interest and importance to determine for solids whether, and how far, the existence of one form of strain influenced the limits of the other whether, for instance, a body subjected to volume extension would require more or less shear to rupture it than when the volume was normal

This point has not, as far as I know, been made the subject of experiment, but for the purpose of this note I shall assume that if a strain which exceeds either of the limits is applied to a solid, rupture will be due to that property of the substance for which the limit is least, and that if the distortion limit is the smaller of the two, breakage will occur at right angles to the lines of greatest extension, whereas if the volume limit is the least the direction of the break will be indeterminate

Consider a solid partly divided by a crack in a certain plane and subject to equal and opposite forces symmetrically applied at points on either side of and equidistant from the plane

Since the plane of the crack is a plane of symmetry, the solid on one side of this plane may be supposed to be absent and its place to be taken by the tractions required to keep the strain unaltered

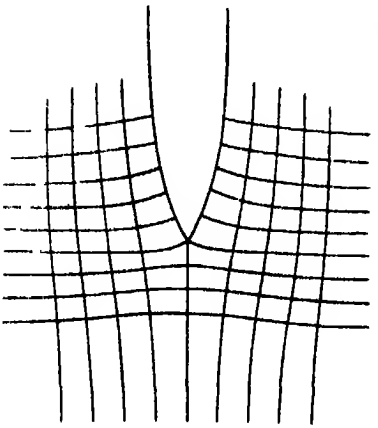
The problem then becomes one regarding the strain produced by a given distribution of stress over a finite area of the plane surface of an otherwise infinite solid

The further development of the crack depends only on the action which takes place in the immediate vicinity of its end for the time being It would be superfluous therefore to refer to the solutions of the above problem which have been given by Boussinesq and others, for these solutions do not apply to points extremely close to the margin of the area of the applied stress, and for the present purpose it is only with regard to such points that information is required. It may be seen, however, that lines of strain near the margin must be of the type sketched in the accompanying diagram, and that the strain and stress would be infinite at the end of the crack if there were no elastic limits Hence one may conclude that even the smallest force when applied to a cracked solid will cause some permanent set at the end of the crack if the material can yield in this

* This may give an explanation of the difference between malleability and ductility Under the hammer the strain is a shear combined with volume compression, while in "drawing" the material undergoes shear combined with volume dilatation. In general, a body which is ductile will also be malleable, but the converse need not hold

way,* or, if rupture ensues when the elastic limit is exceeded, that the crack will be extended

In most homogeneous solids the area over which the stresses exceed the mean stress at some moderate distance from the end of the crack will be of the same order as the square of the width of the crack, an area so small that it is not unlikely that the elastic and other limits of the material within it may be altered by the same cause which produces surface tension in liquids



The proportion of shear to volume extension is dependent on the value of Poisson's ratio for the substance, and increases indefinitely as this ratio approaches one-half, but the limits to which the volume can be altered, or the substance distorted, without rupture do not necessarily involve σ at all

If the conditions of strain at the end of the crack are such that material gives way from over-distortion, the fracture will occur in the plane of the existing crack, which will therefore spread continuously while if the over-dilatation is the origin, the breakage may take place in any direction. If at any place the plane of the new fracture cuts the plane of the crack there will be a rearrangement of stresses, and a relatively considerable length of material will have to be strained before further rupture is possible, and thus the cross fractures will act as a bar to the further extension of the crack.

I conclude therefore that in materials such as glass or other substances in which cracks spread in nearly constant directions that rupture is due to the distortion limit, and that where a crack extends with difficulty in a wandering manner, the dilatation limit is the one which has been exceeded.

The rapid alteration of the direction in which fracture takes place may give rise to the fibrous appearance which often shows itself on broken surfaces in such cases.

When the limits for both μ and κ are reached at nearly the same time, a very small change in either, such as might occur in a body

* The fact that a crack does not close completely when the force which caused it ceases to act is evidence that some permanent set has been set up

nearly but not quite homogeneous, would alter altogether the appearance of a fracture

In this note only isotropic materials are considered, but it seems probable that the same principles might be used to explain the cleavage of crystals

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The Rotation of the Electric Arc in a Radial Magnetic Field

By J Nicol, B A., B Sc, Wheatstone Laboratory, King's College, London

(Communicated by Prof H A. Wilson, F R S Received October 1,—
Read December 10, 1908)

The following paper contains an account of a series of measurements of the velocity of motion of an electric arc in a magnetic field at right angles to its length The experiments are similar to those made by Prof Wilson and Mr. G H Martyn* with the electric discharge in a vacuum tube and were suggested by Prof Wilson

The apparatus consisted of a vertical iron rod (fig 1, A) magnetised by two solenoids B at its ends, wound in opposite directions, so as to give a pole in the middle of the bar With this arrangement the field round the middle of the bar is uniform and radial. The distance between the two solenoids was fixed by a quartz tube C, which also served to protect the iron rod from the heat of the arc

The arc passed between two copper tubes D, 2 cm in diameter, held coaxial with the iron rod by a clamp made of wood and brass The copper tubes were clamped in holes cut in two pieces of thick (5 mm) sheet brass E, fixed to the top and bottom of a block of hard wood F The required arc length was obtained by clamping the electrodes while they were pressed firmly against a gauge of sheet brass held between them.

The base of the iron rod and the stand carrying the arc electrodes each rested on three screws in a hole slot and plane fixed to the table by paraffin wax. Thus the whole apparatus could be quickly taken down and set up again in the same position This was necessary to enable the electrodes to be renewed after each experiment

The arc was in all cases started with the magnetic field already in action, by momentarily bringing a piece of arc-lighting carbon in contact with the two electrodes.

* 'Roy. Soc. Proc.,' A, vol. 79, 1907

Experiments were first made with carbon electrodes, but though the discharge could be started and would occasionally make a few revolutions it never lasted for more than one or two seconds An iron-iron and an iron-mercury arc were tried with equally unsatisfactory results Then a copper

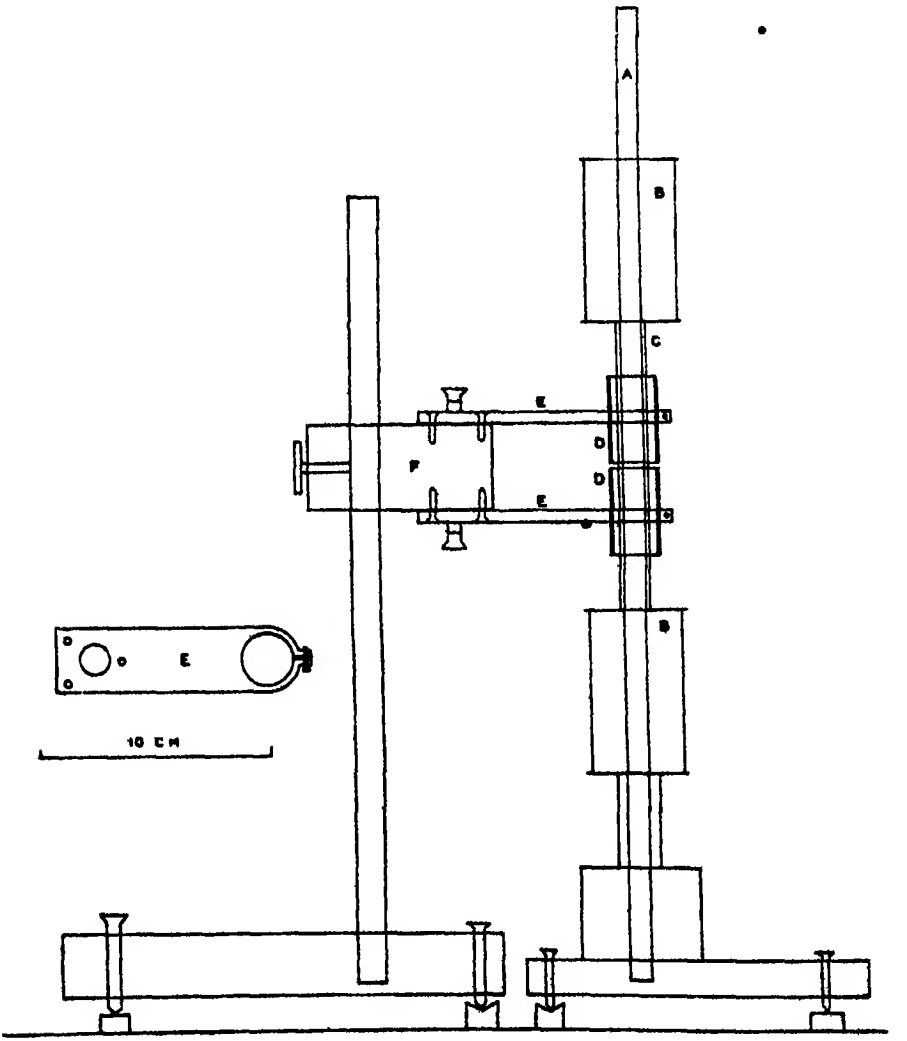

Fig. 1

arc was used and with it all the experiments described in this paper were made

The current for the arc was taken from the electric lighting supply at 200 volts and as the arc used only about 50 volts a large amount of resistance

had to be put in the circuit A few experiments were done with the 100-volt supply, but the arc was then much more liable to go out before the necessary reading had been obtained, it was therefore preferred to use the 200-volt supply The current used was measured by means of an Elliott portable standard ammeter reading up to 15 amperes

Though the copper arc was much more stable than any of the others which had been tried, the discharge seldom lasted for longer than thirty seconds, and then only when brightly polished electrodes were used After every second or third discharge therefore, the apparatus was taken down and the electrodes replaced by fresh ones while the old ones were repolished in a lathe with fine emery paper.

As the discharge only lasted for so short a period, a stroboscopic method of measuring the speed of rotation was out of the question, and in place of it a photographic one involving the use of a rotating mirror was employed The mirror was fixed not quite normally on one end of the shaft of a small electric motor When the motor was working, a point source viewed in the mirror appeared drawn out into a circle of light, and if the point source was intermittent this circle was broken into as many dots as the number of times the source became active during each revolution of the motor The intermittent source was obtained by placing a vertical slit in front of the arc. This was illuminated by the arc once every time it revolved round the iron bar The image of the slit in the rotating mirror was then photographed with an ordinary camera The shutter was adjusted so that the plate was exposed for a little longer than the time taken by the mirror to make one revolution By this means a slight overlapping of the dots at the beginning and end of the exposure was insured and it was consequently easy to count to the nearest tenth the number of revolutions made by the arc during one revolution of the mirror

Imperial Special Rapid plates were used and gave satisfactory results, isochromatic (Imperial N F) plates were tried but did not give nearly so dense an image Three exposures were taken on each plate, the rising front of the camera being moved between each pair of exposures. To count the number of dots the negatives were copied on tracing paper, as the counting was more easily done on these tracings than on the original negatives

To turn the mirror an $\frac{1}{8}$-H P series motor designed for 100 volts was employed It was used, however, as a separately excited machine, the field magnet circuit being connected to the 100-volt mains through an 8-C P. lamp, and the armature windings supplied with current from an 8-volt accumulator. The speed was regulated by an adjustable resistance in the armature circuit. The number of revolutions made by the motor was

registered by a Veeder revolution counter (fig. 2, E) fixed to the opposite end of the shaft from the mirror To determine the speed, the time of 100 revolutions was taken with a stop-watch.

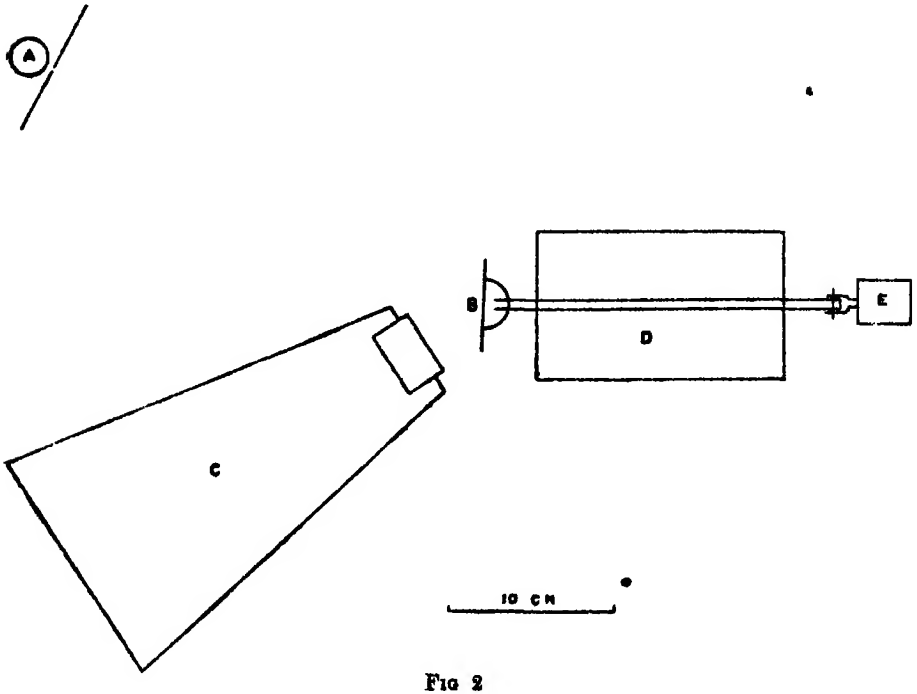

Fig 2

The motor was run at about 250 revolutions a minute, and the camera exposure was about $\frac{1}{3}$ of a second (a nominal second with a Unicum shutter).

The relative positions of the arc A, motor D with mirror B, and the camera C, are shown in fig. 2

Theory of the Rotation of the Arc

If k_1 and k_2 are the velocities of the ions due to unit electric force, the rate at which the arc moves is given by the expression*

$$v = \mathrm{H}\mathrm{X}k_1k_2, \quad \text{or} \quad k_1k_2 = \frac{2\pi n}{\mathrm{H}/r\ \mathrm{X}},$$

if n is the number of revolutions the arc makes per second, and H and X the magnetic and electric forces, and r the radius of the electrode.

The following is a proof of this relation. The velocity of drift of an ion along the arc is $\mathrm{X}k$ This motion of the ion in a magnetic field causes a

* Wilson and Martin, *loc. cit.*

transverse electric force HXk to act on the ion which gives it a transverse velocity HXk^2. Hence the transverse displacement of an ion while it passes from one electrode to the other is $HXk^2 \cdot l/Xk = Hkl$ where l is the arc length.

Although, unless the velocities of the two ions are the same, the arc will not remain parallel to the axis during rotation, the two ends will eventually move at the same speed and their position will be given by

$$y_1 = vt \text{ for anode end}, \qquad y_2 = a + vt \text{ for cathode end.}$$

A positive ion starting from the anode and moving to the cathode in a time l/Xk_1 must then travel a distance $a + v\,l/Xk_1$ transversely.

Similarly, the transverse motion of a negative ion is $-a + v\,l/Xk_2$.

Equating these to the values previously found, we get

$$a + v\,l/Xk_1 = Hk_1l, \qquad -a + v\,l/Xk_2 = Hk_2l$$

thus

$$v = HXk_1k_2$$

Eliminating v instead of a from the above two equations, we get

$$a/l = H(k_1 - k_2)$$

The maximum value of H used was 150, and k the velocity of the ions in unit (C G S electromagnetic) field is not greater than 5×10^{-6}, so that a/l, which measures the inclination of the arc to the axis, is always very small. This explains the observation made by Wilson and Martin that the discharge in their experiments always remained perpendicular to the electrodes during its motion.

Measurement of the Magnetic Field H

The magnetising current was supplied from an 8-volt accumulator, and was measured by means of an ammeter similar to the one used for the measurement of the arc current, but reading only to 5 amperes. The ammeters are issued as correct to 1/5 per cent., and they were found to agree with one another perfectly. A reversing key was included in the magnetising circuit, and the current was frequently reversed when any change was made in the magnetisation. In going from a higher to a lower magnetisation, the iron was first demagnetised by the method of reversals, in order to insure that the field should be a definite function of the magnetising current and independent of the permanent magnetism retained by the bar.

The actual values of the field (or rather of H/r, which is constant for a radial field) for different magnetising currents, were found as follows, the method being the same as that used by Wilson and Martin.

Two coils of 50 turns each were wound in grooves turned on a boxwood cylinder of the same diameter as the copper tubes used as electrodes for the arc discharge. The two coils were first connected in opposition and placed in

the position usually occupied by the arc The coils were connected to a low resistance Broca ballistic galvanometer, and the deflections produced by reversing various currents in the magnetising solenoids of the iron bar observed The connections of the coils were then so altered that they were in series, and that a current would pass round them in the same direction The coils were removed from the iron bar and placed along the axis of a long solenoid at its centre, and the deflection produced by reversing the current in the solenoid observed.

Let n be the number of turns in each coil on the boxwood cylinder,
r radius of mean area of section of these coils,
l their separation,
N number of turns on long solenoid
2L and R its length and radius
i_0 the current in amperes reversed in it
δ and δ_0 the galvanometer swings produced by reversing the field due to the iron bar and to the long solenoid respectively, then

$$\frac{\delta}{\delta_0} = \frac{H 2\pi r l n}{2n\pi r^2 \frac{4\pi N}{2L}\left(1-\frac{R^2}{L^2}\right)\frac{i_0}{10}}, \qquad \frac{H}{r} = \delta \frac{i_0}{10\delta_0}\frac{4\pi N}{2L}\left(1-\frac{R^2}{L^2}\right)\frac{1}{l}$$

Substituting the values used,

$$\frac{H}{r} = \frac{4\,75}{10\times 9\,8}\frac{4\pi 200}{26\,4}\,0\,974\frac{1}{1\,07} = 4\,20\delta$$

The values of δ were plotted on squared paper against the values of the magnetising current in amperes i_H, and were found to lie on a straight line Substituting the value thus found, we get

$$H/r = 4\,20\times 8\,3\,(i_H - 0\,1) = 34\,9\,(i_H - 0\,1)$$

As H/r occurs in the expression for the ionic velocities, it is obvious from the above deduction that it is not necessary to know either the diameter of the coils on the boxwood cylinder, or the number of turns in them The diameter of the coils was, however, made equal to the diameter of the arc electrodes, to avoid any error which would result from the field not being radial The separation of the coils was four or five times the length of arc usually employed, but this does not introduce an error, inasmuch as experiment showed that the field varied very little along the iron bar, the variation being under 1 per cent per centimetre The equality of the number of turns in the two coils was tested by placing them, together with an iron core, inside the long solenoid while they were connected in opposition The deflection obtained on reversing the current in the solenoid was negligible compared with

that obtained when a single turn of wire round the core was substituted for the two coils in opposition

All the connections of the coils were soldered so that the total resistance in the circuit, about 2 ohms, should be the same in both experiments.

Measurement of the Electric Force X

In order to obtain a value of the electric force in the arc, it was assumed that the change in the P D observed when the length of the arc was altered at constant current was equal to the product of the electric force and the change in length This is equivalent to assuming that the potential-drops at the two electrodes are independent of the length of the arc Probably this is not quite true, as lengthening the arc may affect the temperature of the electrodes In any case the method is only a rough one, as the change in the P D used is only the difference between two much larger observed P D 's No better method of measuring the field, however, suggested itself

The value of the arc P D was measured for various currents for arc lengths of 0 272 and 0 360 cm by means of an Elliott portable standard voltmeter reading to 120 volts When the arc was not running, the P D between the electrodes was 200 volts To protect the voltmeter from this high voltage, an automatic switch, actuated by an electromagnet in the main arc-circuit, was used

Very rough experiments showed that the rotation of the arc had a considerable effect on its P D, which was about 10 volts higher when the arc was stationary than when it was moving At first it seemed that the speed of rotation had an effect on the P D, but more careful experiments showed that this was not the case The spurious effect at first noticed was caused by using the electrodes for a second discharge without cleaning them In order to obtain consistent readings, it was found to be essential before each experiment to repolish the electrodes in a lathe If the arc was started again after it had gone out, it was found that the P D. had gone up by 3 to 4 volts. In most cases the P D remained quite steady during the first discharge, however long this lasted, but rose several volts as soon as the discharge was stopped and restarted With currents of about 5 amperes a curious phenomenon was noticed Soon after starting the discharge, the P D would rise steadily 4 or 5 volts and then fall again to its original value, remaining steady until the arc went out This did not happen with larger or smaller values of the current.

The values of the P.D obtained were multiplied by the arc current and the resulting product plotted against the current, as was done by Mrs Ayrton

in the case of the carbon arc The curves obtained were not quite straight lines as they were in Mrs Ayrton's experiments, but were very nearly so Corresponding ordinates of the smooth curves were subtracted, and the differences $\delta(V_i)$ plotted against i The points thus obtained lay with sufficient accuracy on a straight line giving

$$\delta(V_i) = 3{\cdot}75 + 5i, \qquad X = \frac{1}{0{\cdot}088}\left(5 + \frac{3{\cdot}75}{i}\right) = 57 + \frac{42{\cdot}5}{i}$$

The observations on which this formula is based are given in Tables I, II, III Table I shows how V depends on the rotation of the field and the newness of the electrodes Table II contains the experiments with an arc length of 0 272 cm , Table III those with an arc length of 0 360

Table I

a	^{s}H	i	V
1	2 8	7 9	42 5
1	2·0	7 9	42
*1	1·0	7 9	42
*2	2 8	—	45 5
*4	2 8	—	48
*7	0·0	9 5	31
1	2 5	4 3	48 5
1	1 0	4 5	48 5

"a" is the number of the experiment after insertion of new electrodes
* These experiments were done with the same electrodes and the same resistance in circuit

Table II

i	V	V_i
1 45	64·0	93
3 0	53 5	160 5
4 4	48 5	213
5 2	46 75	243
5 4	46 8	252
6 6	43·7	288
7 9	42 2	333
8 45	40·0	338
9 3	40 2	374
11 6	37 5	435
13·0	37·0	481
15 1	35 0	528

Table III

i	V	V_i
1 65	69 0	104
2 03	64 75	131 5
2 95	60 5	178
3 35	57·75	193
4 73	53 75	254
5 13	52 5	269
6 7	48 5	325
7 8	47 0	367
8 4	46 5	390
9 75	44 4	432
11 47	43 0	492
11 7	43·0	503
13 4	42·0	563
14 2	40 75	577

The particulars of the experiments on the speed of rotation of the arc are collected in Tables IV—VII It was found that there was a considerable falling off in the speed of rotation if the same electrodes were used a second

time without cleaning, this was especially noticeable when the arc length was short (under 2 mm) On this account only those experiments which were made with freshly polished electrodes have been tabulated, and all such experiments are included in the tables (Two second experiments are included, but a special allowance is made for them)

Table IV —Arc Length 0 185 cm

No	^1H	t	Dots	Time, 100 motor revs	Arc revs per sec $^1H - 01$	Reduced values r	Reduced values $n/(t - 01)$	Means
11t	3 02	1 80	13 0	26 5	16 8	2	17 9	18 15
6t	2 95	2 10	15 0	27 8	18 95	2	18 4	
B2m	3 61	4 31	27 1	23 6	32 4	4 3	32 3	30 6
9b	2 90	4 30	23 8	26 6	31 9	4 3	31 0	
8m	2 95	4 29	22 6	26 6	29 9	4 3	29 9	
9t	2 93	4 27	21 4	25 8	29 3	4 3	29 5	
10t	2 94	4 27	22 7	26 6	30 0	4 3	30 2	
B1b	2 50	4 27	17 8	25 2	29 4	4 3	29 6	
D6m	2 10	4 45	13 9	21 3	32 6	4 3	32 0	
D6b	2 11	4 43	15 0	21 9	34 1	4 3	33 7	
12t	2 04	4 20	12 2	23 5	26 8	4 3	27 3	
B1t	1 48	4 30	6 4	25 3	18 3	4 3	18 3	omitted
			Mean of 8 2nd experiments			4 3	—	26 6
			„ 1 2nd experiment			6 1	—	33 8 corr 38 9

Table V —Arc Length 0·230 cm.

4Ot	2 50	2 00	12 2	21 9	23 2	2	23 2	22 8
1t	2 03	1·95	10 2	26 9	19·6	2	19 8	
4Ob	1 53	2·00	8 3	22 9	25 3	2	25 3	
9Bt	4 11	4 15	25 8	22 8	29 0	4	28 4	30 9
3Bt	2 96	4 12	22 5	23 8	33 0	4	32 5	
5Bt	2 52	4·04	19 1	24 6	32 1	4	31·9	
11Bm	3 99	6 10	34·0	23 2	37 7	6	37 3	38 1
3Om	3 48	5·98	23 7	22 6	38 2	6	38 3	
3Bb	2·99	6 10	29·2	24 3	38 0	6	37 6	
3Ot	2 49	6·03	22 6	23 3	40 6	6	40 5	
3t	2·05	5 83	16 3	26·9	31 1	6	31 8	
1Ot	2·07	6 21	17 7	22 2	40 5	6	39·7	
12Bm	1 48	6 03	13·9	22 6	44·9	6	44 8	
2Ot	0·98	6·06	7 1	23 1	34 9	6	34 7	
9Bb	3 28	9 15	40 8	24 2	53 5	9	52 9	51 8
4Bm	2·97	8·72	38 1	25 1	53 2	9	54 3	
10Bt	2 51	9 28	30 2	22 4	56·0	9	54·9	
10Bm	2·05	9 11	22 8	22 9	50 8	9	50 4	
10Bb	1 49	9·07	15 8	22·7	50 1	9	49 8	
11Bt	1·04	9·03	7·5	22 1	45 8	9	45·7	

Table VI —Arc Length 0 272

No	t_H	t	Dots	Time, 100 motor revs	Arc revs per sec $-0\,1$	Reduced values t	Reduced values $n/(t_H-0\,1)$	Means
D3m	2 ·08	1 83	9 9	22 3	22 4	2	23 1	23 1
D2t	3 10	4 ·00	21 ·9	22 9	31 ·9	4	31 9	32 1
0t	2 50	4 ·08	17 3	23 5	30 7	4	30 4	
B3m	2 ·05	4 ·03	14 4	24 5	30 1	4	30 0	
D1t	2 11	3 98	14 9	22 6	32 8	4	32 ·9	
D2b	1 15	4 22	9 0	23 7	36 2	4	35 3	
D4t	3 ·06	5 80	24 2	21 5	38 0	6	38 8	39 3
D5b	2 12	6 05	17 4	21 5	40 1	6	39 9	
D5t	1 12	5 87	8 7	22 ·0	38 8	6	39 3	
B7m	2 03	8 71	23 25	24 4	49 4	8	48 6	48 6
Means of all readings—5 1st 5 2nd 3 3rd						4	—	30 ·9
3 1st 3 2nd						6	—	38 6

Table VII —Arc Length, 0 360

B8t	2 ·05	3 ·97	13 8	23 ·7	29 8	4	29 9	29 9
B8m	2 ·04	5 70	16 4	24 2	34 9	5 ·7	34 9	36 2
B8b	2 ·03	3 80	12 8	24 4	27 2	4	27 8	

NOTE —These are 1st, 2nd and 3rd experiments, and 2nd has been corrected to 1st by supposing falling-off takes place uniformly

Most of the experiments were made with arc lengths of 0 230 or 0 272 cm. Arc lengths of 0 185 and 0 360 cm were also used As far as the accuracy of the experiments extends, the arc length appears to have no influence on the speed of rotation

It was provisionally assumed that the speed of rotation was proportional to the magnetic field, and consequently to $t_H-0\,1$ (see determination of magnetic field) The numbers obtained by dividing the number of rotations per second by $t_H-0\,1$ were therefore plotted against the current Before this was done the experiments were averaged in the following manner —As far as possible the current in each experiment was made a whole number of amperes, but to do this exactly was not possible, since there was no time to make any adjustments after the arc had been started, the greater part of

the time it lasted being necessary to read the ammeter and take a photograph The speed of rotation was found to increase with the current approximately linearly, and a rough value of the rate of increase was used to reduce each observation of $n/(i - 0\,1)$ to the value corresponding to the nearest whole number of amperes Means of the readings for each current were then taken and plotted against the current

The values of $n/(i_H - 0\,1)$ obtained for the same current in different experiments showed very considerable variations (10 to 20 per cent in extreme cases), but these variations were irregular, and there was no evidence to show that $n/(i_H - 0\,1)$ depended on the magnitude of $i - 0\,1$, as it would do were the speed of rotation not proportional to the strength of the magnetic field

The final results reduced in the above way are collected in the following table corresponding to fig 3—

i	$n/(i_H - 0\,1)$			
	$l = 0\,185$	0 230	0 272	0 360
2	18 15 (2)	22 8 (3)	23 1 (1)	
4	—	30 9 (3)	32 1 (5)	29 9 (1)
4 3	30 6 (10)			
6	38 3* (1)	38 1 (8)	39 3 (3)	37 3* (1)
8 5	—	—	48 6 (1)	
9	—	51 3 (6)		

The numbers in brackets indicate the number of experiments used in getting the mean value

* These numbers are obtained from experiments made with electrodes used a second time without cleaning

The actual values of the field used varied from 35 to 140 C G S units

The speeds of rotation observed varied from 30 to 170 revs per second corresponding to linear velocities of from 200 to 1100 cm per second

The values of $n/(i_H - 0\,1)$ in the above table have been plotted against the current in fig 3, and it will be seen that the points lie approximately on the straight line

$$n/(i_H - 0\,1) = 4\,1\,i + 14\,2,$$

or substituting the values

$$H/r = 34\,9\,(i_H - 0\,1), \quad \text{and} \quad 2\pi r n = v,$$

$$v = H\,(2\,55 + 0\,74\,i)$$

Substituting the experimental numbers for n, H/r, and X in the equation

$$k_1 k_2 = \frac{2\pi n}{H/r\ X},$$

we obtain the following values of the product of the two ionic velocities per unit electric intensity for different values of the current. If k_1 and k_2

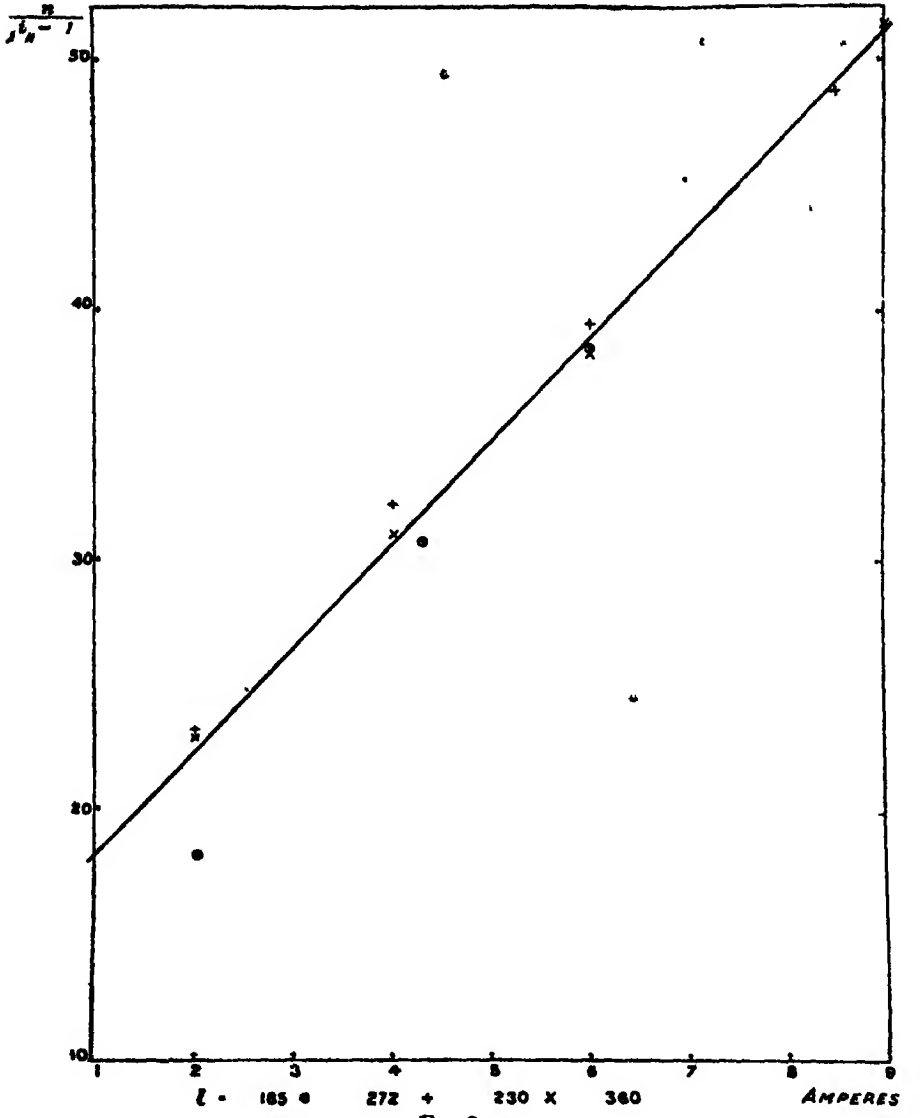

Fig 3

are the velocities per volt per centimetre, the numbers must be multiplied by 10^{16}

t_1	$n/(t_H - 0\ 1)$	$H/r(t_H - 0\ 1)$	$X \times 10^{-8}$	$k_1 k_2 \times 10^9$
2	23	34·9	78	0 53
4	31	34·9	67 5	0·83
6	38	34·9	64	1·07
9	51	34·9	63	1 5

Assuming that the negative ion in these experiments is a corpuscle, its velocity can be calculated from the expression—

$$k = c\lambda/mu^*$$

Neglecting any effect its charge may have, the mean free path λ of a corpuscle in air should, owing to its small size, be four times that of an air molecule; a relation which agrees with experiment† Taking the temperature of the arc to be 2000° C and the mean free path of an air molecule at 0° C as 10^{-5}, we get $\lambda = 10^{-5}\frac{2273}{273}\times 4$ To find u, the mean agitation velocity of the corpuscle, assume that the equation of $mu^2 = m_H u_H^2$ holds between the corpuscles and the gas particles, this gives $u = \sqrt{\frac{m_H}{m}}185000\sqrt{\frac{2273}{273}}$, if we take 1850 metres a second as the velocity of a hydrogen molecule at 0° C

Substituting in the expression for k, we get

$$k = 4\times 10^{-5}\sqrt{\frac{e}{m}}\sqrt{\frac{e}{m_H}}\frac{2273}{273}\frac{1}{185000},$$

but $$e/m = 1\cdot 9\times 10^7, \text{ and } \frac{e}{m_H} = \frac{1}{2\times 0\cdot 000105},$$

therefore $$k_2 = 1\cdot 83\times 10^{-4}$$

Combining this with the value 10^{-9} found for k_1k_2 for a current of 6 amperes, we have a value

$$k_1 = 5\cdot 5\times 10^{-6} = 550 \text{ cm per second per volt per centimetre}$$

This is about one-thirtieth of the velocity of the negative corpuscle, and since from the above deduction of this velocity it is evident that k is proportional to $\sqrt{\frac{1}{m}}$, the mass of the positive ions must be about 900 times that of a corpuscle, or about the same as that of a hydrogen atom The positive ions then cannot be either copper or air molecules, but may be the same as the carriers of positive electricity detected by Prof J J Thomson in the Kanalstrahlen On this view, however, it is rather difficult to give a satisfactory explanation of the way in which k_1k_2 increases with the current For k_1k_2 is only directly proportional to the temperature, and the experiments show that k_1k_2 is trebled by a rise in the current from 2 to 9 amperes This would indicate a rise in temperature from 1000° to 3000°, which is larger than is probable If the positive ions were groups of copper or air molecules the rise could be explained as due to a rise in temperature accompanied by dissociation of these groups

* Langevin, Theses, Paris, 1902, p 47

† Townsend, 'Phil Mag,' 1901, p 198

In this connection it may be mentioned that the negative electrode was always much more oxidised and corroded than the positive, but this may have been due to its being the lower electrode, and thus meeting the upward convection air current and robbing it of its oxygen before it reached the positive electrode

Conclusions—The speed of the electric arc across a transverse magnetic field has been measured and found to be sensibly

(1) independent of the arc length,
(2) proportional to the magnetic field strength, and
(3) to increase linearly with the current

From the experiments the value k_1k_2, the product of the speeds of the ions carrying the current, has been calculated From this value, by assuming the negative ion to be a corpuscle and calculating its velocity, it has been shown that the carriers of positive electricity have a mass of the same order as that of the hydrogen atom

In conclusion, I wish to thank Prof H A Wilson for suggesting to me the subject of this research and for much kind advice and encouragement during its progress.

The Isothermal Layer of the Atmosphere and Atmospheric Radiation

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(Communicated by Dr. W N Shaw, F R S Received October 5,—Read December 10, 1908)

I *The Phenomenon of the Isothermal Layer and the Experimental Evidence of its Existence*

The investigation of the upper air by means of balloons carrying self-recording instruments, which have furnished values for the atmospheric temperature up to heights between 15 and 20 kilometres, has revealed the existence of an abnormal change in the vertical temperature gradient After a fairly uniform fall, with increasing altitude, of about 6° C per kilometre, a height is reached above which the temperature changes very little, sometimes increasing, sometimes diminishing slowly

The phenomenon was first noticed by M Teisserenc de Bort* in a communication to the Société de Physique in June, 1899 He improved his apparatus and made further investigations, in many cases sending up the balloons by night to eliminate any possible insolation effects. He found the average height, at which the change began, to be about 11 kilometres He discovered also that the height was greater near the centre of high pressure areas than in low pressure areas, the average heights for the two cases being 12 5 and 10 kilometres respectively More recently he found that the height increased with approach towards the equator and that near the equator, *ballons-sondes*, ascending to 15 kilometres, had failed to reach this layer if it existed there He proposed to call this layer, in which little temperature change occurred, the "Isothermal Layer of the Atmosphere," and the name has been generally accepted †

The main results have been corroborated by other observers In September, 1906, a series of ascents was made at Milan, in which the isothermal layer was reached at heights varying from 8 to 13 kilometres and at temperatures between −40° C and −65° C The smaller heights and

* 'Comptes Rendus,' vol 134, April, 1902, vol 138, January, 1904, vol 145, July, 1907

† The term "isothermal layer" is slightly misleading, inasmuch as it appears to produce the conception of a definite stratum of uniform temperature lying between two regions where the temperature decreases at a rate approximately adiabatic. The term "isothermal region," suggested by Prof H H Turner, is free from this objection.

higher temperatures were found generally over low pressure areas and the greater heights and lower temperatures accompanied anticyclonic conditions.

Over Berlin,* in 1906, the height at which the layer was reached varied from 10 to 13 kilometres and the temperatures were between −50° C and −65° C., the average being −55° C These ascents were made at different times of the year, but in all cases during anticyclonic conditions or on the outer edge of a cyclone advancing from the west This may account for the fact that the layer was not found at smaller heights in any of the ascents In England,† during 1907, the height of the layer varied between 8 and 15 kilometres and the temperatures between −30° C and −60° C, the mean height being about 11 kilometres and the mean temperature −47° C The height was generally less than the average over low pressure areas, although there were anomalies, notably on November 11, when the height increased slightly and the temperature diminished rapidly in passing from Ditcham Park, which was on the ridge between two low pressure areas, to Manchester, which was under the influence of the northerly system whose centre was between Iceland and Norway

The layer has also been reached in Lapland,‡ near the Arctic circle (and simultaneously at Trappes, near Paris), at heights varying from 8 to 12 kilometres, also by Rotch,§ in America, in latitude 39° N, where the average height appears to be greater than in Europe Further, in an ascent near Brussels in July, 1907, the layer was found to extend with slightly increasing temperature from 12 to 26 kilometres altitude and near Strassburg,‖ in 1905, the temperature recorded at 26 kilometres was 20° C higher than that at 14 kilometres

There can be no question, therefore, of its being merely a local or temporary phenomenon

It is clear that there cannot be convection currents to any marked extent in this region, and I propose to show that in an atmosphere which is not transparent but absorbs and emits radiation, the process of radiation would prevent the establishment of the temperature gradient necessary for convective equilibrium, in the upper layers of the atmosphere, and that in the lower layers of our atmosphere it can be maintained only by transference of energy from the earth to the atmosphere by direct convection or by the process of evaporation of water at the earth's surface and subsequent

* 'Ergebnisse der Arbeiten des Königlich Preussischen Aeronautischen Observatoriums,' 1906.

† Petavel and Harwood, 'Quart Journ Roy. Met. Soc,' January, 1908, Dines, 'Nature,' February 27, 1908

'Met. Zeit.,' 1907, pp 498, 499 § *Loc. cit* ‖ *Ibid*, 1907, p 356.

condensation in the atmosphere The heat necessary for the evaporation of water-vapour at the earth's surface is supplied mainly by absorption of solar radiation and is not taken from the atmosphere, but the heat given up on condensation is added almost entirely to the heat of the atmosphere, and in this way we get a supply of heat to the atmosphere at a rate that may be estimated approximately from the annual rainfall

II. *General Statement of the Possible Temperature Distributions in an Atmosphere Summary of Theoretical Results now reached*

It is clear that if the atmosphere were transparent to radiation of all wave-lengths a state of convective equilibrium would exist up to a certain limit, after which the collision frequency would be too small to admit of mass-agitation and the permanent state of this outer layer would be one of conductive equilibrium But this state could only persist so long as the temperature of the earth was rising, because there would be a continuous flow of heat outwards by conduction This would produce an accumulation of heat in the upper layers of the convective atmosphere, and the adiabatic temperature gradient could be maintained only by warming up the lower layers, since if we assume that there is no radiation from the atmosphere, the difference of temperature could not be renewed by cooling the upper layers If, however, the temperature varied over the surface of the earth a persistent limited convection would be possible, provided that the lowest surface temperatures were below the upper air temperatures, so that conduction of heat took place downwards over these regions and balanced the upward conduction over those places where convective equilibrium prevailed

The temperature of the earth in that case would be such that the heat radiated into space would balance the incident solar radiation absorbed by it The minimum possible value would be obtained by taking the earth to be a full radiator for its own temperature The effect of clouds would be to diminish the mean temperature owing to their greater reflecting power for solar radiation They would either transmit the earth's radiation or act themselves as radiators for the wave-lengths absorbed The reflection of radiation of long wave-length would be small But if we have an atmosphere which absorbs and radiates there will be a theoretical temperature distribution in which there is equilibrium between the radiation and absorption of each element. We may describe such a state as one of radiation equilibrium If the vertical temperature gradient for this state is less than that corresponding to convective equilibrium, it will be impossible for the latter state to persist and the atmosphere will tend to radiation equilibrium

There may, however, be a limited convective equilibrium arising from causes which supply sufficient energy to balance the excess of the radiation in the convective state An example will suffice to explain this Suppose the state of radiation equilibrium were isothermal, and that in this condition the atmospheric radiation just balanced the absorption of solar and terrestrial radiation Consider, now, the effect of a direct communication of heat to the atmosphere at a definite rate at the earth's surface The temperature of the lower layers of the atmosphere would be raised and a state of convective equilibrium would supervene, but only up to a height sufficiently great for the increased atmospheric radiation, due to increased temperature, to balance the energy supplied

The assumptions which form the basis of the theory developed in the following sections are these —

(i) The constituents of the atmosphere radiate for the same wave-lengths for which they absorb, and according to the thermal law

(ii) The curvature of the earth's surface may be neglected in considering radiation in the atmosphere

(iii) Owing to the large portion of the spectrum through which the constituents of the atmosphere radiate, their radiations may be taken to be proportional to the fourth power of the absolute temperature

This I attempt to justify by the experimental data in Section III

(iv) The temperature in the adiabatic state may be represented sufficiently closely by the equation $T^n = kp$, in which n is taken to be 4 instead of 3 5, the theoretical value for dry air.

v) A necessary condition (*v supra*) for convection, which forms the keystone of the present discussion, is that, in the upper part of the convective system, the radiation from any horizontal layer (or any elementary sphere) should exceed the absorption by it

(vi) Where convection is absent the outward and inward radiations across any horizontal plane are equal, conduction being so slow as to be negligible

(vii) The radiating power of the earth's atmosphere diminishes with height owing to the diminution in the proportional amount of water-vapour present, and it may be represented with tolerable approximation by $\alpha/(q-p)$, where α and q are constants and p is pressure

The principal results obtained are as follows —

(*a*) By the use of (i) and (ii) alone, general expressions are found for the intensity of atmospheric, terrestrial, and solar radiation at any point in the atmosphere, and for the absorption and emission by any horizontal layer of finite thickness. The conditions for convection to be possible and for thermal equilibrium in the absence of convection are also found.

(*b*) By the introduction of (iii) to (vi) it is proved that, for an atmosphere uniform in constitution, the adiabatic state could not extend to a height greater than that for which $p = \frac{1}{2}p_0$, where p_0 is the surface pressure. It is also proved that, if the atmosphere were isothermal, the absorption of solar radiation in any layer of it, beginning from $p = 0$, would be equal to the absorption of terrestrial and atmospheric radiation, and each would be equal to the radiation in either direction from the layer

(*c*) By the use of (vii) it is proved that for the earth's actual atmosphere the height to which the adiabatic state can extend is limited Values deduced from the experimental evidence are then substituted for α and q and it is found that if the atmosphere consist of two shells, the inner in the adiabatic, the outer in the isothermal, state (1) the inner cannot extend to a height greater than that for which $p = \frac{1}{4}p_0$ (10,500 metres), (2) the inner must extend to a height greater than that for which $p = \frac{1}{2}p_0$ (5500 metres)

(*d*) It is shown that the radiation from the lower layers of the atmosphere exceeds the absorption by them, and that the deficiency of energy is such that it could be supplied by convection from the earth's surface, and by condensation of water-vapour The deficiency for the layer $\frac{1}{2}p_0$ to $\frac{1}{4}p_0$ is practically negligible, indicating that convection above $\frac{1}{2}p_0$ will be very slight.

(*e*) Minimum possible temperatures for any point in the atmosphere over a place at 300° A (absolute) are 150° A or 200° A, according as the atmosphere radiates throughout the spectrum or only for a part of it containing 75 per cent of the energy of full radiation for its temperature The values are deduced from what would be the radiation intensity across the upper strata of the atmosphere, supposing it were maintained in the adiabatic state throughout For this radiation must correspond to a temperature which is less than that for any other possible temperature distribution, when the surface temperature is unchanged.

III *Experimental Data on Gaseous Radiation and Absorption*

Before proceeding to formulate the conditions of radiation equilibrium and to obtain expressions for the intensity of atmospheric radiation, it will be convenient to describe briefly the results of experiments on the radiation and absorption of the constituents of the atmosphere The pioneer in this region of research was Tyndall,* who conducted a series of careful and elaborate experiments with gases contained in tubes closed by rock-salt plates. He used as his sources a Leslie's cube at 100° C, and a copper plate heated to about 250° C, and measured the radiation by a thermopile He found large

* 'Contributions to Molecular Physics.'

absorption by water-vapour and CO_2, but practically no absorption by oxygen and nitrogen He found also that ozone exercised a remarkable absorbing power Except for CO_2 the results of his observations on these gases are mainly qualitative in character and designed to show the greater absorbing power of damp air

Arrhenius* used a tube 50 cm long and examined the absorption of CO_2 at various pressures for radiation from sources at 100° and −80° C His results agree fairly well with Tyndall's, the absorption for a path length of 50 cm at atmospheric pressure being about 10 per cent. for the 100° C source and 15 per cent for the −80° C source J Koch† measured the absorption by CO_2 in tubes of different lengths at three different pressures, 380, 760, and 1520 mm

Koch's results proved what had previously been pointed out by Ångstrom,‡ that the absorption depends on the density of the gas as well as on the total mass of gas in the path of the radiation Ångstrom found that a tube 1 metre long, containing CO_2 at a pressure of 4 atmospheres, absorbed 16 2 per cent of the radiation from a source at 300° C, while a tube 4 metres long, containing CO_2 at a pressure of 1 atmosphere, absorbed only 13 2 per cent of the radiation from the same source The effect of this remarkable result can also be seen in Tyndall's observations, but it is complicated by his use of a different source in his experiments at constant pressure

The distribution of CO_2 absorption bands in the spectrum has been the subject of researches by Ångstrom,§ Paschen,‖ and Rubens and Aschkinass ¶ Absorption bands were discovered from 2 4 to 3 0 μ, 4 2 to 4 5 μ, and 12 5 to 16 μ The last is the most important for radiation at terrestrial temperatures The absorption in a path 22 cm long for radiation from a zirconium mantle was 75 per cent at the maximum and 30 per cent on the average for the range 12 5 to 16 μ For a path of 65 cm there was total absorption from 14 to 15 5 μ, the absorbing gas being at atmospheric pressure and temperature

The absorption by 33 cm of gas at atmospheric pressure and temperature for the band 4 2 to 4 5 was 90 per cent, and for the band 2 4 to 3 0 μ and the same layer it was 40 per cent, the source being hot blackened platinum

Schlaefer** has investigated the effect on the absorption bands of varying the pressure He found that the bands were widened by increasing the

* 'Ann der Physik,' vol 4, 1901, p 690
† 'Öfversigt af K Vet Akad Forhandlingar 1901, p 478
‡ 'Öfversigt,' 1901, p 378 § *Ibid*, 1890
‖ 'Ann der Physik,' 1894, vols 52, 53
¶ 'Astrophysical Journal,' 1898
** 'Ann der Physik,' vol 16, 1905

pressure, while an increase in the length of path sufficient to make the mass of gas in the path of the radiation the same as when the pressure was increased did not produce this widening He concludes that a variation in the amount of CO_2 in the atmosphere would not materially affect its absorbing power for solar radiation, since the amount present, equivalent for vertical transmission to a path of 250 cm at a pressure of 760 mm, is much more than sufficient to exert full absorption for the width of band corresponding to the density of CO_2 in the atmosphere

The principal feature of water-vapour absorption is the large number of spectral regions in which it occurs The bands up to 5 μ in the solar spectrum have been carefully observed by Abney, Langley,* and others Paschen observed the bands up to 10 μ, using as source a blackened iron plate at 400° C Rubens and Aschkinass measured the absorption between 10 μ and 20 μ, and at 24 μ, using as source a zirconium mantle

The following table gives the percentage absorption for different wave-lengths by a 7 cm layer of water-vapour at 100° C and atmospheric pressure, deduced from Paschen's observations †

λ	8·1	8·0	7·8	7·6	7·4	7·2	7·0	6·8	6·6	6·5	
A	6	6	10	10	20	35	43	49	75	82	per cent
λ	6·4	6·2	6·0	5·9	5·8	5·6	5·4	5·2	5·0		
A	78	31	61	69	66	54	31	12	0		„

If the absorption by the water-vapour in the atmosphere were given by $100\,(1-e^{-km})$, where m is the mass of water-vapour in the path of the radiation and k is a constant, different for different wave-lengths, deduced from these results, we find that for an atmosphere of average humidity containing water-vapour equivalent to a layer of liquid water 2 cm thick, there would be total absorption for wave-lengths 5 μ to 8 μ.

From the observations of Rubens and Aschkinass we deduce the following values for the absorption by 75 cm of water-vapour at 100° C and atmospheric pressure

λ	A. per cent	λ	A per cent
8—12	5	15—16	50
12—13	15	16—17	75
13—14	20	17—19	85
14—15	40	19—20	98

On the same hypothesis as above, this would imply total absorption by the water-vapour in the atmosphere for radiations from 12 to 20 μ.

* 'Annals of the Astrophysical Observatory of the Smithsonian Institution,' vol 1, 1900.
† 'Ann. der Physik,' 1894, vol. 52, p 212

Of the remaining constituents of the atmosphere, ozone is the only one which shows considerable absorption. Tyndall* found ozone exercised a remarkable absorption for his low temperature radiation. E. Meyer† observed the absorption by ozone of short wave-length radiation Taking the ozone in the atmosphere to be 1·3 milligrammes per kilogramme of air, the following table gives approximately the percentage absorption for vertical transmission for different wave-lengths, according to Meyer's results.—

λ	A per cent.	λ.	A. per cent.
200	10	260	84
210	15	270	81
220	24	280	64
230	50	290	43
240	78	300	37
250	83		

Ångstrom‡ found no ozone absorption bands between wave-lengths 0·6 μ and 4·6 μ He found bands at

4·8 μ .	Sharp,	6·7 μ	Uncertain,
5·8 μ .	Weaker,	9 to 10 μ ..	Very strong,

and no further bands up to 14 μ

The band 9 to 10 μ corresponds to an absorption band found by Langley in his work on the temperature of the moon, and as there are no CO_2 bands there, and the water-vapour absorption is weakest in that part of the spectrum, the existence of this band is incidentally strong evidence of the presence of a considerable quantity of ozone in the atmosphere The origin of this is possibly indicated by an observation of Fr Fischer,§ who found that ozone was formed by the action of ultra-violet light as well as by electric discharge

Ladenberg and Lehmann‖ verified Ångstrom's results for ozone absorption at 4·8 μ and 9 to 10 μ, and obtained additional maxima of absorption at 7·6 μ, 11·3 μ, but these were weak compared with the 9 to 10 μ band

Eva von Bahr¶ measured the absorption of low temperature radiation by ozone. The percentage of the total radiation absorbed approached an asymptotic value as the ozone in the path was increased. This asymptotic value agreed with the assumption that the absorption took place in two bands, 4·8 to 4·9 μ and 9·2 to 10·2 μ

* Tyndall, 'Contributions,' p 102.
† 'Ann der Physik,' vol 12, 1903, p. 856.
‡ 'Arkiv för Matematik Ast och Fysik,' vol 1, 1903—4, pp. 347, 395
§ 'Ann der Physik,' 1903
‖ *Ibid*, 1906, vol 21, p 313.
¶ 'Arkiv för Matematik Ast och Fysik,' vol 3, 1907, No 15.

Langley* measured the absorption by 100 metres of air, relative humidity 60 per cent., dew point 18° C., giving water-vapour in the path equivalent to 0·15 cm of liquid water. He found 21 per cent. of the radiation from a 100° C. source was absorbed. With air of different degrees of dampness he found the following results, where t is the equivalent amount of liquid water in the path:—

t	A
1·66 mm	24·3 per cent
0·96 „	14·6 „
2·05 „	32·9 „

The observations of the absorbing power of the different gases are not entirely in accordance with one another, and it appears that the absorption by water-vapour, as well as that by CO_2, depends on the density as well as on the total absorbing mass.

If we use the results obtained, we find that for CO_2, at a pressure less than 760 mm., the maximum possible absorption for radiation from a 15° C. source in the bands discovered is about 18 per cent. of the complete energy in the spectrum of a perfect emitter, as given by Planck's formula,

$$J = C\lambda^{-5}/(e^{c/\lambda T}-1),$$

where $C = 8346$, $c = 14{,}350$, λ being measured in terms of μ (10^{-3} mm.).

This agrees nearly with the value obtained by Ekholm for the absorption by CO_2 in the atmosphere, by using Koch's results for the absorption of the total radiation †.

If we apply the results of Paschen and Rubens and Aschkinass, we find that the water-vapour in the atmosphere would absorb 95 per cent. of the earth's radiation for vertical transmission, but if we assume that the apparent absorption between 8 and 12·5 μ is spurious, we find that 25 per cent. is transmitted. If we use Langley's observations and assume that the absorption takes place throughout the spectrum, we find 94 per cent. of earth radiation absorbed in vertical transmission. Assuming no absorption between 8 to 12·5 μ, we find 73 per cent. of the total radiation absorbed.

To obtain an estimate of the possible absorption of solar radiation, it is necessary to know the distribution of energy in the solar spectrum. I have for this purpose assumed that the effective temperature of solar radiation varies with the wave-length and is given by the equation $T = 6600 - 1000e^{-\lambda}$, where λ is measured in terms of μ. With this value of T and Planck's formula we find a value 2·7 for the solar constant (the value according to the best recent

* 'Memoirs Nat. Acad.,' vol. 4, Ninth Memoir, p. 184.

† See 'Met. Zeit.,' 1902, p. 496.

determinations is 2 2),* and a maximum intensity at wave-length 0 52 μ agreeing with Langley's estimate

The following table gives the values of the intensity of radiation at different wave-lengths calculated on this assumption —

λ	J	λ	J	λ	J
0 10 μ	0·01	1 0 μ	927	2 8 μ	40 8
0 15	6 3	1 1	739	2 9	36·0
0 20	104	1 2	591	3·0	31 9
0 25	445	1 3	476	3 2	25 4
0 30	979	1 4	386	3 4	20 4
0 35	1540	1 5	316	3·6	16 6
0 40	1927	1·6	260	3 8	13 6
0 45	2161	1 7	216	4·0	11 2
0 50	2239	1 8	180	5 0	4 6
0 55	2207	1·9	152	6 0	2 5
0 60	2103	2 0	128	7 0	1 4
0 65	1956	2 1	109	8 0	0 82
0 70	1789	2 2	93 5	9 0	0 52
0 75	1621	2 3	80 4	10·0	0 34
0 80	1458	2 4	69 6	11·0	0 24
0 85	1305	2 5	60 5	12·0	0 17
0·90	1166	2 6	52 8	15 0	0 07
0 95	1040	2 7	46 3	20 0	0 02

With these values of J we find the following values for the percentage amount of solar radiation absorbed in vertical transmission, the water-vapour in the path being equivalent to 2 cm of liquid water —

Absorbing gas	Amount of absorption	Possible absorption
	per cent	per cent
CO_2	1 5	1 5
H_2O	9·5	11
Ozone	1 2	2 2

Thus the absorption of solar radiation would be about 12 per cent for zenith sun, and about 15 per cent. for low sun, when the absorption in the bands would be full The largest observed percentage absorption of solar radiation by water-vapour, given in the 'Annals,' etc,† is 11·3 per cent., the transmission being through 3 4 atmospheres and the surface vapour pressure being 0 68 mm

The radiation from gases has formed the subject of researches by Maurer,‡

* 'Annals of the Astrophysical Observatory of the Smithsonian Institution,' vol 2, 1908

† Vol. 2, 1908, p 131

‡ See 'Met. Zeit,' 1901, p 223

Hutchins and Pearson,* Paschen,† and Very‡ From meteorological observations, Maurer deduced that an air layer 1 cm thick radiated to another at 1° C lower temperature 4×10^{-5} gm cal per hour from each cm^2 of surface This agrees with the value 5×10^{-5} deducible from Langley's observations on absorption for air of 60 per cent relative humidity and 18° C dew-point, on the assumption that absorption and emission follow the same law and that the intensity of radiation varies as the fourth power of the absolute temperature From the experiments of Hutchins and Pearson on the radiation from a hot air column, the corresponding radiation would be 95×10^{-5} gm cal

Paschen's results are chiefly of importance as identifying the spectral regions of emission with those of absorption for the same gases

Very found from his experiments a value 22×10^{-5} gm cal per hour, a value intermediate between that of Maurer, and Hutchins and Pearson The experiments on which Very relies for his results were made by enclosing the air in a tube with a moveable radiating disc and a rock salt face The tube was heated from below by Bunsen burners and the radiation was observed first with the disc near to the rock salt and secondly with the disc drawn back so that a layer of air intervened in the path of the radiation It is assumed that the radiation from this air is measured by the difference, corrected for absorption, between the total radiation in the two positions, no radiation from the walls of the tube reaching the bolometer Now if the gas and the disc have the same temperature and the disc radiates fully (the disc used was blackened copper), it appears certain that the gas will absorb just as much disc radiation as it itself radiates Any additional radiation, therefore, in the second position, must have been due to a change in the temperature of the radiating solid parts or to an excess in the temperature of the gas over that of the disc, and of this no account is taken.

We shall therefore assume that the absorption results are the more correct and utilize them in the application to the atmosphere

IV *General Expressions for the Radiation from the Atmosphere, and Conditions of its Equilibrium*

Let us consider the radiation and absorption in a gas stratified in horizontal layers in which the pressure at any point is due to the weight of the gas above it Since the radiations are thermal, the emission will follow the same law as the absorption. Thus if an element of gas of mass dm

* 'Amer Journ of Sci.,' 1904, p. 277

† Paschen, 'Ann der Physik,' 1894.

‡ Very, 'Atmospheric Radiation'

occupying an element of volume of unit sectional area and thickness dz emits radiation of wave-length λ equal to $I_\lambda b dm d\lambda$, where I_λ is the intensity of radiation of wave-length λ for a full radiator at the temperature of the gas, then the same element will absorb radiation $Ib dm d\lambda$ if radiation $Id\lambda$ of wave-length λ is incident normally on unit area For the absorption by a length z of the gas we find at once, on putting $dm = \rho\, dz$, the expression

$$Id\lambda(1-e^{-W_1}), \quad \text{where} \quad W_1 = \int_0^z b\rho dz,$$

and a corresponding value for the emission.

Divide up the gas into horizontal layers across which the pressure change is dp so that the mass of gas in each layer is proportional to dp The intensity of radiation from a layer at pressure p emitted in a direction inclined at an angle θ to the normal will be, per unit area,

$$I_\lambda\{1-e^{-bdp\sec\theta}\}\, d\lambda \cos\theta$$

The intensity of this at a point at pressure p_1 and distant h from the layer will be

$$I_\lambda(1-e^{-bdp\sec\theta})\, d\lambda\cos\theta \;\; e^{-u(p,p_1)}/h^2\sec^2\theta,$$

where $u(p,p_1) = \int_p^{p_1} bdp\sec\theta$

This will also be the amount entering a spherical element of radius $\pi^{-\frac{1}{2}}$ at the point, and we get, for the whole amount of radiation from the layer entering such an element per unit time,

$$\int_0^{\frac{1}{2}\pi} I_\lambda d\lambda(1-e^{-bdp\sec\theta})\cos\theta \;\; e^{-u(p,p_1)}\frac{2\pi h^2\tan\theta\sec^2\theta\, d\theta}{h^2\sec^2\theta}$$

$$= \int_0^{\frac{1}{2}\pi} 2\pi I_\lambda d\lambda \sin\theta \frac{d}{dp} e^{-u(p,p_1)} dp\, d\theta = \int_0^{\frac{1}{2}\pi} \phi(p,p_1)\, dp\, d\theta, \text{ say}$$

Therefore from the whole mass of gas situated above the layer p we get radiation of wave-length λ entering the element per unit time,

$$A_\lambda d\lambda = \int_0^{p_1} dp \int_0^{\frac{1}{2}\pi} \phi(p,p_1)\, d\theta$$

Similarly, from the gas below the element we get,

$$B_\lambda d\lambda = -\int_{p_1}^{p_0} dp \int_0^{\frac{1}{2}\pi} \phi(p_1,p)\, d\theta$$

For the radiation crossing unit area of a horizontal plane at p_1 downwards and upwards we find respectively,

$$P_\lambda d\lambda = \int_0^{p_1} dp \int_0^{\frac{1}{2}\pi} \phi(p,p_1)\cos\theta\, d\theta, \quad Q_\lambda d\lambda = -\int_{p_1}^{p_0} dp \int_0^{\frac{1}{2}\pi} \phi(p_1,p)\cos\theta\, d\theta \quad .$$

For the radiation from the earth entering the spherical element and crossing unit area of a horizontal plane, upwards, we get

$$F_\lambda d\lambda = 2\pi \int_0^{\frac{1}{2}\pi} E_\lambda d\lambda \sin\theta \, d\theta \, . \, e^{-u(p_1, p_0)},$$

$$G_\lambda d\lambda = 2\pi \int_0^{\frac{1}{2}\pi} E_\lambda d\lambda \sin\theta \cos\theta \, d\theta \; e^{-u(p_1, p_0)}$$

The corresponding values arising from the average direct intensity of solar radiation are

$$U_\lambda d\lambda = \frac{1}{\pi} \int_{\phi-\delta}^{\frac{1}{2}\pi} S \, d\theta, \qquad V_\lambda = \frac{1}{\pi} \int_{\phi-\delta}^{\frac{1}{2}\pi} S \cos\theta \, d\theta,$$

where

$$S = S_\lambda d\lambda \sin\theta \, e^{-u(o, p_1)} (1 - \sin^2\phi - \sin^2\delta - \cos^2\theta + 2 \sin\phi \sin\delta \cos\theta)^{-\frac{1}{2}},$$

and ϕ is latitude and δ declination

Now the absorption, by a spherical element, of a stream of radiation J is $Jb\int z dS$, where z is the length of path in the sphere, *i e*, it is $Jb \times$ volume of sphere But the radiation from the element is $4\pi Ib \times$ vol

The condition of radiation equilibrium is therefore

$$\int (U_\lambda + A_\lambda + B_\lambda + F_\lambda) \, b d\lambda = 4\pi \int I_\lambda b d\lambda, \qquad (I)$$

where b may vary with λ, expressing that the radiation absorbed in the spherical element must be equal to that emitted, it is included in the condition

$$\int (P_\lambda + V_\lambda) \, d\lambda = \int (Q_\lambda + G_\lambda) \, d\lambda, \qquad (II)$$

expressing that the outward and inward radiations across any horizontal plane must be equal to each other. If we take account of the scattering and reflexion of direct solar radiation, we must subtract from V_λ the average vertical component of this and add to U_λ its average intensity.

If, instead of using the spherical element, we wish to deal only with horizontal layers, we must find the absorption of radiation in such a layer Let us then consider the absorption of the radiation from a layer extending from p_2 to p_1 by a layer p_1 to p' The radiation entering the second layer in a direction inclined at an angle θ to the vertical is per unit area

$$P_\theta d\theta \, d\lambda = 2\pi \int_{p_2}^{p_1} I_\lambda d\lambda \sin\theta \cos\theta \, d\theta \frac{d}{dp} e^{-u(p, p_1)} dp,$$

and the total amount absorbed will be

$$X_\lambda d\lambda = \int_0^{\frac{1}{2}\pi} P_\theta \, d\theta \, (1 - e^{-u(p_1, p')}) d\lambda$$

Putting $\sec\theta = x$, $\sin\theta\, d\theta = dx/x^2$, we find

$$X_\lambda d\lambda = 2\pi d\lambda \int_{p_2}^{p_1}\int_1^\infty I_\lambda \frac{dx}{x^3}\frac{d}{dp} e^{-u(p,p_1)}dp\,(1-e^{-u(p_1,p')}),$$

or

$$X_\lambda = 2\pi\int_{p_2}^{p_1}\int_1^\infty I_\lambda \frac{bdx}{x^3} e^{-u(p,p_1)}dp\,(1-e^{-u(p_1,p')})$$

$$= 2\pi\int_{p_2}^{p_1}\int_1^\infty I_\lambda \frac{bdx}{x^3} e^{-u(p,p_1)}dp - 2\pi\int_{p_2}^{p'}\int_1^\infty I_\lambda \frac{bdx}{x^3} e^{-u(p,p')}dp + 2\pi\int_{p_1}^{p'}\int_1^\infty I_\lambda \frac{bdx}{x^3} e^{-u(p,p')}dp$$

Therefore $$X_\lambda = f(p_1,p_2)+f(p',p_1)-f(p',p_2),$$

where $$f(p_1,p_2) = \int_0^{\frac{1}{2}\pi} P_\theta d\theta = 2\pi\int_{p_2}^{p_1}\int_1^\infty I_\lambda \frac{bd\lambda}{x^3} e^{-u(p,p_1)}dp$$

Similarly, for the absorption of radiation from a layer p_3 to p' below the absorbing layer, we find

$$Y_\lambda = g(p_3,p')+g(p',p_1)-g(p_3,p_1),$$

where $$g(p_3p_1) = 2\pi\int_{p_1}^{p_3}\int_1^\infty I_\lambda \frac{bd.x}{x^3} e^{-u(p_1,p)}dp$$

If we put $p_2 = 0$, $p_3 = p_0$, we obtain the expressions for the total atmospheric radiation absorbed in any layer. We will use X_θ, Y_λ with this special meaning

The earth radiation absorbed in the layer is

$$E'_\lambda d\lambda = 2\pi E_\lambda d\lambda \int_1^\infty \frac{dx}{x^3}(e^{-u(p',p_0)} - e^{-u(p_1,p_0)}),$$

and the solar radiation absorbed is

$$S'_\lambda d\lambda = \frac{1}{\pi}\int_{\phi-\delta}^{\frac{1}{2}\pi}(S-S')\cos\theta\, d\theta,$$

where S' is S (*vide supra*) with p' substituted for p_1

The radiation from the layer is, per unit area,

$$\{f(p',p_1)+g(p',p_1)\}\,d\lambda = K_\lambda d\lambda, \text{ say}$$

Thus we may replace the first condition by

$$\int(X_\theta + Y_\lambda + E'_\lambda + S'_\lambda)\,d\lambda = \int K_\lambda d\lambda, \qquad \text{(III)}$$

where the integration extends to all wave-lengths. We shall need both forms of the condition *infra*.

The writer has verified analytically that I can be deduced from III and that II follows from III if we introduce the condition that the downward and upward radiations balance at the earth's surface.

If convection takes place in the gas it must always involve a flow of heat upwards from the lower layers Further, any upper layer receives at least as much heat as it loses by convection, and generally more Consequently, the effect of convection would be to add a positive term to the right-hand side of II and to the left-hand side of III, *i e*, convection is only possible if

$$\int (P_\lambda + Y_\lambda)\, d\lambda > \int (Q_\lambda + G_\lambda)\, d\lambda$$

everywhere in the actual state, and at the same time

$$\int (X_\lambda + Y_\lambda + E'_\lambda + S'_\lambda)\, d\lambda < \int K_\lambda d\lambda$$

for the upper layers

V *Application to an Atmosphere of Uniform Constitution Impossibility of Convective Equilibrium throughout such an Atmosphere*

If we have an isothermal atmosphere, we find the following expressions for A_λ, etc, on putting sec $\theta = x$ —

$$A_\lambda = 2\pi I_\lambda \left(1 - \int_1^\infty x^{-4} e^{-u(o, p_1)} dx\right), \qquad B_\lambda = 2\pi I_\lambda \left(1 - \int_1^\infty x^{-2} e^{-u(p_1, p_0)} dx\right),$$

$$P_\lambda = 2\pi I_\lambda \left(\tfrac{1}{2} - \int_1^\infty x^{-3} e^{-u(o, p_1)} dx\right), \qquad Q_\lambda = 2\pi I_\lambda \left(\tfrac{1}{2} - \int_1^\infty x^{-3} e^{-u(p_1, p_0)} dx\right),$$

$$F_\lambda = 2\pi E_\lambda \int_1^\infty x^{-2} e^{-u(p_1, p_0)} dx, \qquad G_\lambda = 2\pi E_\lambda \int_1^\infty x^{-3} e^{-u(p_1, p_0)} dx,$$

$$X_\lambda = 2\pi I_\lambda \int_1^\infty x^{-4} dx \left(1 - e^{-u(o\ p_1)} - e^{-u(p_1, p')} + e^{-u(o, p')}\right),$$

$$Y_\lambda = 2\pi I_\lambda \int_1^\infty x^{-3} dx \left(1 - e^{-u(p', p_0)} - e^{-u(p_1, p')} + e^{-u(p_1, p_0)}\right),$$

$$K_\lambda = 2\pi I_\lambda \int_1^\infty x^{-3} dx \left(2 - 2e^{-u(p_1\ p')}\right)$$

It is to be noted that F and G are independent of the temperature-distribution in the gas

Substituting in equation III, we find

$$\int S'_\lambda d\lambda + \int 2\pi I_\lambda d\lambda \int_1^\infty x^{-3} dx \left\{e^{-u(o, p')} + e^{-u(p_1, p_0)} - e^{-u(o\ p_1)} - e^{-u(p', p_0)}\right\}$$
$$+ \int 2\pi E_\lambda d\lambda \int_1^\infty \left\{e^{-u(p', p_0)} - e^{-u(p_1, p_0)}\right\} x^{-3} dx = 0,$$

or if $E_\lambda = I_\lambda$,

$$\int S'_\lambda d\lambda + \int 2\pi I_\lambda d\lambda \int_1^\infty \{e^{-u(o,p')} - e^{-u(o,p_1)}\} x^{-3} dx = 0$$

If we put $p_1 = 0$, $p' = p_0$, we see that the solar radiation absorbed in the atmosphere must be equal to the terrestrial radiation absorbed and each must be equal to one-half the radiation from the atmosphere

Further, the solar radiation absorbed in any layer of the atmosphere, starting from $p_1 = 0$, must be equal to one-half the radiation from that layer

Again, if we substitute in equation II, we find

$$\int V_\lambda d\lambda = \int 2\pi I_\lambda d\lambda \int_1^\infty x^{-2} e^{-u(o,p_1)} dx,$$

and if p_1 is small this becomes

$$\int V d\lambda = \int \pi I_\lambda d\lambda = \pi I,$$

or the temperature for the isothermal state must be such that a full radiator at that temperature would radiate with an intensity equal to the average vertical component of the intensity of solar radiation.

For a dry atmosphere in convective equilibrium we have, if θ be temperature, $\theta \propto p^n$ nearly, where $n = 1/3.5$, and consequently

$$I_\lambda = \frac{C}{\lambda^5} \Big/ \left(\frac{c}{e^{\lambda p^n} - 1}\right)$$

We cannot integrate our results for this value of I_λ, but remembering that near the maximum $I_\lambda \propto \theta^5$ and that $\int I_\lambda d\lambda \propto \theta^4$, we may put as an approximation $I \propto p$, corresponding to a temperature diminution rather slower than that for dry air in the convective state Let us also take b to be constant, *i.e.*, assume that there is no change in the constitution of the atmosphere with change of height.

We find, then, for the values of A_λ, etc, dropping subscripts,

$$A = \frac{2\pi I}{p_0} \int_1^\infty x^{-2} dx \{p_1 - \frac{1}{bx}(1 - e^{-bp_1 x})\},$$

$$B = \frac{2\pi I}{p_0} \int_1^\infty x^{-2} dx \left\{p_1 - p_0 e^{-b(p_0 - p_1)x} + \frac{1}{bx}(1 - e^{-b(p_0 - p_1)x})\right\},$$

$$P = \frac{2\pi I}{p_0} \int_1^\infty x^{-3} dx \left\{p_1 - \frac{1}{bx}(1 - e^{-bp_1 x})\right\},$$

$$Q = \frac{2\pi I}{p_0} \int_1^\infty x^{-3} dx \left\{p_1 - p_0 e^{-b(p_0 - p_1)x} + \frac{1}{bx}(1 - e^{-b(p_0 - p_1)x})\right\},$$

where I refers to the temperature at p_0.

Taking E = I and substituting in equations I and II, we find

$$\int U b d\lambda + 2\pi \int I d\lambda \int_1^\infty \frac{dx}{p_0 x^3}(e^{-bp_1 x} - e^{-b(p_0-p_1)x}) = 0 \qquad \text{(i)}$$

and

$$\int V d\lambda = \int \frac{2\pi I d\lambda}{b p_0} \int_1^\infty x^{-4} dx (2 - e^{-bp_1 x} - e^{-b(p_0-p_1)x}) \qquad \text{(ii)}$$

But if $p_1 < \frac{1}{2} p_0$, the left-hand side of (i) is always positive, and, consequently, for all altitudes at which the pressure is less than half the surface pressure, the absorption exceeds the radiation and the state of convective equilibrium could not persist Moreover, the right-hand side of (ii) has the same value at both limits of the atmosphere, and therefore there could be no absorption of solar radiation in an atmosphere of uniform constitution in convective equilibrium

The values of X, Y, K in this case are as follows —

$$X = \frac{2\pi I}{p_0} \int_1^\infty x^{-3} dx \left\{ p_1 (1 - e^{-b(p'-p_1)x}) - \frac{1}{bx}(1 - e^{-bp_1 x} + e^{-bp'x} - e^{-b(p'-p_1)x}) \right\}$$

$$Y = \frac{2\pi I}{p_0} \int_1^\infty x^{-3} dx \left\{ p'(1 - e^{-b(p'-p_1)x}) + \frac{1}{bx}(1 - e^{-b(p'-p_1)x} - e^{-b(p_0-p')x} + e^{-b(p_0-p_1)x} + p_0(e^{-b(p_0-p_1)x} - e^{-b(p_0-p')x}) \right\}$$

$$K = \frac{2\pi I}{p_0} \int_1^\infty x^{-3} dx (p_1 + p')(1 - e^{-b(p'-p_1)x})$$

Therefore, putting E = I, we get for (X + Y + E − K) the value

$$\frac{2\pi I}{p_0} \int_1^\infty bx^{-4} dx \{ e^{-bp_1 x} - e^{-bp'x} + e^{-b(p_0-p_1)x} - e^{-b(p_0-p')x} \}$$

or

$$\frac{2\pi I}{p_0} \int_1^\infty bx^{-4} dx \{ e^{-b(p_1+p')x} - e^{-bp_0 x} \} \{ e^{bp'x} - e^{bp_1 x} \}$$

Now this represents the excess of the absorption over the radiation for any layer, apart from absorption of solar radiation. But by hypothesis $p' > p_1$, and therefore any layer for which $p_1 + p' < p_0$ absorbs more radiation than it emits.

Clearly, if the radiation decreases with height at a rate faster than that assumed, the decrease in the radiation received by any layer will be less than the decrease in that emitted by the layer, so that in the upper half of the atmosphere the absorption would exceed the radiation and the temperature would rise. Moreover, if the temperature of the upper layers rises, the absorption of radiation near $p_1 = \frac{1}{2} p_0$ will also rise, and therefore the rise in temperature must extend at least as far as $p_1 = \frac{1}{2} p_0$ if the lower layers are to be in convective equilibrium Therefore, in an atmosphere of uniform

constitution, the rate of temperature diminution corresponding to convective equilibrium cannot be maintained to a height greater than that for which the pressure is half the surface pressure

VI *Application to the Earth's Atmosphere, taking into account the Diminution of Water-vapour with Height Limits to which Convective Equilibrium can subsist*

We proceed to take into account the fact that in the atmosphere itself there is a rapid diminution in the proportion of water-vapour present as the height increases There is therefore a decrease in the value of b with increasing height We may represent this approximately by taking b equal to $a/(q-p)$, where a and q are constants We get, then,

$$A_\lambda = 2\pi \int_0^{p_1} I_\lambda dp \int_1^\infty \frac{d\lambda}{\nu^2} \frac{d}{dp}\left(\frac{q-p_1}{q-p}\right)^{a\nu}, \qquad B_\lambda = 2\pi \int_{p_1}^{p_0} I_\lambda dp \int_1^\infty \frac{dx}{\nu^2} \frac{d}{dp}\left(\frac{q-p}{q-p_1}\right)^{a\nu},$$

and similar expressions for P_λ, Q_λ, while

$$F_\lambda = 2\pi \int_1^\infty E_\lambda \frac{dx}{x^2}\left(\frac{q-p_0}{q-p_1}\right)^{a\nu}, \qquad G_\lambda = 2\pi \int_1^\infty E_\lambda \frac{d\nu}{\nu^2}\left(\frac{q-p_0}{q-p_1}\right)^{a\nu}$$

If I_λ is constant,

$$A_\lambda = 2\pi I_\lambda \int_1^\infty \frac{d\nu}{\nu^2}\left\{1-\left(\frac{q-p_1}{q}\right)^{a\nu}\right\}, \qquad B_\lambda = 2\pi I_\lambda \int_1^\infty \frac{dx}{x^2}\left\{1-\left(\frac{q-p_0}{q-p_1}\right)^{ax}\right\},$$

with similar expressions for P and Q

Writing $k_1 = \log_e \frac{q}{q-p_1}$, $k_2 = \log_e \frac{q-p_1}{q-p_0}$, and taking $I_\lambda \propto p$, we get

$$A_\lambda = \frac{2\pi I_\lambda}{p_0} \int_1^\infty \frac{dx}{x^2}\left\{p_1 - \frac{q-p_1}{a\nu-1}(1-e^{-k_1(a\nu-1)})\right\},$$

$$B_\lambda = \frac{2\pi I_\lambda}{p_0} \int_1^\infty \frac{dx}{x^2}\left\{p_1 - p_0 e^{-k_2 ax} + \frac{q-p_1}{ax+1}(1-e^{-k_1(ax+1)})\right\},$$

and similar expressions for P_λ, Q_λ, while

$$F_\lambda = 2\pi E_\lambda \int_1^\infty \frac{dx}{x^2} e^{-k_2 ax}, \quad G_\lambda = 2\pi E_\lambda \int_1^\infty \frac{d\nu}{x^2} e^{-k_2 ax}$$

Substituting in equation (I), we get, after reduction,

$$\int U_\lambda b d\lambda + \int \frac{2\pi I_\lambda}{p_0} \int_1^\infty \frac{dx}{x^2}\left\{\frac{q-p_1}{ax+1}(1-e^{-k_2(ax+1)}) - \frac{q-p_1}{ax-1}(1-e^{-k_1(ax-1)})\right\} = 0$$

Now near $p_1 = 0$,

$$k_1 = 0, \text{ and } \int_1^\infty \frac{dx}{x^2} \frac{q-p_1}{ax-1}(1-e^{-k_1(ax-1)}) = 0,$$

so that the term involving I_λ becomes positive, indicating that, even apart

from absorption of solar radiation, there is an excess of absorption over radiation The upper layers would therefore be warmed up, so that with the modified value of b also the state $I \propto p$ cannot exist throughout the atmosphere

To find X_λ, Y_λ we notice that

$$f(p_1 p_\lambda) = \frac{2\pi I_\lambda}{p_0} \int_1^\infty \frac{dx}{x^3} \left\{ p_1 - p_2 \left(\frac{q-p_1}{q-p_2}\right)^{\alpha x} - \frac{q-p_1}{\alpha x - 1}\left[1 - \left(\frac{q-p_1}{q-p_2}\right)^{\alpha x - 1}\right]\right\}$$

and

$$q(p_2 p_1) = \frac{2\pi I_\lambda}{p_0} \int_1^\infty \frac{dx}{x^3} \left\{ p_1 - p_3 \left(\frac{q-p_3}{q-p_1}\right)^{\alpha x} - \frac{q-p_{11}}{\alpha x + 1}\left[1 - \left(\frac{q-p_3}{q-p_1}\right)^{\alpha x + 1}\right]\right\}$$

Substituting these values, we find

$$X_\lambda + Y_\lambda + E'_\lambda - K_\lambda = \frac{2\pi I_\lambda}{p_0} \int_1^\infty \frac{dx}{x^3} \left\{ \frac{q-p'}{\alpha x - 1}\left[1 - \left(\frac{q-p'}{q}\right)^{\alpha x - 1}\right] + \frac{q-p_1}{\alpha x + 1}\left[1 - \left(\frac{q-p_0}{q-p_1}\right)^{\alpha x + 1}\right] - \frac{q-p_1}{\alpha x - 1}\left[1 - \left(\frac{q-p_1}{q}\right)^{\alpha x + 1}\right] - \frac{q-p'}{\alpha x + 1}\left[1 - \left(\frac{q-p_0}{q-p'}\right)^{\alpha x + 1}\right]\right\}$$

If we put $p_1 = 0$, this gives the excess of absorption over radiation in the atmospheric layer from the outer limit to a place at pressure p' On simplifying, we find that the sign of the integrand is always positive Thus, under the condition $I_\lambda \propto p$, any such layer of the atmosphere would absorb more radiation than it emitted, even apart from solar radiation It is therefore certain that if in the lower part of the atmosphere $I_\lambda \propto p$, in the upper part the temperature must be considerably greater than would correspond to such a radiation law

If we put $p' = p_0$, we get for the excess of absorption over radiation in the layer extending from the earth to a point at pressure p_1,

$$\frac{2\pi I_\lambda}{p_0} \int_1^\infty \frac{dx}{x^3} \left\{ \frac{q-p_1}{\alpha x - 1}\left[1 - \left(\frac{q-p_0}{q}\right)^{\alpha x - 1}\right] + \frac{q-p_1}{\alpha x + 1}\left[1 - \left(\frac{q-p_0}{q-p_1}\right)^{\alpha x + 1}\right] - \frac{q-p_1}{\alpha x - 1}\left[1 - \left(\frac{q-p_1}{q}\right)^{\alpha x - 1}\right]\right\}$$

and this must be negative if the state $I \propto p$ is to hold up to p_1 Also, if it is negative, its value must not be greater than the absorbed solar radiation plus the energy convected from the earth's surface

The values of q and α will vary for different wave-lengths, but to obtain an approximate result we may take mean values for these quantities We therefore put $q = \frac{2}{3}p_0$, which gives a diminution in the absorption more rapid than the observed diminution in the proportion of water-vapour present We will consider also the case $q = \frac{1}{4}p_0$, for which the rate of decrease is slower than that of water-vapour.

The intensity of the transmitted radiation for a vertical path bears to the initial intensity the ratio $\{(q-p_0)/q\}^{\alpha}$, or for the values of q taken above, $(\frac{1}{3})^{\alpha}$ and $(\frac{1}{5})^{\alpha}$ respectively

Langley* estimates the transmitted radiation to be 0 806 and 0 75 to 0 96, but states that his estimate is probably too high

Lowell† takes the absorption of terrestrial radiation to be 0 5, and is followed by Poynting‡

(i) Let us assume that 25 per cent. of the radiation is transmitted freely without absorption, and that two-thirds of the remainder is absorbed in vertical transmission This gives $\alpha = 0\,5$ and 0 68 for the values of q taken

(ii) Let us take the absorption to extend throughout the spectrum and to be such that at the surface 100 metres of air of average humidity absorbs 20 per cent of the low temperature radiation passing through it, in accordance with Langley's observations This gives $\alpha = 2$ and 4 for the two values of q

For the purpose of calculation we use the following table, giving the values of

$$\int_1^{\infty} x^{-n}e^{-tx}dx = \mathrm{H}_n \quad \text{for} \quad n = 1, 2, 3$$

and different values of t The values for $n = 1$ are given by J W L. Glaisher§ —

t	H_1	H_2	H_3	t	H_1	H_2	H_3
0·01	4·03793	0 94967	0 49027	0 90	0 26018	0 17241	0 12570
0·02	3 35471	0 91311	0 48097	0 95	0 23874	0 15994	0 11740
0·03	2·95912	0 88168	0 47200	1·0	0 21938	0·14850	0 10969
0·04	2·68126	0 85354	0 46332	1 1	0 18599	0 12828	0·09589
0·05	2 46790	0 82784	0 45492	1 2	0 15841	0 11110	0 08394
0·06	2 29531	0 80404	0 44676	1 3	0 13545	0 09644	0 07358
0 07	2 15084	0 78184	0 43883	1 4	0 11622	0·08389	0·06458
0·08	2·02694	0 76096	0 43112	1 5	0 10002	0 07310	0·05674
0·09	1·91874	0 74125	0 42361	1 6	0·08631	0·06380	0 04991
0 10	1 82292	0 72255	0 41629	1 7	0·07465	0·05578	0·04393
0 15	1 46446	0 64104	0 38227	1 8	0·06471	0·04882	0·03871
0 20	1 22265	0 57420	0 35195	1·9	0·05620	0·04279	0·03413
0 25	1 04428	0 51773	0 32468	2·0	0 04890	0 03754	0·03013
0 30	0·90568	0 46912	0 30004	2 2	0·03719	0 02898	0·02352
0 35	0 79422	0 42671	0 27767	2 4	0·02844	0 02246	0·01841
0 40	0·70238	0 38937	0 25728	2 5	0 02491	0·01962	0·01627
0 45	0 62533	0 35623	0 23866	2 6	0·02185	0·01746	0·01444
0 50	0 55977	0 32665	0 22160	2·8	0·01686	0·01360	0·01136
0 55	0 50336	0 30010	0 20595	3 0	0·01305	0·01064	0·00894
0 60	0 45438	0 27619	0 19155	3·5	0·00697	0·00580	0·00495
0 65	0 41152	0 25456	0 17829	4·0	0·00378	0·00320	0 00276
0 70	0 37377	0 23495	0 16606	4 5	0·00207	0·00179	0 00158
0 75	0 34034	0 21711	0 15477	5·0	0·00115	0·00099	0·00090
0 80	0 31060	0 20085	0 14432	6 0	0·00036	0·00032	0·00028
0·85	0 28402	0 18599	0 13466				

* "Temperature of the Moon," 'Memoirs Nat Acad Washington,' vol. 4, pp. 38, 184.

† 'Phil Mag.,' July, 1907 ‡ *Ibid*, November, 1907.

§ 'Phil Trans,' vol 160, 1870

We observe also that

$$\frac{1}{x^3(\alpha x+1)} = \frac{1}{x^3} - \frac{\alpha}{x^2} + \frac{\alpha^2}{x} - \frac{\alpha^3}{x+c}, \quad \text{where} \quad c = \frac{1}{\alpha}$$

and $$\int_1^\infty \frac{e^{-k(\alpha x+1)}dx}{x+c} = \int_1^\infty \frac{e^{-kx(1+\alpha)}}{x}dx, \qquad \int_1^\infty \left(\frac{1}{x} - \frac{1}{x+c}\right) = \log(1+c)$$

We have, too, $$\frac{1}{x^3(\alpha x-1)} = -\frac{1}{x^3} - \frac{\alpha}{x^2} - \frac{\alpha^2}{x} + \frac{\alpha^3}{x-c},$$

and if $\alpha > 1$,

$$\int_1^\infty \frac{e^{-k(\alpha x-1)}}{x-c}dx = \int_1^\infty \frac{e^{-kx(\alpha-1)}}{x}dx \quad \text{and} \quad \int_1^\infty \left(\frac{1}{x} - \frac{1}{x-c}\right)dx = \log(1-c)$$

But if $\alpha < 1$, we have

$$\int_1^\infty \left(\frac{1}{x-c} - \frac{1}{x}\right)\left[1-e^{-k\alpha(x-c)}\right]dx = \underset{\epsilon=0}{Lt}\left\{\int_1^{c-\epsilon} + \int_{c+\epsilon}^\infty\right\} dx \left(\frac{1}{x-c} - \frac{1}{x}\right)\left[1-e^{-k\alpha(x-c)}\right]$$

$$= \underset{\epsilon=0}{Lt}\left[-\log\frac{\epsilon}{c+\epsilon} + \log\frac{\epsilon}{c-\epsilon} - \log(c-1) - \int_1^{c-\epsilon}\frac{e^{-k\alpha(x-c)}dx}{x-c} - \int_{c+\epsilon}^\infty \frac{e^{-k\alpha(x-c)}dx}{x-c}\right.$$

$$\left. + \int_1^\infty x^{-1}e^{-k\alpha(x-c)}dx\right]$$

$$= -\log(c-1) - \underset{\epsilon=0}{Lt}\left[\log\epsilon - \log(c-1) - k\alpha(c-\epsilon-1) - \frac{k^2\alpha^2}{2\ 2!}\{(c-1)^2 - \epsilon^2\}\right.$$

$$\left. - \quad + \int_\epsilon^\infty x^{-1}e^{-k\alpha x}dx + \int_1^\infty x^{-1}e^{-k\alpha(x-c)}dx\right]$$

$$= -\log(c-1) + \log(c-1) + C + \log k\alpha + k\alpha(c-1) + \quad . + \int_1^\infty x^{-1}e^{-k\alpha(x-c)}dx,$$

since $\underset{\delta=0}{Lt}\left(\log\delta + \int_\delta^\infty t^{-1}e^{-t}dt\right) = -C$, where C is Euler's constant *

Thus putting again α^{-1} for c, we find for the value of the integral

$$C + \log k\alpha + k(1-\alpha) + \frac{k^2}{2}\frac{(1-\alpha)^2}{2!} + \quad + \int_1^\infty x^{-1}e^{-k(\alpha x-1)}dx$$

We denote by P, Q, etc, the values of $\int P\lambda d\lambda$, etc, and we calculate values for I constant and for $I \propto p$ The values are given in order corresponding to the four values of α, *i.e.*, 0 5, 2 0, 0 68, 4 0, the first pair corresponds to $q = \frac{8}{17}p_0$, and the second pair to $q = \frac{4}{7}p_0$

At $p = 0$, $k_1 = 0$, $k_2 = \log_e 9$ or $\log_e 5 = 2\ 2$ or 1·61.

For constant I,

$$P = 0, \quad Q = 0\ 61\pi I, \ \pi I, \ 0\ 61\pi I, \ \pi I.$$

For $I \propto p$,

$$P = 0, \quad A = 0$$

$$Q = 0\ 32\pi I, \ 0\ 26\pi I, \ 0\ 27\pi I, \ 0\ 18\pi I,$$

$$B = 0{\cdot}55\pi I, \ 0\ 42\pi I, \ 0\ 51\pi I, \ 0\ 26\pi I$$

* Bromwich, 'Infinite Series,' p. 460

We notice here a somewhat surprising result Since the value of b in case (4)—*i e*, $4/(\frac{4}{3}p_0-p)$—is always greater than the value for the corresponding pressure in case (2)—*i e*, $2/(\frac{2}{3}p_0-p)$,—the radiation from any element of an atmosphere corresponding to case (4) is greater than the radiation for the corresponding element in case (2) Yet the outward radiation from the whole atmosphere in case (4) is considerably less than in case (2)

The values of F and G for the four cases are given by

$$F = 0\,26\pi E,\ 0\,00\pi E,\ 0\,26\pi E,\ 0\,00\pi E,$$
$$G = 0\,39\pi E,\ 0\,00\pi E,\ 0\,39\pi E,\ 0\,00\pi E$$

Now the values of $(F+A+B)$ at $p = 0$ when $I \propto p$ are less than for any other part of the atmosphere for any possible temperature distribution with the same surface temperature. But the radiation from any element must be greater than the amount of $(F+A+B)$ absorbed Thus if $E = I$ and we remember that in cases (1), (3) only 75 per cent of the radiation suffers absorption, we find as lower limits for I′,

$$0{\cdot}19\,I,\ 0\,11\,I,\ 0{\cdot}17\,I,\ 0\,07\,I$$

where I′ is the radiation intensity of a full radiator at the temperature of the element

If I correspond to a temperature 300° A, the lower limits for the temperature at any point in the atmosphere are therefore

$$198°,\ 173°,\ 193°,\ 154°\ A.$$

At the earth's surface, $p = p_0$, $k_1 = 2\,2$ or $1\,61$ and $k_2 = 0$

For constant I,

$$P = 0\,61\pi I,\ \pi I,\ 0\,61\pi I,\ \pi I,\ \text{and } Q = 0$$

For $I \propto p$,

$$P = 0\,49\pi I,\ 0{\cdot}93\pi I,\ 0\,47\pi I,\ 0\,95\pi I,\ \text{and } Q = 0$$

At $p = \frac{1}{2}p_0$, $k_1 = 0\,6$ or $0\,5$, $k_2 = 1\,61$ or $1\,1$

For constant I,

$$P = 0\,30\pi I,\ 0\,83\pi I,\ 0\,33\pi I,\ 0\,94\pi I,$$
$$Q = 0\,54\pi I,\ 0\,98\pi I,\ 0\,52\pi I,\ 1\,00\pi I$$

For $I \propto p$,

$$P = 0\,10\pi I,\ 0\,30\pi I,\ 0\,15\pi I,\ 0\,34\pi I,$$
$$Q = 0\,40\pi I,\ 0\,63\pi I,\ 0\,38\pi I,\ 0\,60\pi I$$

Also if P_1 denote the radiation to earth from the layer extending up to $\frac{1}{2}p_0$ and Q_1, the radiation to space of the upper half of the atmosphere, we find for $I \propto p$,

$$P_1 = 0\,47\pi I,\ 0\,93\pi I;\ 0\,45\pi I,\ 0\,95\pi I,$$
$$Q_1 = 0\,08\pi I,\ 0\,16\pi I,\ 0\,08\pi I,\ 0{\cdot}15\pi I$$

By applying the results of § IV for $_{\lambda}Y_{\lambda}$ we find for the atmospheric radiation absorbed in this upper layer,

$$X_1 = 0\,16\pi I,\ 0\,53\pi I,\ 0\,19\pi I,\ 0\,57\pi I$$

The terrestrial radiation absorbed in the layer is

$$E_1 = 0\,07\pi E,\ 0\,02\pi E,\ 0\,09\pi E,\ 0\,00\pi E$$

Hence if $E = I$, the total absorption, apart from that of solar radiation, is

$$0\,23\pi I,\ 0\,55\pi I,\ 0\,28\pi I,\ 0\,57\pi I.$$

The radiation from the layer in case $I \propto p$ is

$$0\,10\pi I,\ 0\,30\pi I,\ 0\,15\pi I,\ 0\,34\pi I \text{ downwards,}$$

$$0\,08\pi I,\ 0\,16\pi I,\ 0\,08\pi I,\ 0\,15\pi I \text{ upwards,}$$

or $$0\,18\pi I,\ 0\,46\pi I,\ 0\,23\pi I,\ 0\,49\pi I \text{ in all.}$$

As we expect, the total radiation is less than the total absorption If the layer above $\frac{1}{2}p_0$ were isothermal the amount of absorption by it would be the same, but the radiation would be in each direction,

$$0\,30\pi I_1,\ 0\,83\pi I_1,\ 0\,33\pi I_1,\ 0\,94\pi I_1,$$

where I_1 is the intensity of radiation corresponding to the temperature of the layer, *i e* is $\frac{1}{2}I$, since $I \propto p$ in the lower layer

The radiation would therefore exceed the absorption by

$$0\,07\pi I,\ 0\,28\pi I,\ 0\,05\pi I,\ 0\,37\pi I$$

These amounts are too large to be supplied by the absorption of solar radiation in the upper half of the atmosphere, and we conclude that we could not have the isothermal state for so large a part of the upper atmosphere If such a state existed at any time the temperature of the layer would fall, and this would allow the convection currents of the lower atmosphere to penetrate to greater heights and establish the state $I \propto p$ in the lower part of the layer

We proceed to consider the case when the division is taken to be at $p = \frac{1}{4}p_0$ For $p = \frac{1}{4}p_0$,

$$k_1 = 0\,251 \text{ or } 0\,222,\ k_2 = 1\,95 \text{ or } 1\,39$$

For constant I,

$$P = 0\,15\pi I,\ 0\,56\pi I,\ 0\,18\pi I,\ 0\,75\pi I,$$

$$Q = 0\,58\pi I,\ 0\,99\pi I,\ 0\,57\pi I,\ 1\,00\pi I.$$

For $I \propto p$,

$$P = 0{\cdot}015\pi I,\ 0\,08\pi I,\ 0\,02\pi I,\ 0\,09\pi I,$$

$$Q = 0\,37\ \pi I,\ 0\,46\pi I,\ 0{\cdot}34\pi I,\ 0\,39\pi I$$

If P_2 denote the radiation to earth of the layer $\frac{1}{4}p_0$ to p_0 and Q_2 the radiation to space of the layer $\frac{1}{4}p_0$ to 0, we find

$$P_2 = 0\,48\pi I,\ 0\,93\pi I,\ 0\,47\pi I,\ 0\,95\pi I,$$

$$Q_2 = 0\,02\pi I,\ 0\,06\pi I,\ 0\,02\pi I,\ 0\,11\pi I$$

We find as above for the atmospheric and terrestrial radiation absorbed in the upper layer,

$$X_2 = 0\,07\pi I,\ 0\,26\pi I,\ 0\,09\pi I,\ 0\,32\pi I,$$
$$E_2 = 0\,03\pi E,\ 0\,01\pi E,\ 0\,04\pi E,\ 0\,00\pi E,$$

giving for the total absorption, apart from that of solar radiation if $E = I$,

$$0\,10\pi I,\ 0\,27\pi I,\ 0\,13\pi I,\ 0\,32\pi I$$

The radiation from the layer if the law $I \propto p$ still held would be

$$0\,035\pi I,\ 0\,14\pi I,\ 0\,04\pi I,\ 0\,20\pi I,$$

which is much less than the absorption

If this layer were isothermal, the radiation from it in each direction would be

$$0\,15\pi I_c,\ 0\,56\pi I_c,\ 0\,18\pi I_c,\ 0\,75\pi I_c,$$

where I_c corresponds to the temperature of the layer, *i.e.*, $I_c = \frac{1}{4}I$

The radiation is therefore

$$0\,07\pi I,\ 0\,28\pi I,\ 0\,09\pi I,\ 0\,37\pi I,$$

and does not exceed the absorption apart from that of solar radiation. We conclude that if the outer layer of the atmosphere is isothermal it must extend at least until the layer $p = \frac{1}{4}p_0$ is reached It appears also that the greater the absorbing power of the atmosphere for terrestrial radiation the greater will be the height at which the isothermal condition begins, apart from other considerations

The radiation from the layer p_0 to $\frac{1}{4}p_0$ is

$$0\,48\pi I,\ 0\,93\pi I,\ 0\,47\pi I,\ 0\,95\pi I \text{ down},$$

$$0\,37\pi I,\ 0\,46\pi I,\ 0{\cdot}34\pi I,\ 0\,39\pi I \text{ up},$$

or $$0\,85\pi I,\ 1\,39\pi I,\ 0\,81\pi I,\ 1\,34\pi I \text{ in all.}$$

The absorption in it is from the outer layer,

$$0{\cdot}12\pi I_c,\ 0\,55\pi I_c,\ 0\,14\pi I_c,\ 0\,75\pi I_c,$$

and $$0\,58\pi E,\ 0\,99\pi E,\ 0\,57\pi E,\ 1\,00\pi E \text{ from the earth,}$$

or $$0\,61\pi I,\ 1\,13\pi I,\ 0\,61\pi I,\ 1\,19\pi I \text{ if } E = I$$

The radiation therefore exceeds the absorption in this layer by

$$0\,24\pi I,\ 0\,26\pi I,\ 0\,20\pi I,\ 0{\cdot}15\pi I,$$

and this must be made up by absorption of solar radiation and by convection of energy from the earth's surface either in the form of warmed air or water vapour

If we take the mean value to be $0\,21\pi I$, and assume that solar radiation and convection are absent, and that in consequence the temperature of the layer falls uniformly, the fall per minute is $\frac{0\,21\times 0\cdot 6J}{57\times 13\,6\times 0\,24}$, or about 1° C in 24 hours if I correspond to a temperature 300° A The radiation to earth from the atmosphere if $I \propto p$ up to $p = \frac{1}{2}p_0$, and is constant afterwards, is

$$0\,51\pi I,\ 0\,94\pi I,\ 0\,51\pi I,\ 0\,95\pi I$$

From measurements made at Zurich by Maurer,* it was found that the radiation from the atmosphere was 0 37 calories per minute, the air temperature being 15° C, and the height of the observing station being 440 metres above M S L This gives a down radiation equal to $0\,70\pi I$

Similar measurements made at Rauris (950 metres), at a temperature of −6° C, gave for the atmospheric radiation 0 21 calories per minute or $0\,41\pi I$

Very† found values for the effective temperature of the sky from radiation measurements varying from −25° C to −60° C With a cirrus haze he found −15° C If we take a mean value −40° C, we get for the ratio of sky radiation to that from a black body at temperature 15° C, the value 0 42

These values agree best with the values obtained on assumption (1) regarding the absorbing power of the atmosphere

The radiation from the layer p_0 to $\frac{1}{2}p_0$ is

$$0\,87\pi I,\ 1\,56\pi I,\ 0\,83\pi I,\ 1\,55\pi I$$

The radiation absorbed comes from the earth, from the layer $\frac{1}{2}p_0$ to $\frac{1}{4}p_0$ and from the outer layer $\frac{1}{4}p_0$ to 0

The amounts from these three sources are respectively

$$0\,54\pi E,\ 0\,98\pi E,\ 0\,52\pi E,\ 1\,00\pi E,$$

$$0\,08\pi I,\ 0\,28\pi I,\ 0\,12\pi I,\ 0\,33\pi I$$

and

$$0\,07\pi I_c,\ 0\,16\pi I_c,\ 0\,03\pi I_c,\ 0\,12\pi I_c.$$

Thus if $E = I$, the total absorption in the layer p_0 to $\frac{1}{2}p_0$ is

$$0\,64\pi I,\ 1\,30\pi I,\ 0\,65\pi I,\ 1\,36\pi I$$

The balance of energy to be supplied by solar radiation and convection is

$$0\,23\pi I,\ 0\,26\pi I,\ 0\,18\pi I,\ 0\,19\pi I$$

* Hann, 'Lehrbuch der Meteorologie,' p 36.

† "Temperature of the Moon."

By subtracting these values from those found for the more extensive layer p_0 to $\frac{1}{4}p_0$, we get the energy to be supplied by convection and solar radiation to the layer $\frac{1}{2}p_0$ to $\frac{1}{4}p_0$, *i e*,

$$0\,01\pi I,\ 0\,00\pi I,\ 0\,02\pi I,\ -0\,04\pi I$$

These amounts could be supplied by absorption of solar radiation alone, so that there will be little vertical convection above $\frac{1}{4}p_0$ In fact, in case (4) the layer is absorbing more energy than it radiates

The outward radiation across the outer limit of the atmosphere in the case when the layer above $\frac{1}{4}p_0$ is isothermal is

$$0\,15\pi I_e + 0\,39\pi E + 0\,32\pi I - 0\,02\pi I,$$
$$0\,56\pi I_e + 0\,00\pi E + 0\,26\pi I - 0\,08\pi I,$$
$$0\,18\pi I_e + 0\,39\pi E + 0\,27\pi I - 0\,02\pi I,$$
$$0\,75\pi I_e + 0\,00\pi E + 0\,18\pi I - 0\,09\pi I,$$

or if $E = I$,

$$0\,73\pi I,\ 0\,32\pi I,\ 0\,68\pi I,\ 0\,28\pi I$$

But this radiation must be equal to the difference between the incident and the reflected solar radiation, including in the reflected, the part of the diffused radiation which is returned to space, say one-half According to Abbott and Fowle,* clouds reflect 65 per cent. of the solar radiation We may take the amount of diffused radiation to be 40 per cent † or to the cloud level 16 per cent Thus, if there were no clouds present the incident solar radiation which the above radiations ought to balance would be 80 per cent of the total incident solar radiation, and if clouds were present the amount would be

$$100\,(1 - \tfrac{1}{2} \times 0\,16 - 0\,84 \times 0\,65 + \tfrac{1}{2} \times 0\,16 \times 0\,84 \times 0\,65) \text{ per cent} = 41 \text{ per cent},$$

so that if the average intensity of solar radiation $= \pi I$, the temperature of the upper layer in the cases of greater absorption must be higher than that corresponding to the isothermal state from $\frac{1}{4}p_0$ upwards

The presence of clouds will not materially affect the terrestrial and atmospheric radiation, since they reflect it but little and if they absorb they will also radiate for the same wave-lengths

VII *Application to the Consideration of the Day and Night Temperatures of the Earth's Surface*

If we would apply our results to a consideration of the day and night

* 'Annals of Astrophysical Observatory of the Smithsonian Institution,' 1908, p. 144

† *Ibid.*, p. 129.

temperature of the earth's surface as Poynting* has done, we find, if we neglect conduction,

$$E_{max} = Q + tS - E', \qquad E_{min} = Q - E'_1,$$

where E_{max}, E_{min} are the earth-radiations at the time of maximum and minimum temperature, Q is the downward atmospheric radiation, and E', E'_1 are the rates at which energy is being convected from the surface or is being lost by evaporation E'_1 is probably small and may be positive or negative, but E' will be considerable To produce an annual rainfall of 200 cm, which we may take as the value for the equatorial zone, the rate at which energy is being lost by evaporation is 0 22 calories per minute per cm^2 If the process goes on only during the day, the average rate is 0 44 calories per minute, and it seems fair to take the rate at the time of maximum temperature to be at least twice the average rate, so that we may put E' = 0 9 calories per minute

Putting† tS = 1 4 and taking the mean air temperature to be 300° A, so that $\pi I = 0{\cdot}62$, we get

$$E_{max} = 0\,51\pi I + tS - E' = 0{\cdot}32 + 1\,4 - 0\,9 = 0\,82 \qquad (i)$$

or $$= 0\,95\pi I + tS - E' = 0\,59 + 1\,4 - 0\,9 = 1\,09 \qquad (ii)$$

and $$E_{min} = 0\,51\pi I = 0\,32 \qquad (i)$$

or $$= 0\,95\pi I = 0\,59 \qquad (ii)$$

the two cases corresponding to the two assumptions as to the absorbing power of the atmosphere

The corresponding temperatures would be

$$\theta_{max} = 322° \text{ A} \quad \text{or} \quad 346° \text{ A}.$$

$$\theta_{min} = 254° \text{ A} \quad \text{or} \quad 296° \text{ A}$$

At an altitude of 5 5 kilometres, corresponding to a pressure equal to half the surface pressure, we have

$$Q = 0\,14\pi I \qquad (i)$$

or $$= 0\,35\pi I \qquad (ii)$$

in the two cases

Further, tS = 1 7 nearly, and in order to obtain an approximate value for E', let us put it equal to $x(\theta - T)$, where T is the mean air temperature The value of x will be 0 04 or 0 02, assuming that the surface values are correct

The equations for θ_{max} then become

$$\alpha\theta^4_{max} = 0\,09 + 1\,7 - 0\,04(\theta_{max} - T) \qquad (i)$$

* 'Phil Mag,' December, 1907

† See Angström, 'Met. Zeit.,' 1901, p 187.

or $$\alpha\theta^4_{max} = 0\,22 + 1\,7 - 0\,02\,(\theta_{max} - T),^* \qquad \text{(ii)}$$

where $T = 252°$ A, corresponding to a sea-level temperature of 300° A. These give

$$\theta_{max} = 284° \text{ A (i)} \quad \text{or} \quad 312° \text{ A (ii)},$$

and the value of E′ is 1 2 nearly.

The values of θ_{min} are 184° A. (i) and 231° A. (ii).

These results indicate that the effect of solar radiation would be to set up convective streams in the atmosphere for a considerable height above an elevated plateau. Thus in case (i), at the time of the maximum, it would be necessary to ascend 3 5 kilometres with an adiabatic gradient before reaching a place where the temperature was equal to the mean daily temperature at the surface, and the effect of radiation in the atmosphere would not be in the direction of destroying the adiabatic fall, since this fall would be in the lower layers of the atmosphere over the plateau, and would, in fact, be consistent with radiation equilibrium to a height at least equal to that at which the pressure was equal to half the pressure at the surface of the plateau.

Note on the Effect of Hydrogen on the Discharge of Negative Electricity from Hot Platinum.

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(Received October 6,—Read November 19, 1908)

In a recent paper on "The Effect of Hydrogen on the Discharge of Negative Electricity from Hot Platinum,"* I gave a calculation of the thickness of the double layer on the surface and of the number of free electrons inside the platinum Professor O W Richardson has pointed out to me that two terms in one of the equations, one of which I discarded as being small compared with the other, are really of the same order of magnitude The results of the calculation are consequently wrong, and the estimate of the number of free electrons is considerably too high The difficulty mentioned in the paper, that the energy required to raise the temperature of the electrons is apparently greater than that required to raise the temperature of the platinum, consequently disappears

In the equation†

$$V = -\beta \log\left(1+\frac{4\pi\sigma^2}{\rho_0\beta}e^{-4\pi\sigma t/\beta}+\frac{\rho_0\beta}{16\pi\sigma^2}e^{+4\pi\sigma t/\beta}\right)$$
$$+\beta \log\left(1+\frac{4\pi\sigma^2}{\rho_0\beta}+\frac{\rho_0\beta}{16\pi\sigma^2}\right)$$

the terms $\frac{\rho_0\beta}{16\pi\sigma^2}e^{4\pi\sigma t/\beta}(=10^{-14})$ and 1 are quite negligible compared with $\frac{4\pi\sigma^2}{\rho_0\beta}e^{-4\pi\sigma t/\beta}(=10^{14})$, so that we get

$$V = -\beta \log\frac{4\pi\sigma^2}{\rho_0\beta}e^{-4\pi\sigma t/\beta}+\beta \log\left(1+\frac{4\pi\sigma^2}{\rho_0\beta}+\frac{\rho_0\beta}{16\pi\sigma^2}\right),$$

which gives, without further approximation,

$$V = 4\pi\sigma t+2\beta \log\left(1+\frac{\rho_0\beta}{8\pi\sigma^2}\right)$$

Substituting $R = NVe/J$, $Q = 4\pi\sigma te N/J$, and $\beta = \beta_0\theta$, this gives

$$R = Q+\frac{2\beta_0\theta e N}{J}\log\left(1+\frac{2\rho_0\beta_0\theta\pi t^2e^2N^2}{Q^2J^2}\right)$$

Comparing this with $R = Q+2\theta \log D/A$, we get

$$\log\frac{D}{A} = \frac{\beta_0 eN}{J}\log\left(1+\frac{2\pi\rho_0\beta_0\theta t^2e^2N^2}{Q^2J^2}\right).$$

* 'Phil. Trans.,' vol. 208, A 432

† *Loc cit*, p. 270

Now, since $p = -\rho\beta_0\theta = -ne\beta_0\theta$, we see that $-\beta_0 eN$ is equal to the gas constant $R = 2J$, hence $\beta_0 eN/J = -2$ Hence

$$t^2 = \frac{Q^2\beta_0}{8\pi\rho_0\theta}\left\{\left(\frac{A}{D}\right)^{\frac{1}{2}} - 1\right\}$$

If we take two values of Q, Q_1, and Q_2, and the corresponding values A_1 and A_2, we get

$$D = \left\{\frac{Q_1^2 A_1^{\frac{1}{2}} - Q_2^2 A_2^{\frac{1}{2}}}{Q_1^2 - Q_2^2}\right\}^2$$

This equation, with the values found for A and Q, gives $D = 3\,7 \times 10^8$, hence $\rho_0 = -7 \times 10^{12}$ The expression for t then gives the following values —

Q	A	t
145,000	$1\,14 \times 10^8$	$9\,6 \times 10^{-8}$ cm
131,000	$6\,9 \times 10^7$	9 9 „
110,000	10^8	10 7 „
90,000	5×10^4	9 0 „
56,000	2×10^2	5 6 „

The five values of t agree as well as could be expected

Since $\rho_0 = ne$ and e is about 3×10^{-10}, we get $n = 2 \times 10^{22}$ Patterson* calculated the number of free electrons per cubic centimetre of platinum from the change in its resistance due to a magnetic field, on J J Thomson's theory, and got $n = 2\,8 \times 10^{22}$.

It is interesting to apply the formula for t to platinum polarised with hydrogen in dilute sulphuric acid The potential fall in this case is about 0 9 volt, which corresponds to a value of Q about $2\,1 \times 10^4$ If, then, we suppose A/D to be small, which is the case in H_2 at high pressures, we get $t_2 = Q^2\beta_0/8\pi\rho_0\theta$, which gives, at $\theta = 300$, $t = 4\,8 \times 10^{-8}$ cm. The thickness of the double layer in this case has been estimated by several observers from the polarisation capacity and found to be about 2×10^{-8} cm

* 'Phil Mag,' 6, vol 3, p 643.

The Yielding of the Earth to Disturbing Forces.

By A E H Love, F R S, Sedleian Professor of Natural Philosophy in the University of Oxford.

(Received November 28, 1908,—Read January 14, 1909)

1 Any estimate of the rigidity of the Earth must be based partly on some observations from which a deformation of the Earth's surface can be inferred, and partly on some hypothesis as to the internal constitution of the Earth The observations may be concerned with tides of long period, variations of the vertical, variations of latitude, and so on The hypothesis must relate to the arrangement of the matter as regards density in different parts, and to the state of the parts in respect of solidity, compressibility, and so on In the simplest hypothesis, the one on which Lord Kelvin's well-known estimate* was based, the Earth is treated as absolutely incompressible and of uniform density and rigidity This hypothesis was adopted to simplify the problem, not because it is a true one No matter is absolutely incompressible, and the Earth is not a body of uniform density It cannot be held to be probable that it is a body of uniform rigidity. But when any part of the hypothesis, *e.g.*, the assumption of uniform density, is discarded, the estimate of rigidity is affected Different estimates are obtained when different laws of density are assumed Again, whatever hypothesis we adopt as regards the arrangement of the matter, so long as we consider the Earth to be absolutely incompressible and of uniform rigidity, different estimates of this rigidity are obtained by using observations of different phenomena Variations of the vertical may give one value, variations of latitude a notably different value It follows that "the rigidity of the Earth" is not a definite physical constant But there are two determinate constant numbers related to the methods that have been used for obtaining estimates of the rigidity of the Earth. One of these numbers specifies the amount by which the surface of the Earth yields to forces of the type of the tide-generating attractions of the Sun and Moon The other number specifies the amount by which the potential of the Earth is altered through the rearrangement of the matter within it when this matter is displaced by the deforming influence of the Sun and Moon If we adopt the ordinarily-accepted theory of the figure of the Earth, the so-called theory of "fluid equilibrium," and if we make the very probable assumption that the physical constants of the

* For Lord Kelvin's work on the subject, reference may be made to his 'Math and Phys. Papers,' vol. 3, or to Thomson and Tait's 'Nat. Phil.,' Part 2, §§ 735—737 and 833—847.

matter within the Earth, such as the density or the incompressibility, are nearly uniform over any spherical surface having its centre at the Earth's centre, we can determine both these numbers without introducing any additional hypothesis as to the law of density or the state of the matter. We shall find, in fact, that observations of variations of latitude lead to a determination of the number related to the inequality of potential, and that, when this number is known, observations of variations of the vertical lead to a determination of the number related to the inequality of figure.

[*Note added*, December 15, 1908.—This statement needs, perhaps, some additional qualification. It is assumed that, in calculating the two numbers from the two kinds of observations, we may adopt an equilibrium theory of the deformations produced in the Earth by the corresponding forces. If the constitution of the Earth is really such that an equilibrium theory of the effects produced in it by these forces is inadequate, we should expect a marked discordance of phase between the inequality of figure produced and the force producing it. Now Hecker's observations, cited in § 6 below, show that, in the case of the semidiurnal term in the variation of the vertical due to the lunar deflexion of gravity, the agreement of phase is close. If, however, an equilibrium theory is adequate, as it appears to be, for the semidiurnal corporeal tide, a similar theory must be adequate for the corporeal tides of long period and for the variations of latitude.]

2. We take a to represent the mean radius of the Earth and ρ to represent the mean density. In the undisturbed state the Earth will be taken to rotate as if rigid about a principal axis. We shall denote by ρ_0, p_0, V_0 the density, pressure, and potential in the undisturbed state. In the theory of fluid equilibrium it is supposed that, for a first approximation, ρ_0 may be taken to be a function of r, the distance of a point from the centre. If ρ_0 were known, V_0 would be known, and then p_0 would be determined by the equation

$$\frac{1}{\rho_0}\frac{\partial p_0}{\partial r} - \frac{\partial V_0}{\partial r} = 0,$$

and the condition that p_0 vanishes at $r = a$. In the same theory it is supposed that, for a second approximation, the surfaces of equal density, which are also equipotential surfaces and surfaces of equal pressure, may be regarded as oblate ellipsoids of revolution about the polar axis, the ellipticity E of the ellipsoids being a function of r, which is small for all values of r in the range $a \geqslant r > 0$. Thus a surface of equal density, given, to a first approximation, by the equation $r = R$, is given, to a second approximation, by the equation

$$r = R\{1 + E(\tfrac{1}{3} - \cos^2\theta)\},$$

where θ is the colatitude measured from the polar axis Then the theory leads to the equation*

$$\int_0^a \rho_0 \frac{d}{dr}(r^5 \mathrm{E})\, dr = \tfrac{1}{3}\rho a^5 \left(\epsilon - \frac{a\omega^2}{2g}\right), \tag{1}$$

where ϵ denotes the value of E at $r = a$, ω denotes the angular velocity of rotation, and g denotes the value of gravity at the surface, so that

$$g = \tfrac{4}{3}\pi\gamma\rho a,$$

the constant of gravitation being denoted by γ Further, if C, A denote the moments of inertia of the rotating body about its axis of figure and a perpendicular axis passing through its centre, there is no difficulty in obtaining the equation

$$\mathrm{C} - \mathrm{A} = \frac{8\pi}{15}\int_0^a \rho_0 \frac{d}{dr}(r^5 \mathrm{E})\, dr \tag{2}$$

In calculating the constant of precession $(\mathrm{C} - \mathrm{A})/\mathrm{A}$, we may, with sufficient approximation, take A to be given by the formula

$$3\mathrm{A} = 8\pi \int_0^a \rho_0 r^4 dr \tag{3}$$

The values of a, ρ, ω, g, ϵ, $(\mathrm{C}-\mathrm{A})/\mathrm{A}$ are pretty accurately known by observation

This outline of the theory is sufficient for our purpose We shall not need to take account of those further refinements in which quantities of the order ϵ^2 are retained

3 Now consider the rotating body subjected to deforming forces With a view to calculating the deformation we may disregard the effects of rotation and consider the undisturbed body as spherical and of density ρ_0 at a poin distant r from its centre Let the forces be derived from a potential which, in the region $0 < r \leqslant a$, is expressible as a sum of spherical solid harmonics of positive integral degrees. Let W_n be a single spherical harmonic term of this sum, the degree n of this term being a positive integer Under the action of the forces derived from the potential W_n the body will be deformed Let U denote the radial component of the displacement, and Δ the cubical dilatation at any point P The density at P in the disturbed state will not be the value of ρ_0 which belongs to the point P, but it will be

$$\rho_0 - \mathrm{U}\frac{d\rho_0}{dr} - \rho_0\Delta \tag{4}$$

Whatever theory of the constitution of the Earth may be adopted,† provided that the physical constants, such as density or incompressibility, are

* *Cf* J H Pratt, 'Figure of the Earth,' 1865 edition, p. 79

† See the note added at the end of § 1

functions of r, the quantities U and Δ will be products of the spherical solid harmonic W_n and some functions of r The potential V of the disturbed body at a point will not be V_0 but it will be the sum of V_0 and the potentials due to (i) a volume distribution of density $-U(d\rho_0/dr)-\rho_0\Delta$ throughout the region $0 < r < a$, and (ii) a surface distribution of density $[\rho_0 U]_{r=a}$ over the surface $r = a$ Hence the potential will differ from V_0 by terms which are the products of the spherical solid harmonic W_n and functions of r In the most important cases $n = 2$, and we may write

$$U = H(r)\frac{W_2}{g}, \quad \Delta = f(r)\frac{W_2}{g}, \quad V = V_0 + K(r)W_2, \tag{5}$$

where $H(r)$, $f(r)$, $K(r)$ are functions of r, and the constant $1/g$ has been inserted in the expression for U for the sake of convenience If, in fact, W_2 is a periodic term of the tide-generating potential, W_2/g is the "true equilibrium height" of the corresponding tide, that is to say, it is the height of the harmonic inequality which the forces answering to W_2 would produce in an ocean covering a rigid spherical nucleus, of the same size and mass as the Earth, if the depth and density of the ocean were negligible From the definition of $K(r)$ we easily find the formula

$$K(a) = \frac{3}{5\rho a^5}\int_0^a \rho_0\left[\frac{d}{dr}\{r^5 H(r)\} - r^5 f(r)\right]dr \tag{6}$$

In what follows we shall write h for $H(a)$ and k for $K(a)$, so that the equation of the disturbed surface is

$$r = a + hW_2/g,$$

and the disturbance of potential at the surface is kW_2. The numbers referred to above as specifying the inequality of figure and the inequality of potential are the numbers h and k

4 The horizontal pendulum is an instrument for measuring small displacements of the vertical The deflexion is proportional to the gravitational attraction in a direction at right angles to the axis of the instrument. The attraction in question may be regarded as due to the Earth, the Sun, and the Moon The axis of the instrument may be taken to be normal to the disturbed surface $r = a + hW_2/g$ The forces acting on the pendulum are: (i) a force derived from the potential W_2, (ii) a force derived from the potential kW_2, (iii) the component of undisturbed gravity tangential to the surface $r = a + hW_2/g$ If W_2 is the disturbing potential due to the Moon alone, the three forces act in the plane containing the Earth's centre, the Moon's centre, and the bob of the pendulum. We calculate the forces in this case, beginning with the force (iii). We have

$$W_2 = \frac{\gamma M}{D^3} r^2(\tfrac{3}{2}\cos^2\theta' - \tfrac{1}{2}),$$

where M denotes the Moon's mass, D the distance of the Earth's centre from the Moon's centre, r the distance of a point from the Earth's centre, θ' the angle between the radius vector drawn from the Earth's centre to the point and the radius vector drawn from the Earth's centre to the Moon's centre The surface $r = a + hW_2/g$ is approximately an ellipsoid of revolution, and its eccentricity e is given, correctly to the first order, by the equation

$$e^2 = \frac{3ha\gamma M}{gD^3}$$

The angle between the central radius vector and the normal at any point of the ellipsoid is, to the first order, $e^2 \sin\theta' \cos\theta'$ Hence the component of gravity tangential to the surface is, to the first order,

$$\frac{3ha\gamma M}{D^3} \sin\theta' \cos\theta', \quad \text{or} \quad -\frac{h}{a}\frac{\partial W_2}{\partial \theta'}$$

The sense of this force is that in which θ' increases The magnitudes of the other forces can be written down; and the resultant force acting at right angles to the axis of the instrument is

$$\frac{1}{a}\frac{\partial W_2}{\partial \theta'} + \frac{k}{a}\frac{\partial W_2}{\partial \theta'} - \frac{h}{a}\frac{\partial W_2}{\partial \theta'} \tag{7}$$

If the Earth were absolutely rigid, the force would be $a^{-1}\partial W_2/\partial\theta'$ Hence the ratio of the observed deflexion of the pendulum to that which would occur if the Earth were absolutely rigid is

$$1 + k - h \,.\, 1$$

This result* has been obtained by regarding W_2 as the Moon's tide-generating potential, but it is clear that the same expression would be arrived at if W_2 were any term of the tide-generating potential due to the action of both Sun and Moon, for the numbers h and k are the same for all disturbing potentials expressed by spherical solid harmonics of the second degree

5 An approximate expression for the height of any oceanic tide which follows the equilibrium law may be calculated by neglecting the depth and density of the ocean Let W_2 be a term of the tide-generating potential Then the corresponding inequality of the surface of the ocean, calculated in the way described, is $(1+k)W_2/g$, and the inequality of the nucleus is hW_2/g Thus the height of the corresponding tide† is calculated as

* An equivalent result for an incompressible sphere with a special law of density is given by W Schweydar, 'Beiträge zur Geophysik,' vol 9, 1907, p 41, but his argument is, in one place, so briefly expressed that I find it difficult to follow

† This result is, of course, well known

$+k-h)W_2/g$, and the ratio of the actual height to the true equilibrium height is $1+k-h \cdot 1$ Apart, therefore, from a correction depending on the depth of the ocean, the self-attraction of the waters, and the pressure of the tidal wave upon the nucleus, the quantity which is measured by means of observations of the long period tides is the same as that which is measured by means of the horizontal pendulum It is, therefore, not surprising that estimates of the rigidity of the Earth which are based on observations of the fortnightly tide are nearly the same as those which are based on observations of the variations of the vertical

6 Lord Kelvin's estimate was based chiefly on observations of the fortnightly tide in the Indian Ocean, from which observations it was inferred that

$$1+k-h = \tfrac{2}{3} \qquad (8)$$

very nearly This result has been confirmed by W. Schweydar,* by an analysis of much more numerous observations of fortnightly tides, and also by an analysis of several sets of observations made with horizontal pendulums. But the most striking confirmation is to be found in the investigations of O Hecker,† conducted by means of horizontal pendulums His very precise and consistent results show that the equation (8) is remarkably exact.

7 An analysis of many series of observations of variations of latitude led S C Chandler‡ to the conclusion that such motions are roughly periodic with a period of about 427 days If the Earth were an absolutely rigid body, having the figure of an oblate ellipsoid of revolution, with ellipticity equal to that found by geodetic observations, and having that ratio of moments of inertia about polar and equatorial axes which is deduced from the observed amount of precession, it would execute a free oscillation about the steady motion of rotation in a period of about 306 days, and the variations of latitude would have this period The lengthening of the period from 306 days to 427 days is due to the defect of rigidity The result that a yielding of the Earth would lengthen the period was obtained by S. Newcomb,§ and the period was investigated theoretically by S. S. Hough.‖ The theory has been placed in a very clear light by G Herglotz ¶

8 We take the undisturbed axis of figure as axis of z, and choose axes of x and y in the equatorial plane Let l, m denote the cosines of the angles which the instantaneous axis of the Earth's rotation makes with the axes of

* *Loc cit.*

† "Beobachtungen an Horizontalpendeln ," 'Veröffentlichung d. k Preuss. geodätischen Institutes,' No 22, Berlin, 1907

‡ 'Astronomical Journal,' vols. 11, 12, 15, 19, 21, 22 (1891—1902)

§ 'Mon Not. R. Astr Soc.,' 1892.

‖ 'Phil Trans,' A, vol 187, 1896

¶ 'Zeitschr f Math u. Phys,' vol. 52, 1905, p 275

x and y The yielding of the Earth may be computed as the yielding of a sphere to forces derived from a potential W_2, where

$$W_2 = -\omega^2(lx+my)z \tag{9}$$

It is, therefore, expressed by a radial displacement U and a dilatation Δ, where

$$U = H(r)\,W_2/g, \qquad \Delta = f(r)\,W_2/g$$

This displacement of the matter is to be superposed upon the inequality due to uniform rotation about the axis of z The moments and products of inertia of the deformed rotating body with respect to the axes of x, y, z can be calculated The angular velocity of the frame of moving axes of x, y, z has components $l\omega$, $m\omega$, ω referred to their instantaneous positions The equations of motion of the body with the calculated moments and products of inertia referred to the moving axes can be formed by the ordinary methods of dynamics, and the period of free nutation can be deduced This is Herglotz' method, generalised by the inclusion of the effect of compressibility

9 It is found that, to the first order, the moments of inertia, A, C are unaltered, but that small products of inertia are introduced The values of these products are given, correctly to the first order, by the equations

$$\begin{aligned}
\int yz\,dm &= -\frac{8\pi}{15}\,m\,\frac{\omega^2}{2g}\int_0^a \rho_0\left[\frac{d}{dr}\{r^5H(r)\} - r^5 f(r)\right]dr,\\
\int zx\,dm &= -\frac{8\pi}{15}\,l\,\frac{\omega^2}{2g}\int_0^a \rho_0\left[\frac{d}{dr}\{r^5H(r)\} - r^5 f(r)\right]dr,\\
\int xy\,dm &= 0,
\end{aligned} \tag{10}$$

where the integrations in the left-hand members are taken through the mass of the disturbed body By means of the result (2) we obtain the products of inertia in the forms*

$$-m\alpha(C-A), \qquad -l\alpha(C-A), \qquad 0, \tag{11}$$

where

$$\alpha\int_0^a \rho_0\frac{d}{dr}(r^5E)\,dr = \frac{\omega^2}{2g}\int_0^a \rho_0\left[\frac{d}{dr}\{r^5H(r)\} - r^5 f(r)\right]dr \tag{12}$$

The period τ of free nutation is given by the equation

$$\tau = \frac{2\pi A}{\omega(1-\alpha)(C-A)} \tag{13}$$

* The formulæ (11) and (13), and a formula equivalent to (12) in the case of incompressibility, were given by Herglotz (*loc cit.*) The constant α is equivalent, in this case, to the constant which he denotes by ν The result expressed by (12) and (13) agrees with that obtained by Hough if the initial density ρ_0 is uniform and the matter incompressible.

If the Earth were absolutely rigid the period τ_0 would be

$$\tau_0 = \frac{2\pi A}{\omega(C-A)}, \tag{14}$$

so that the period is lengthened by the yielding in the ratio $1 : (1-\alpha)$.

Now if we introduce the results (1) and (6), we find

$$1-\frac{\tau_0}{\tau} = \alpha = k\frac{a\omega^2}{2g}\Big/\left(\epsilon-\frac{a\omega^2}{2g}\right) \tag{15}$$

It appears, therefore, that the constant k is determined.

10 The period denoted by τ_0 is known from the constant of precession to be about 306 days, the period denoted by τ is known from observations of variations of latitude to be about 427 days The value of $a\omega^2/g$ is about $\frac{1}{289}$, and the value of ϵ is about $\frac{1}{577}$ Hence we find

$$k = \tfrac{4}{15} \text{ nearly} \tag{16}$$

Since $h-k = \frac{1}{3}$ nearly, and $k = \frac{4}{15}$ nearly, we find

$$h = \tfrac{3}{5} \text{ nearly} \tag{17}$$

These results may be expressed as follows —The inequality produced in the potential of the Earth, near its surface, by the action of the Sun and Moon, is about four-fifteenths of the tide-generating potential, and the inequality produced in the surface of the Earth is about three-fifths of the true equilibrium height of the tide The results hold for each of the partial tides answering to the several periodic terms of the tide-generating potential

11 If the matter within the Earth is assumed to be absolutely incompressible and of uniform density ρ, we should have $k = \frac{3}{5}h$ If, further, it is assumed to be of uniform rigidity μ, the theory of the deformation of an elastic sphere would give the result

$$h = \tfrac{5}{2}\left(1+\tfrac{19}{2}\frac{\mu}{g\rho a}\right)^{-1} \tag{18}$$

But the actual relation $h-k = \frac{1}{3}$, obtained from the tides and the variations of the vertical, then gives $h = \frac{5}{8}$, or $\mu = \frac{4}{19}g\rho a$ Since $g\rho a = 3\,5\times10^{12}$ nearly in C G S units, we have $\mu = 7\,6\times10^{11}$ nearly Thus the rigidity calculated by this method is about the rigidity of steel (Kelvin's estimate). If, instead of using the result $h-k = \frac{1}{3}$, we use the value $\frac{3}{5}$ for h, we find from (18) the value $\mu = \frac{1}{10}g\rho a$, or about $1\,2\times10^{12}$ The approximate agreement of this estimate of rigidity with that which has been deduced from the rate of transmission of the second phase of earthquake waves to great distances should not, I think, be regarded as of much importance.

12. One way of deducing an estimate of rigidity from observations of

variations of latitude combined with the hypotheses of absolute incompressibility and uniform density and rigidity is to give to τ_0, τ, ϵ, $a\omega^2/2g$ in equation (15) their actual values, so that $k = \frac{4}{5}$ nearly. This method gives $h = \frac{4}{3}$ nearly, and then, by equation (18), we find $\mu = 1{\cdot}7 \times 10^{12}$ nearly. A second way is to give to τ_0/τ and $a\omega^2/2g$ their actual values, and to ϵ the value $\frac{1}{230}$, which it would have, according to the theory of fluid equilibrium, if the matter were homogeneous This method* gives $k = 0\,4226$ and $\mu = 9\,2 \times 10^{11}$ nearly A third way is to give to τ and $a\omega^2/2g$ their actual values, and to τ_0 and ϵ the values which, according to the theory of fluid equilibrium and the theory of precession, they would have if the matter were homogeneous We should then find $k = 0\,685$ and $\mu = 4\,4 \times 10^{12}$ nearly In view of these results the approximate agreement of Hough's result with Kelvin's estimate does not seem to have much importance.†

13 In order to obtain an estimate of the rigidity of the Earth from observations of variations of latitude, it seems to be necessary to combine the observations with some hypothesis which will admit of the ellipticity ϵ and the constant of precession $(C-A)/A$ having their actual values. One of the simplest admissible constitutions is that which has been proposed by E. Wiechert‡ He takes the Earth to consist of a solid nucleus of density 8 206 enclosed in a solid shell of density 3 2, the ratio of the radius of the nucleus to the outer radius of the shell being 0 78 If we make the further assumptions that there is no slipping at the interface between nucleus and shell, and that the matter is absolutely incompressible and of uniform rigidity, we can calculate the numbers h and k in terms of the rigidity If, then, the rigidity is adjusted so that the period of free nutation may be 427 days, there results $\mu = 11\,68 \times 10^{11}$ nearly If, however, the rigidity is adjusted so that the displacement of a horizontal pendulum by tide-generating forces may be two-thirds of what it would be if the Earth were absolutely rigid, there results $\mu = 6{\cdot}3 \times 10^{11}$ nearly The first of these results was obtained by Herglotz by transforming the equations of elastic equilibrium for a body of variable density, and solving the transformed equations. The second result was obtained by Schweydar by using some

* It is, in effect, the method adopted by Hough (*loc cit*)

† This statement is not meant to suggest any doubt as to Kelvin's general conclusion that the Earth, as a whole, is a very rigid body All the astronomical evidence confirms this conclusion Kelvin's rather precise numerical estimate of rigidity, as some 7 or 8 times 10^{11} dynes per square centimetre, appears to be confirmed rather well by Hough's estimate of about 9 times the same Hough described his estimate as founded upon a "reasonable hypothesis" Two other reasonable hypotheses lead to $4\frac{1}{2}$ and 17, where Hough's leads to 9 These numbers confirm the general conclusion, not the numerical estimate

‡ 'Göttingen Nachrichten,' 1897

analysis due to Chree.* By an extension of the analysis which was applied to such problems by Kelvin, I have found the values $11\cdot52\times10^{11}$ and $6\cdot3\times10^{11}$ The discrepancy between the two estimates of rigidity, deduced respectively from the period of free nutation and the displacement of horizontal pendulums, shows, as we should expect, that the assumption of uniform rigidity is untenable Schweydar has proposed to assume that there is one uniform rigidity for the nucleus and another for the shell, and finds that the results of the two kinds of observations can be reconciled by taking—

$$\mu \text{ for the nucleus} = 2\cdot02\times10^{12},$$
$$\mu \text{ for the shell} = 0\cdot9\times10^{11}$$

The values which I have found are $2\cdot15\times10^{12}$ and $0\cdot86\times10^{11}$ The slight disagreement between my results and those of Herglotz and Schweydar is due to slight differences in the assumed data The general conclusion, that the rigidity of the nucleus may be much greater than that of ordinary materials at the surface, and the rigidity of the shell smaller than that of most rocks,† is more important than the numerical values

14 The results which have just been described may be taken as indicating the effect of heterogeneity on the estimate of rigidity It appears that increase of density towards the centre compensates to some extent for defect of rigidity, and that increase of rigidity towards the centre can compensate for a considerable defect of rigidity in the superficial portions The effect of compressibility is not known,‡ but it seems improbable that the yielding of a compressible sphere with an assigned rigidity should be less than that of an incompressible one The result that the rigidity of Wiechert's shell may be less than that of most surface rocks led Schweydar to adopt Wiechert's suggestion that there may exist a plastic sheet between the nucleus and the shell I think it may be regarded as certain that there is not within a depth of 1400 km a continuous layer of molten matter, separating the inner portions of the Earth's body from the outer portions, and behaving as a fluid in respect of forces of the type of tide-generating forces In order that the astronomical motions may be performed as we know they are, and that the surface may not yield to such forces more than we know it does, the portions of the Earth which are outside such a sheet, if it exists, must be

* 'Cambridge Phil Soc. Trans.,' vol 14, 1889.

† The rigidities of many kinds of granite and marble lie between 2×10^{11} and 3×10^{11} C G S units See the memoir by F D Adams and E G Coker, 'Publications of the Carnegie Institution,' No 46.

‡ My attempt to determine this effect in 'Cambridge Phil. Soc Trans.,' vol. 15, 1900, does not now appear to me to be satisfactory

much more rigid than we can reasonably conceive them to be No amount of rigidity of the nucleus would enable us to satisfy the conditions To illustrate this statement quantitatively it will be necessary to set down some of the analysis of the problem

15 We consider a sphere of gravitating matter stratified in concentric spherical layers. The matter in any layer is taken to be of uniform density ρ_0 and rigidity μ, these quantities varying from layer to layer, and it is taken to be absolutely incompressible. If we discard the assumption of incompressibility, the problem becomes much more difficult We suppose the sphere to be deformed by body forces derived from a potential W_n, which is a spherical solid harmonic of the nth degree The initial pressure p_0 and potential V_0 in the layer become p_0+p and $V_0+V_n+W_n$, where V_n is the potential due to the inequalities at the disturbed boundaries of all the layers. Let (u, v, w) denote the displacement at any point (x, y, z) within a layer The equations of equilibrium are three equations of the type

$$-\frac{\partial}{\partial x}(p_0+p)+\mu\nabla^2 u+\rho_0\left(\frac{\partial V_0}{\partial x}+\frac{\partial V_n}{\partial x}+\frac{\partial W_n}{\partial x}\right)=0, \tag{19}$$

and, since the undisturbed state is one of equilibrium, we can replace these by equations of the type

$$-\frac{\partial p}{\partial x}+\mu\nabla^2 u+\rho_0\frac{\partial}{\partial x}(V_n+W_n)=0 \tag{20}$$

We have also the condition of incompressibility

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{21}$$

To solve these equations, we put

$$p=\rho_0(V_n+W_n)-\mu\Sigma\frac{2i+1}{i}\psi_i, \qquad u=\Sigma\left(u_i-\frac{r^2}{2i}\frac{\partial\psi_i}{\partial x}\right), \tag{22}$$

where i is an integer, positive or negative, v and w are to be derived from u by replacing u_i successively by v_i and w_i, and x successively by y and z, also u_i, v_i, w_i are spherical solid harmonics of the ith degree, and ψ_i is a spherical solid harmonic of the ith degree defined by the equation

$$\psi_i=\frac{\partial u_{i+1}}{\partial x}+\frac{\partial v_{i+1}}{\partial y}+\frac{\partial w_{i+1}}{\partial z}. \tag{23}$$

The summation refers to all the values of i that need be taken

16 With a view to the expression of the radial displacement U and the components of traction across a spherical surface $r=R$, it is convenient to introduce a spherical solid harmonic of the $(-i-2)$th degree defined by the equation

$$\phi_{-i-2}=\frac{\partial}{\partial x}\left(\frac{u_i}{r^{2i+1}}\right)+\frac{\partial}{\partial y}\left(\frac{v_i}{r^{2i+1}}\right)+\frac{\partial}{\partial z}\left(\frac{w_i}{r^{2i+1}}\right). \tag{24}$$

Then we find $$rU = -\Sigma\left[\frac{2i-1}{2(2i+1)}r^2\psi_{i-1}+\frac{r^{2i+3}}{2i+1}\phi_{-i-2}\right] \quad (25)$$

The value of U at $r = R$ can be expressed by means of (25) as a sum of spherical surface harmonics of positive degrees, say ΣS_j. We shall write $\Sigma\zeta_j$ for the corresponding sum of spherical solid harmonics of positive degrees, so that $\zeta_j = (r/R)^j S_j$. If $r = R$ is a boundary of a layer in the undisturbed state, and ρ_0', ρ_0'' are the densities just inside and just outside this surface, the contribution of this boundary to the inequality of potential V_n is

$$\Sigma\frac{4\pi\gamma(\rho_0'-\rho_0'')}{2j+1}R\zeta_j \quad \text{or} \quad \Sigma\frac{4\pi\gamma(\rho_0'-\rho_0'')}{2j+1}\frac{R^{2j+2}}{r^{2j+1}}\zeta_j, \quad (26)$$

according as the point at which V_n is estimated is within or without the surface $r = R$. The value of V_n is the sum of such contributions from all the bounding surfaces.

The x-component of traction across the surface $r = R$ is the value at $r = R$ of the expression

$$-\frac{x}{r}(p_0+p)+\frac{\mu}{r}\left\{\frac{\partial}{\partial x}(rU)+r\frac{\partial u}{\partial r}-u\right\},$$

or

$$-\frac{x}{r}\left\{p_0+\rho_0(V_n+W_n)-\mu\Sigma\frac{2i+1}{i}\psi_i\right\}$$

$$+\frac{\mu}{r}\Sigma\left\{(i-1)u_i-r^2\frac{\partial\psi_{i-1}}{\partial x}+\frac{r^{2i+1}}{2i+1}\frac{\partial}{\partial x}\left(\frac{\psi_{i-1}}{r^{2i-1}}\right)-\frac{1}{2i+1}\frac{\partial}{\partial x}(r^{2i+3}\phi_{-i-2})\right\} \quad (27)$$

The radial component of the traction is the value at $r = R$ of the expression

$$-\{p_0+\rho_0(V_n+W_n)\}+\mu\Sigma\left\{(2i+3)\left(\frac{1}{i+1}-\frac{i+2}{2i+5}\right)\psi_{i+1}\right.$$

$$\left.-\frac{2i}{2i+1}r^{2i+1}\phi_{-i-2}\right\} \quad (28)$$

The tangential traction across the surface $r = R$ is a vector quantity which has a definite magnitude and direction at each point of this surface, the direction being at right angles to the central radius vector. We may resolve this vector into components parallel to the axes. The component parallel to the axis of x is obtained by subtracting the x-component of the radial traction from the x-component of the resultant traction. We find that the x-component of the tangential traction is the value at $r = R$ of the expression

$$\frac{\mu}{r}\Sigma\left\{(i-1)u_i-\frac{i+1}{2i+1}r^2\frac{\partial\psi_{i-1}}{\partial x}-\frac{i-1}{2i+1}r^{2i+1}\frac{\partial}{\partial x}\left(\frac{\psi_{i-1}}{r^{2i-1}}\right)\right.$$

$$\left.-\frac{3}{(2i+1)(2i+3)}\frac{\partial}{\partial x}(r^{2i+3}\phi_{-i-2})-\frac{2i}{(2i+1)(2i+3)}r^{2i+3}\frac{\partial\phi_{-i-2}}{\partial x}\right\}. \quad (29)$$

17 The conditions to be satisfied at a disturbed boundary of a layer are conditions of continuity of displacement and traction. Except in the case of the normal traction, it is sufficient to form the conditions at the undisturbed boundary. Let letters with single accents denote the values of quantities such as u, and μ just inside a boundary $r = R$, and letters with double accents denote the values of the corresponding quantities just outside the same boundary The condition of continuity of the x-component of displacement is

$$u'_i - \frac{R^2}{2(i+1)}\frac{\partial \psi'_{i+1}}{\partial x} + u'_{-i-1} + \frac{R^2}{2i}\frac{\partial \psi'_{-i}}{\partial x}$$
$$= u''_i - \frac{R^2}{2(i+1)}\frac{\partial \psi''_{i+1}}{\partial x} + u''_{-i-1} + \frac{R^2}{2i}\frac{\partial \psi''_{-i}}{\partial x}, \qquad (30)$$

where we have picked out all the terms which contain spherical surface harmonics of the same degree i We may now regard i as positive The spherical solid harmonic of the ith degree,

$$u'_i - u''_i - \frac{R^2}{2(i+1)}\left(\frac{\partial \psi'_{i+1}}{\partial x} - \frac{\partial \psi''_{i+1}}{\partial x}\right) + \frac{r^{2i+1}}{R^{2i+1}}(u'_{-i-1} - u''_{-i-1})$$
$$+ \frac{1}{2i}\frac{r^{2i+1}}{R^{2i-1}}\left(\frac{\partial \psi'_{-i}}{\partial x} - \frac{\partial \psi''_{-i}}{\partial x}\right)$$

vanishes at the spherical surface $r = R$, and therefore vanishes for all finite values of r We have two other like expressions which vanish in virtue of the conditions of continuity of the y- and z-components of displacement Hence, by the usual method, we deduce the two equations

$$\psi'_{i+1} + \frac{\phi'_{i+1}}{R^{2i+5}} - \frac{2i+5}{2}\frac{r^{2i+3}}{R^{2i+3}}\psi'_{-i-2} = \text{same with doubly accented letters}, \qquad (31)$$

and

$$\frac{r^{2i+3}}{R^2}\phi'_{-i-2} - \frac{2i+1}{2}\frac{\phi'_{i+1}}{R^{2i+5}} + \frac{(2i+3)^2}{4}\frac{r^{2i+3}}{R^{2i+3}}\psi'_{-i-2} = \text{same with doubly accented letters} \qquad (32)$$

The condition of continuity of the x-component of tangential traction is

$$\mu'\Big[(i-1)u'_i - (i+2)\frac{r^{2i+1}}{R^{2i+1}}u'_{-i-1} - \frac{i+3}{2i+5}R^2\frac{\partial \psi'_{i+1}}{\partial x} - \frac{-2}{2i-3}\frac{r^{2i+1}}{R^{2i-1}}\frac{\partial \psi'_{-i}}{\partial x}$$
$$- \frac{i-1}{2i+1}r^{2i+1}\frac{\partial}{\partial x}\left(\frac{\psi'_{i-1}}{r^{2i-1}}\right) - \frac{i+2}{2i+1}\frac{1}{R^{2i+1}}\frac{\partial}{\partial x}(r^{2i+3}\psi'_{-i-2})$$
$$- \frac{3}{(2i+1)(2i+3)}\frac{\partial}{\partial x}(r^{2i+3}\phi'_{-i-2})$$
$$- \frac{3}{(2i+1)(2i-1)}\frac{r^{2i+1}}{R^{2i+1}}\frac{\partial}{\partial r}\left(\frac{\phi'_{i-1}}{r^{2i-1}}\right) - \frac{2(i-2)}{(2i-3)(2i-1)}\frac{r^{2i+1}}{R^2}\frac{\partial \phi'_{-i}}{\partial x}$$
$$+ \frac{2(i+3)}{(2i+5)(2i+3)}R^{2i+3}\frac{\partial \phi'_{i+1}}{\partial x}\Big]$$

same with doubly accented letters (33)

From the three equations (two independent) of this type we obtain in the usual way the equation

$$\mu'\Big[(i+1)(i+3)\psi'_{i+1}+\frac{i(i+2)(2i+5)}{2i+1}\frac{r^{2i+3}}{R^{2i+3}}\psi'_{-i-2}+\frac{2i(i+2)(2i+5)}{(2i+1)(2i+3)}\frac{r^{2i+3}}{R^3}\phi'_{-i-2}-\frac{2(i+1)(i+3)}{2i+3}\frac{\phi'_{i+1}}{R^{2i+5}}\Big]=\text{same with doubly accented letters} \qquad (34)$$

In forming the condition of continuity of normal traction at a disturbed boundary $r=R+U$, we may calculate all the terms that arise from the additional stress as if the boundary were the sphere $r=R$ The only terms in which we need take account of the displacement of the boundary are those that arise from the initial stress The initial stress being hydrostatic pressure, the corresponding normal component of traction across any surface at a point is the pressure at that point with its sign changed The initial pressure at a point on the surface $r=R+U$ is expressible as the value of $p_0+U(\partial p_0/\partial r)$ at a point on the surface $r=R$ When $r=R$ is a boundary of a layer p_0 is continuous in crossing the boundary, but $\partial p_0/\partial r$ is not, for its value is $\rho_0(\partial V_0/\partial r)$, and ρ_0 is discontinuous Hence the condition of continuity of normal traction at the surface $r=R+U$ is

$$-\rho_0'\Big(U\frac{\partial V_0}{\partial r}+V_n+W_n\Big)$$

$$+\mu'\Sigma\Big[(2i+3)\Big(\frac{1}{i+1}-\frac{i+2}{2i+5}\Big)\psi'_{i+1}+(2i+3)\Big(\frac{1}{i+2}+\frac{i+1}{2i+1}\Big)\frac{r^{2i+3}}{R^{2i+3}}\psi'_{-i-3}-\frac{2i}{2i+1}\frac{r^{2i+3}}{R^3}\phi'_{-i-2}-\frac{2(i+3)}{2i+5}\frac{\phi'_{i+1}}{R^{2i+5}}\Big]=\text{same with doubly accented letters.}$$

It is convenient to eliminate ϕ'_{-i-2} and ϕ''_{-i-2} by means of the equation (34) We thus obtain the equation

$$-\rho_0'\Big(U\frac{\partial V_0}{\partial r}+V_n+W_n\Big)$$

$$+\mu'\Sigma\frac{2i+3}{(i+2)(2i+5)}\Big\{\frac{2i^2+8i+9}{i+1}\psi'_{i+1}+(i+3)(2i+5)\frac{r^{2i+3}}{R^{2i+3}}\psi'_{-i-2}-2(i+3)\frac{\phi'_{i+1}}{R^{2i+5}}\Big\}$$

$$=\text{same with doubly accented letters} \qquad (35)$$

It is clear from these equations that in general the only functions of type ϕ, ψ which occur are ϕ_n, ψ_n, ϕ_{-n-1}, ψ_{-n-1}, and the only functions of type u_n which occur are u_{n-1}, u_{n+1}, u_{-n}, u_{-n-2} and the corresponding v's and w's In the central portion of the sphere the only functions which occur are those of

the types ψ_n, ϕ_{-n-2}, u_{n-1}, u_{n+1} All these can be expressed in terms of W_n by solving linear equations It appears that U and V_n are multiples of the surface harmonic $W_n r^{-n}$

I obtained the results stated in § 13 above by applying the above analysis to the case of a central spherical nucleus and an enclosing shell with Wiechert's values for the densities and radii

18 We proceed to the example of an absolutely rigid nucleus, of density ρ_1 and radius a_1, separated by a layer of fluid* of density ρ_2 from a solid shell of density ρ_3, rigidity μ, and inner and outer radii a_2 and a_3, the whole being subjected to body force derived from a potential W_2, which is a spherical solid harmonic of the second degree Let ζ', ζ denote the spherical solid harmonics of the second degree to which the radial displacement U becomes equal at the surfaces $r = a_2$, $r = a_3$ respectively Then at any point in the solid enclosing shell we have

$$V_0 = \tfrac{4}{3}\pi\gamma(\rho_1-\rho_2)\frac{a_1^3}{r} + \tfrac{4}{3}\pi\gamma(\rho_2-\rho_3)\frac{a_2^3}{r} + \tfrac{2}{3}\pi\gamma\rho_3(3a^2-r^2) \qquad (36)$$

and
$$V_2 = \frac{4\pi\gamma}{5}\left\{(\rho_2-\rho_3)\frac{a_2^5}{r^5}\zeta' + \rho_3 a_3\zeta\right\} \qquad (37)$$

The displacement at any point in the solid enclosing shell is expressed in terms of the functions ϕ_2, ϕ_{-3}, ψ_2, ψ_{-3} Equation (25) gives

$$\left.\begin{aligned}\zeta' &= -a_2\left(\tfrac{1}{11}\psi_2 + \tfrac{1}{3}\frac{r^5}{a_2^2}\phi_{-3} + \tfrac{5}{6}\frac{r^5}{a_2^5}\psi_{-3} - \tfrac{1}{7}\frac{\phi_2}{a_2^7}\right),\\ \zeta &= -a_3\left(\tfrac{5}{14}\psi_2 + \tfrac{1}{4}\frac{r^5}{a_3^5}\phi_{-3} + \tfrac{5}{6}\frac{r^5}{a_3^5}\psi_{-3} - \tfrac{1}{7}\frac{\phi_2}{a_3^7}\right)\end{aligned}\right\} \qquad (38)$$

The tangential traction vanishes at $r = a_2$ and at $r = a_3$, and hence we have the equations

$$\left.\begin{aligned}8\psi_2 + \tfrac{14}{5}\frac{r^5}{a_2^2}\phi_{-3} + 7\frac{r^5}{a_2^5}\psi_{-3} - \tfrac{14}{5}\frac{\phi_2}{a_2^7} &= 0,\\ 8\psi_2 + \tfrac{14}{5}\frac{r^5}{a_3^2}\phi_{-3} + 7\frac{r^5}{a_3^5}\psi_{-3} - \tfrac{14}{5}\frac{\phi_2}{a_3^7} &= 0\end{aligned}\right\} \qquad (39)$$

The condition of continuity of normal traction at $r = a_2 + \zeta'$ is obtained from equation (35) in the form

$$\begin{aligned}&\gamma^5\Gamma\mu\left(\tfrac{19}{3}\psi_2 + 28\frac{r^5}{a_2^5}\psi_{-3} - 8\frac{\phi_2}{a_2^7}\right)\\ &+ 4\pi\gamma(\rho_2-\rho_3)\left[\frac{W_2}{4\pi\gamma} + \tfrac{1}{5}(\rho_2-\rho_3)a_2\zeta' + \tfrac{1}{5}\rho_3 a_3\zeta\right.\\ &\qquad\left. - \tfrac{1}{3}\left\{(\rho_1-\rho_2)\frac{a_1^3}{a_2^3} + \rho_2\right\}a_2\zeta'\right] = 0, \qquad (40)\end{aligned}$$

* In regard to the question of the adequacy of the statical theory to determine the behaviour of the supposed fluid layer, reference may be made to the Note at the end of § 1.

and the condition that the normal traction at $r = a_3 + \zeta$ vanishes is obtained in the same way in the form

$$\tfrac{5}{2}\pi\mu\left(\tfrac{9}{2}\psi_2 + 28\frac{r^5}{a_3^5}\psi_{-3} - 8\frac{\phi_2}{a_3^7}\right)$$
$$-4\pi\gamma\rho_3\left[\frac{W_2}{4\pi\gamma} + \tfrac{1}{5}(\rho_2-\rho_3)\frac{a_2^5}{a_3^5}\zeta' + \tfrac{1}{5}\rho_3 a_3\zeta - \tfrac{1}{5}\left\{(\rho_1-\rho_2)\frac{a_1^3}{a_3^3}\right.\right.$$
$$\left.\left. + (\rho_2-\rho_3)\frac{a_2^3}{a_3^3} + \rho_3\right\}a_3\zeta\right] = 0 \quad (41)$$

19 It is sufficient to consider the case in which $\rho_2 = \rho_3$, or the density of the fluid layer is the same as that of the solid enclosing shell Writing g for the value of gravity at the surface and ρ for the mean density, so that

$$g = \tfrac{4}{3}\pi\gamma\rho a_3 = \tfrac{4}{3}\pi\gamma\left\{(\rho_1-\rho_2)\frac{a_1^3}{a_3^3} + \rho_2\right\}a_3,$$

and putting

$$\zeta = h\frac{W_2}{g}, \qquad \frac{a_2}{a_3} = f, \quad (42)$$

we find that the value of h is given by the equation

$$1 = h\left[1 - \tfrac{3}{5}\frac{\rho_2}{\rho} + \tfrac{1}{5}\frac{\mu}{g\rho_2 a_3}\frac{456 - 1800f^3 + 2688f^5 - 1800f^7 + 456f^{10}}{24 + 40f^3 - 45f^7 - 19f^{10}}\right] \quad (43)$$

20 Suppose the fluid layer to be thin, and consider the case where the density is distributed according to Wiechert's law We have then $\rho_2 = 3\cdot2$, $\rho = 5\cdot58$, $f = 0\cdot78$ Remembering that $g\rho a_3 = 3\cdot5\times10^{12}$ nearly, and adjusting μ so that h may have its actual value $\frac{3}{8}$, we find $\mu = 3\cdot5\times10^{12}$ nearly It appears, therefore, that, even if the solid nucleus were absolutely rigid, and the enclosing shell were 1400 km thick, the presence of a layer of fluid separating the nucleus from the enclosing shell would increase very much the yielding of the surface To prevent the surface from yielding more than it actually does, the rigidity of the enclosing shell would have to be nearly five times that of steel If the enclosing shell were thinner, a still higher rigidity would be needed For example, if it were 64 km thick, and of density half the mean density, or about 2·8, the requisite rigidity of the enclosing shell, the nucleus being absolutely rigid and the fluid layer thin, would be about 50×10^{12} dynes per square centimetre, or about sixty-six times the rigidity of steel These numbers seem to me to be decisive against the hypothesis of the fluid layer This conclusion does not negative the possible existence of areas of continental dimensions beneath which there may be molten matter It means that such areas must be isolated, the molten matter beneath them cannot form a continuous sheet separating a central body from an enclosing crust. The conclusion does not negative the possible existence of a layer of comparatively small rigidity, but, if there is such a layer, it must be rigid enough to prevent a finite slipping of the enclosing crust over the central body

The Relation of the Earth's Free Precessional Nutation to its Resistance against Tidal Deformation.

By Prof J Larmor, Sec R S

(Received December 18, 1908,—Read January 14, 1909)

The modern investigation of the wandering of the Earth's axis of rotation, considered as a physical problem relating to the actual non-rigid Earth, may be said to have been initiated in Lord Kelvin's address to the Physical Section of the British Association in 1876 After referring* to the scrutiny of the recorded observations of change of latitudes, conducted by Peters in 1841 and independently by Maxwell in 1851, in search of the regular Eulerian free period of 306 days which would belong to a rigid Earth, with negative results, he insisted that the irregular motions brought out in these analyses are not merely due to instrumental imperfections, but represent true motions of the Pole, due to displacement of terrestrial material For example, he estimates that existing shifts of material, of meteorological type, are competent to produce displacements of the axis of rotation ranging from 1/2 to 1/20 of a second of arc A sudden shift of material on the Earth will not at once affect the axis of rotation, but will start it into motion round the altered axis of inertia, with a period of 306 days if the Earth were rigid, which will go on displacing the Pole until it is damped out by the frictional effects of the tidal motions thus originated A radius of rotation of 1 second of arc would raise an ocean tide of the same period as the rotation, having as much as 11 cm of maximum rise and fall Thus the motion of the Pole is to be considered as continually renewed by meteorological and other displacements, as it is damped off by tidal and elastic friction, it was therefore, perhaps, not to be expected that it would show much periodicity, though the movements were eminently worthy of close investigation Their nature was examined more closely by Newcomb at Kelvin's request, but not much more had been done regarding their cause when Chandler announced that the records of changes of latitude did actually indicate a period of precession—of 427 days, however, instead of the Eulerian period of 306 days, which, if any, had previously been taken for granted Soon after, in 1890, observations were organised systematically by the International Geodetic Union on the motion of Prof Foerster, of Berlin; and already, in 1891, he was able to inform Lord Kelvin that a comparison

* Reprint in 'Popular Lectures and Addresses,' vol. 2, see pp 262—272

of European observations with synchronous ones made at Honolulu gave direct proof of his conclusion of 1876 (*supra*), "that irregular movements of the Earth's axis to the extent of half a second may be produced by the temporary changes of sea level due to meteorological causes"*

In the following year the synchronous observations had already indicated periodicity, apparently in about 385 days, considerably less than Chandler's estimate, which, however, longer observation has since confirmed substantially Lord Kelvin remarks in his next annual address as follows† —"Newcomb, in a letter which I received from him last December, gave what seems to me undoubtedly the true explanation of this apparent discrepance from dynamical theory, attributing it to elastic yielding of the Earth as a whole He added a suggestion, especially interesting to myself, that investigation of the periodic variations of latitude may prove to be the best means of determining approximately the rigidity of the Earth As it is, we have now for the first time what seems to be a quite decisive demonstration of elastic yielding of the Earth as a whole, under the influence of a deforming force, whether of centrifugal force round a varying axis, as in the present case, or of tide-generating influences of the Sun and Moon, with reference to which I first raised the question of elastic yielding of the Earth's material many years ago" But "when we consider how much water falls on Europe and Asia during a month or two of rainy season, and how many weeks or months must pass before it gets to the sea, and where it has been in the interval, and what has become of the air from which it fell, we need not wonder" that the amplitudes of the polar wanderings "should often vary by 5 or 10 metres in the course of a few weeks or months"

It will be recalled that the main object of the original calculations of Lord Kelvin, which assigns to the Earth as a whole an effective rigidity of the same order as that of steel, was to combat the view then prevalent which assumed for the Earth a fluid interior Even a solid shell of very considerable thickness, enclosing a fluid core, was thus ruled out, unless its materials were preternaturally rigid, and it is clear that placing a solid core in the middle of the fluid interior cannot affect this conclusion so long as an equilibrium theory is applicable, *i e*, so long as the layer of fluid material is not so thin or viscous as to prevent its adjusting itself immediately by flow to the alternating tidal stresses impressed upon it from its solid walls. By passing to the other limit, and thus taking it so thin that the outer shell

* 'Presidential Address R S,' Nov 30, 1891, 'Popular Lectures ,' vol 2, p 504 Lord Kelvin's investigations up to 1876 are collected in 'Math. and Phys Papers,' vol. 3, especially pp. 312—350

† 'Presidential Address R.S,' Nov. 30, 1892, *loc cit*, p 525

practically rides on the solid nucleus, but without effective tangential stress-connection, we obtain a hypothesis to which this objection does not apply

In a brief note in 'Monthly Notices R.A.S.' this year (1892), Newcomb showed, by a general estimate, that the effect of elastic yielding is competent to prolong the free period to about the amount required by observation. A formal mathematical discussion on the bases of calculation of the elastic deformation of a homogeneous sphere was first given by Mr S. S. Hough, now H.M. Astronomer at the Cape of Good Hope, in a memoir on "The Rotation of an Elastic Spheroid," in 'Phil. Trans.,' 1896

He concluded that the Chandler free period required an effective rigidity of the whole Earth of the order of that of steel, agreeing with Lord Kelvin's previous estimates from tidal phenomena, and his result seems to have been substantially confirmed by more recent calculations, giving for the average effective rigidity estimates derived from various possible hypotheses and simplifying assumptions ranging between extreme values 17×10^{11} and $4{\cdot}4 \times 10^{11}$, while Hough's estimate was put at $8{\cdot}98 \times 10^{11}$ This shows an even striking degree of agreement in calculations necessarily vague on account of the unknown constitution of the Earth's interior, especially in so far as observations of the equilibrium tides of long periods, and of the deviation of sea level due to tidal attraction which is essentially the same thing, lead to results of the same order as those of free precessional rotation * It, indeed, suggests, as we shall actually recognise, that this internal terrestrial constitution really is not involved in these various phenomena, except in the common feature of determining the surface effects arising from a given tidal or rotational stress † The key to the matter, from the general point of view, is contained in the remark of Hough that the free precession of the yielding Earth is the same as that of a rigid one of the shape that would result when the bulging arising from the centrifugal force of diurnal rotation is removed It is not difficult to show, from geometrical considerations regarding momentum,‡ that this result is general, and extends to an Earth of any degree of heterogeneity or plasticity. The argument may be reproduced in analytical form and rather wider scope, from another place§ (with definition of I rewritten), as follows —

Let ω be the angular velocity of the Earth about the instantaneous axis,

* *Cf* Prof A. E. H. Love, 'Roy. Soc. Proc.,' *supra*, p. 73. To this paper I am indebted for information as to results of recent calculations.

† The identity of these two types runs through the discussions in Thomson and Tait's 'Natural Philosophy'

‡ 'Proc. Camb. Phil. Soc.,' May, 1896, p. 185.

§ E. H. Hills and J. Larmor, "The Irregular Movement of the Earth's Axis of Rotation," 'Monthly Notices R.A.S.,' Nov., 1906, p. 24.

ω_1, ω_2, ω_3 its components referred to the principal axes in the configuration that the Earth would have if the motion were steady The Earth is deformed from this configuration by the inequality of centrifugal force due to the deviation of the instantaneous axis from the principal axis, with which it would coincide if the motion were steady. This deforming force is the resultant of the centrifugal force, directed outwards from the instantaneous axis, and the reversed centrifugal force, directed inwards towards the principal axis in question A linear law of elasticity applies to the small resultant of these two forces If the same law applied to the two forces separately, the reversed centrifugal force would change the moments of inertia A, B, C to certain values A′, B′, C′, which might, under simplifying hypotheses, be calculated from the theory of the deformation of an elastic sphere, and the centrifugal force directed outwards from the instantaneous axis would produce a certain change of density at each internal point, and would raise a certain protuberance on the surface, which might be calculated by the same theory Let I denote the moment of inertia (about the instantaneous axis) of a mass arranged as specified by this change of density and this protuberance The instantaneous axis is a principal axis of this mass, and therefore the contributions of this mass to the components of moment of momentum are $I\omega_1$, $I\omega_2$, $I\omega_3$. The complete expressions for the components h_1, h_2, h_3 of moment of momentum are therefore

$$h_1 = A'\omega_1 + I\omega_1, \quad h_2 = B'\omega_2 + I\omega_2, \quad h_3 = C'\omega_3 + I\omega_3.$$

The equations of motion referred to the rotating axes are of the well-known vector type,

$$dh_1/dt - h_2\omega_3 + h_3\omega_2 = L$$

When A and B are equal, the third of them is

$$\frac{d}{dt}(C\omega_3) = N,$$

where C is the effective moment of inertia $C' + I$ when N is null ω_3 is thus constant, say Ω, up to the first order The other two equations are

$$\frac{d}{dt}\ (A' + I)\,\omega_1 + (C' - B')\,\Omega\omega = L,$$

$$\frac{d}{dt}\ (B' + I)\,\omega_2 - (C' - A')\,\Omega\omega_1 = M,$$

which in the case of approximate symmetry involve a free period $2\pi(A' + I)/(C' - A')\,\Omega$, and similarly in the general case, thus depending only on A′, B′, C′ when I is small

The result is that the period of the free precession is not C/(C−A) days, as it would be for a rigid Earth, but approximately C/(C′−A′), where the

denominator is that difference of principal moments of inertia which would remain after the imposition of a bodily force having as potential

$$W = -\tfrac{1}{2}\omega^2 r^2 \sin^2\theta = -\tfrac{1}{3}\omega^2 r^2 (1 - P_2),$$

namely, that of the centrifugal force reversed, P_2 representing the zonal harmonic $\frac{1}{2}(3\cos^2\theta - 1)$

The first part of W, the term $-\frac{1}{3}\omega^2 r^2$, corresponds to slight contraction of volume, which is immaterial as regards the desired quantity C′−A′ The other part, $\frac{1}{3}\omega^2 r^2 P_2$, will produce an extension, of the same harmonic type as itself, along the polar axis, which will in turn alter the potential of the Earth's attraction at its own surface by $k\ \frac{1}{3}\omega^2 r^2 P_2$, where the value of k depends on its effective resistance to deformation Moreover the Earth's potential is at distant points, by Laplace's formula,

$$V = \gamma\left(\frac{E}{r} + \frac{A+B+C-3I}{2r^3} + \ldots\right),$$

which gives

$$V = \gamma\left(\frac{E}{r} - \frac{C-A}{r^3}P_2 + \quad\right)$$

in the present special case, and if, as in the actual circumstances, further harmonics do not occur to sensible amount, this expression holds right up to the Earth's surface The free surface, of ellipticity e, is

$$r = a(1 + e\sin^2\theta)$$
$$= a(1 - \tfrac{2}{3}eP_2),$$

where $a = a(1 + \frac{2}{3}e)$ The value of e is determined by the constancy over the ocean surface of the total potential V−W, as −W is the potential of the centrifugal force, viz, of

$$\gamma\left\{\frac{E}{a}(1 + \tfrac{2}{3}eP_2) - \frac{C-A}{a^3}P_2\right\} + \tfrac{1}{3}\omega^2 a^2(1 - P_2),$$

whence, equating to zero the coefficient of P_2,

$$\tfrac{2}{3}ga\left(e - \frac{\omega^2 a}{2g}\right) - \frac{\gamma}{a^3}(C-A) = 0,$$

thus deriving from data of the distribution of gravity, or of the form of the Earth's surface, the value of C−A, which determines the astronomical precession. Again, if taking off the centrifugal force would change C−A to C′−A′, it would alter V by $\gamma r^{-3}\{(C-A)-(C'-A')\}P_2$, which must, according to the above specification of k, be equal to $k \cdot \frac{1}{3}\omega^2 r^2 P_2$. Thus

$$\frac{C'-A'}{C-A} = 1 - \frac{\frac{1}{3}k\omega^2 a^5 \gamma^{-1}}{C-A}$$
$$= 1 - \frac{k\,\omega^2 a/2g}{e - \omega^2 a/2g},$$

Hence, if τ is the periodic time of actual free precession and τ_0 is what it would be if the Earth were rigid,

$$1-\frac{\tau_0}{\tau}=k\frac{\omega^2 a}{2g}\Big/\left(\epsilon-\frac{\omega^2 a}{2g}\right)$$

This is the formula (15) in Prof Love's paper before referred to, it is there deduced from a hypothesis of concentric spheroidal stratification of the Earth's interior, after the manner of Laplace We have found that, like Clairaut's formula for gravity, this relation is independent of any hypothesis as to the Earth's internal structure, except such as is involved in the definition and value of k

As $\omega^2 a/g$ is 1/289 and τ is found to be 428 days, and τ_0 is 306 days, this relation makes k equal to 4/15

The values of k corresponding to various moduli of rigidity and compressibility of the Earth considered as a homogeneous globe might perhaps be deduced and tabulated for comparison, from Lord Kelvin's and similar elastic analysis

The height of the long-period equilibrium tides provides different data, corresponding to an extraneous tide-producing potential W_2 of this type, the absolute rise of the water is $(1+k)W_2/g$, from which has to be subtracted hW_2/g for the rise of the solid Earth due to this tide-producing potential, thus leaving a factor $1+k-h$ for the relative tide which alone can be the subject of observations The reductions of tidal data for the Indian Ocean gave Kelvin and G H Darwin the value 2/3 for this factor, which is confirmed by more recent discussions the observations of Hecker with a horizontal pendulum at the bottom of a well, which obviously determine the same thing, viz, the change of level due to tide-producing potential, concur in a remarkable manner Thus $h=3/5$

These values of k and h, as defined in the last paragraph, would not be independent for a homogeneous incompressible globe they would, in general, require for their consistency both elasticity of volume and of form The phenomena of free precession give the value of k, but with reference to compression along the polar axis, those of tidal change of level give the value of $h-k$, or rather its mean value, with reference to compression along axes in the neighbourhood of the equator* This statement is the purest and simplest expression of the information relating to the solid Earth's resistance to deforming forces that the data of periodic change of latitude

* *Cf* Prof Love, *supra*, p. 81, to whom this proposition is substantially due, having been reached by him through analysis appropriate to a centrically stratified body. The quantities h and k, in other notation, enter essentially into the tidal discussions by Kelvin and Darwin in Thomson and Tait's 'Nat. Phil.'

and of equilibrium (*i e*, long-period) tides can supply, prior to any hypothesis regarding the internal distribution and the effective elasticity or plasticity of its materials

[*Added February* 2.—It has been remarked above, after Lord Kelvin, that a sudden shift of material from one part of the Earth's surface to another would alter the position of the principal axis of inertia round which the free precession of the Earth's axis of rotation takes place, and thus cause a sharp bend in the path of the Pole. If the shift were merely local, such as an earthquake may be expected to produce, the effect would be inappreciable. The connection of sharp curvature in the path of the Pole with seismic disturbance, if it really exists, would thus be indirect, the earthquake being itself started possibly by the slight changes, meteorological or other, of distribution of surface load, which are indicated by the disturbance of the free precession.

But it is to be noticed that a *submarine* seismic subsidence, if uncompensated by adjacent elevation, or *vice versâ*, would be competent to produce sensible direct disturbance of the path of the Pole, for water would have to flow, in part from distant regions, to fill up the defect of level thus produced. The same would be true for earthquake subsidence near coast lines, if it is compensated by rise of the land. In reply to an inquiry on this subject, Prof. Milne writes as follows:—"When a very large earthquake occurs on land, we find vertical and lateral displacements of, let us say, 20 feet, along lines which may be one or two hundred miles in length. The majority of big earthquakes, however, are sub-oceanic in their origin, along lines parallel to mountain ridges, as, for example, at the bottom of the trough which runs parallel to the Andes. The mass movement appears to result in the deepening of the trough and the rise of the coast line. We have measurements where depth has increased as much as 200 fathoms: see 'Brit. Assoc. Seismic Report,' 1897, for a number of these measurements."

An estimate of the effect of such displacements is easily made. Thus, an uncompensated subsidence of the ocean floor, of volume corresponding to a fall of one foot over a thousand miles square, in middle latitudes, would produce* a direct shift in the Pole of rotation amounting to about one-eighth of a second of arc, and at the same time the Pole of the principal axis of inertia, round which the 428-day precession of the axis of rotation takes place, would be displaced in the opposite direction through an angle of the same order of magnitude.

In connection with the possibility of irregularity in the Earth's diurnal

* *Loc. cit.*, 'Monthly Notices R.A.S.,' Nov., 1906, p. 26.

rotation due to causes of this kind, similar considerations arise.* A slight subsidence, due to shrinkage around the equator, unless it extended downward a long way toward the Earth's centre, would have negligible direct effect on the moment of inertia and, therefore, on the length of the day, but if it were under sea it would involve transference of water from regions nearer the Earth's axis, in order to make up the deficiency, and if the equatorial regions were all under water, a contraction of 50 cm in equatorial radius would in this way alter the length of the year by an amount of the order of half a second of time, which would be astronomically of high importance]

* Lord Kelvin, *loc cit*, § 38

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Notes on Observations of Sun and Stars in some British Stone Circles. Fourth Note.—The Botallek Circles, St. Just, Cornwall.

By Sir Norman Lockyer, Sc D, K.C B., F R S., Director Solar Physics Observatory

(Received December 8, 1908,—Read January 14, 1909)

Borlase, in his "Antiquities of Cornwall" (p 199), published in 1769, refers to what he terms "the curious cluster" of circles at Botallek, the seeming confusion of which led him to write "I cannot but think that there was some mystical meaning, or, at least, distinct allotments to particular uses."

Fortunately for science, he accompanies his account with a plan evidently carefully prepared (fig. 1), which is now the only thing that remains; every stone has been utilised in building an engine house, or in other ways. Only the site is shown on the ordnance map

As the "cluster" of circles exceeds in elaboration anything of the kind with which I am acquainted, it was of great interest to see if anything could be made of it in the light of other researches in Cornwall already referred to in previous communications to the Royal Society.† The first point of

* Lord Kelvin, *loc. cit*, § 38.

† 'Roy Soc. Proc.,' A, vol. 76, 1905, p. 177; A, vol. 77, 1906, p. 465; A, vol. 80, 1908, p. 285.

inquiry concerned the N point given on the plan—whether it was true or magnetic. A perusal of Borlase's volume showed that he was fully acquainted with the necessity of referring in such descriptions to the true north, instead of, as he says, "such an inconstant and fluctuating index as the

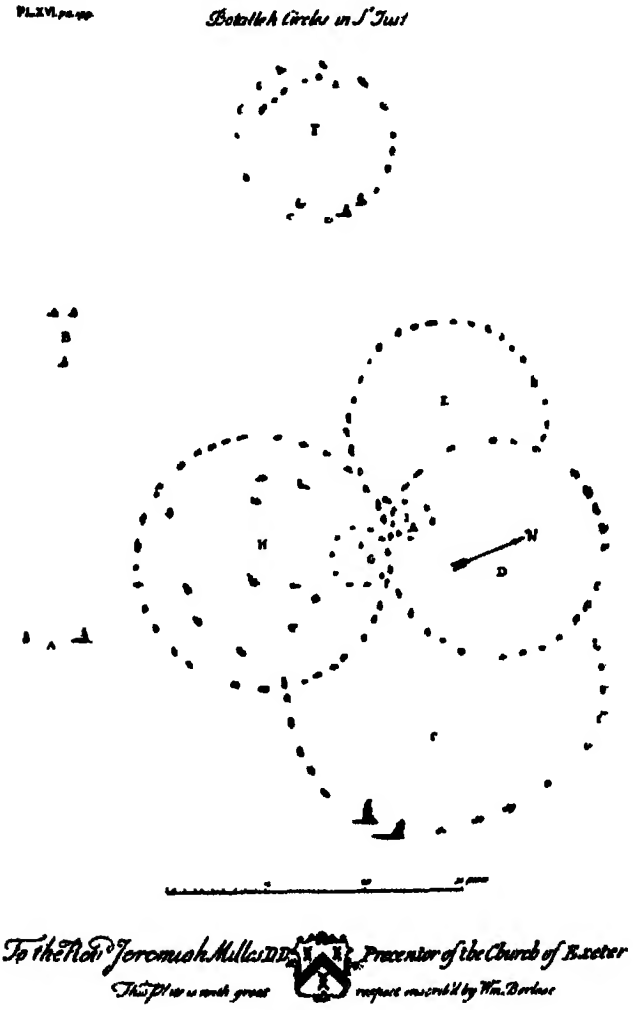

FIG. 1

declination of the needle, which is not only different in different places, but varies also at different times in one and the same place" (p 115).

When the point was settled, it became evident, when the circles were completed and lines drawn from centre to centre, that approximately the same azimuths were in question as those before noted.

Borlase does not give the heights of hills for the various azimuths to complete his plan. I therefore asked Mr. Thomas, an active member of the Cornish Society for the Astronomical Study of Ancient Monuments, to observe them for me Taking Borlase's orientation as being near the truth, lines were drawn joining up the approximate centres of the various circles and a list of the various azimuths was sent to Mr Thomas, who was good enough to comply with my request at once

Among the azimuths were two, the first from the approximate centre of the circle F to the approximate centre of E, N 83° E, and the second, from the approximate centre of F to that of H, S. 66° E In sending his results to me Mr Thomas remarked that the former line passes over the Carn Bean barrow and the latter passes 2¼° to the N of the Goon Rith barrow, thus the azimuth of the Goon Rith barrow would be S 63¾° E.

It struck me that this circumstance would enable us to check the accuracy of Borlase's N. point. It is much easier to make a careful survey of a monument than to indicate its true orientation, so some slight error has to be expected. Borlase in all probability employed a compass in making his surveys and was, therefore, dependent, for accurate orientation, on a knowledge of the value for the local magnetic variation, for this he would have to depend upon the results of some general survey Even at the present day it is a matter of great uncertainty to obtain the variation for any one place without making a special determination on the spot, and we should expect a possible error of several degrees in any orientation made in Borlase's time

The two directions to two still existing monuments pointed out by Mr Thomas are common to Botallek and other monuments in Cornwall, we seem justified therefore in accepting them as such. On this assumption, Borlase's orientation was true, and not magnetic, and, also, was not far from the mark

The next step was to make a very careful determination of the centres of the circles and it was found that the line, centre of F to centre of H, coincided with the line S. 63° 45′ E. from the former to the Goon Rith barrow In other words, the difference between the azimuth we had provisionally determined from the circles and that of Goon Rith barrow was due to an error of centring, and no doubt was left that the line between the centres of F and H was really directed to the barrow. Similarly the line N. 83° E. joining the centres of F and E was directed to the Carn Bean barrow. Both these lines were recognised as familiar, giving, approximately, the November sunrise and the heliacal rising of the Pleiades in May respectively. In the case of the S.E. azimuth there is an alternative

explanation of the sight-line. Both in Cornwall and Wales we have found that azimuth-marks (barrows, etc.), were sometimes erected so that they gave the direction of sunrise a fortnight or three weeks before the critical date. I therefore decided to adopt the Pleiades azimuth, N. 83° E., as the fundamental line by which to fix the N point, and it followed that Borlase's N. point was less than 3° to the west.

Working on this basis, I joined up the centres of the circles, as shown on the plans (figs. 2 and 3), and carefully measured the resulting azimuths. These I sent to Mr Thomas, asking him if the slight modifications that I had introduced had sensibly altered his values for the corresponding angular elevations. After a second series of observations, he replied that the elevations were the same for the modified azimuths as they were before.

It at once became obvious that the alignments divided themselves naturally into two groups—the one erected for the observations of May-year, the other for solstitial, phenomena—and with each group there is associated a clock-star which affords a means of determining the approximate date of that group. For this reason I give two separate plans (figs 2 and 3) showing the separate

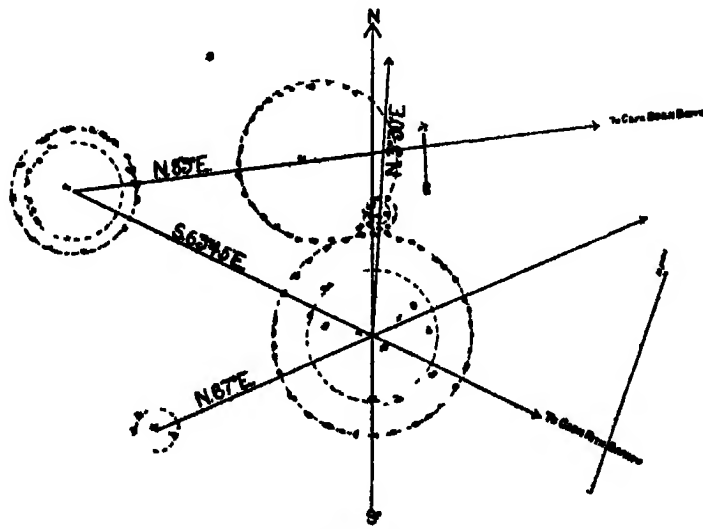

Fig. 2

groups of alignments, and two separate tables giving the respective results I will deal with the May-year circles first (fig. 2).—

May-year Alignments at Botallek (lat. 50° 8′ N).

Alignment	Azimuth.	Hill (Mr Thomas's) measures	Declination	Object.	Date.
	° ′	° ′	° ′		
Centre of circ B to cent of circ. H	N 67 0 E	3 0	16 31 N	May sun	May 6; Aug 7
Cent. of circ F to cent of circ H to Goon Rith barrow	S 63 45 E	2 44	14 48 S	Nov sun (possibly a warner)	Nov 2; Feb 10
Cent. of circ F to cent of circ E to Carn Bean barrow	N 83 0 E	3 35	7 2 N	Pleiades (warning May sun)	1680 B.C
Cent of circ H to cent. of circ I	N 3 30 E	0 0	39 14 N.	Arcturus (clock star)	1780 B.C

These results agree in a wonderful way with the May-year results previously obtained from the study of other Cornish circles, and to illustrate this I bring together a selection of the results previously published —

Similar May-year Alignments in Cornwall (for comparison)

Monument	Lat. N	Alignment	Azimuth	Hill	Declination.	Object	Date
	° ′		° ′	° ′	° ′		
Merry Maidens	50 4	Circ to Fougou	N 64 0 E	0 30	16 21 N	May sun	May 5; Aug 7
Boscawen-Un	50 5	„ stone	S 66 30 E	1 0	14 22 S	Nov sun	Nov. 2; Feb 10
The Hurlers	50 31	S. circ to N E stone	N 78 47 E	0 12	7 23 N	Pleiades	1610 B.C.
Trippet stones	50 33	Cent of circ. to Rough Tor	N 15 0 E	1 30	39 1 N	Arcturus	1700 B.C

Examination of fig. 2 shows that the azimuths given in the table are exactly those obtained by joining up the true centres of the circles and adopting the N—S. line derived from Mr. Thomas's two measures of direction. The results certainly justify the 3° change of the orientation.

The Solstitial Year.

Joining up the centres of H, G, D, and C, as shown in fig. 3, we obtain the results given in the following table, results which are obviously connected *inter se* and with the solstitial year:—

Solstitial Alignments at Botallek (lat. 50° 8′ N.).

Alignment	Azimuth.	Hill (Mr Thomas's) measures	Declination	Object.	Date.
	° ′	° ′	° ′		
Cent of circ H to cent of circ C	N 53 0 E	1 45	23 41 N	Solstitial sun (summer)	
Cent of circ D to cent. of circ. C	S 49 30 E	1 35	23 44 S	Solstitial sun (winter)	
Cent. of circ H to cent of small circ G	N 16 0 E	0 0	37 28 N	Arcturus (clock-star)	1490 B.C

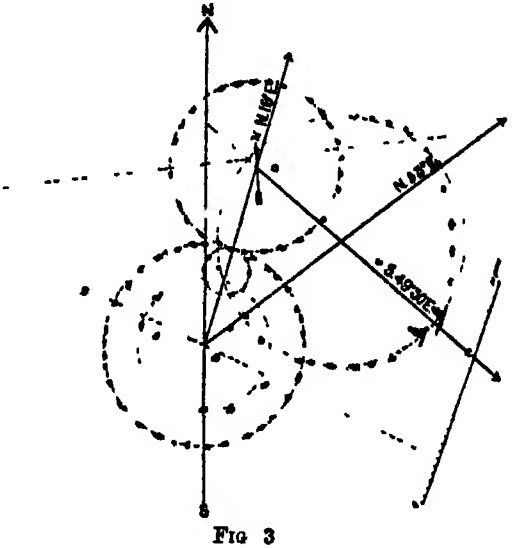

Fig 3

As before, I give a selection from previous results, showing that the alignments we are now dealing with have become familiar by reason of their occurrence at the Cornish monuments investigated earlier :—

Similar Solstitial Alignments in Cornwall (for comparison)

Monument	Lat N.	Alignment	Azimuth.	Hill.	Declination.	Object.	Date
	° ′		° ′	° ′	° ′		
Boscawen-Un .	50 5	Circ. to Fine Menhir	N 53 30 E.	2 23	23 59 N	Solstitial sun (summer)	
The Hurlers .	50 31	N circ to S E. stone	S 50 50 E.	1 18	24 17 S.	Solstitial sun (winter)	
Tregeseal . .	50 9	Longstone to Chûn Castle	N 23 30 E	1 35	37 9 N	Arcturus	1350 B.C.

It will probably be remarked that I attach no dates to these solstitial sight-lines. This is because the data available are not sufficiently certain to justify dating. The solstitial variation takes place so slowly and between such restricted limits that, until the most accurate observations possible have been made at a monument, it is merely conjecture to apply a date, only at Stonehenge, so far, has this been possible. Under the Botallek conditions, a difference of half a degree in the azimuth would produce a variation of more than 2000 years in the resulting date, and one cannot assume that accuracy in the present case.

From the results given above it is evident that in this "curious cluster" of circles at Botallek we have an epitome of the chief sight-lines found in Cornwall. May-year sun, clock-star, warning-star, and solstitial sun are all represented.

The occurrence of star circles is fortunate, as it enables us to attempt to arrange the groups in order of date. As shown above, the May-year group, F, H, B, and E, with the clock-star circle I, was probably the first, by something like 300 years, to be erected, and it should be noted that the date for the Pleiades circle E is coincident, within our probable error, with the date of the clock-star alignment H—I.

Borlase's plan (fig. 1) affords us evidence on this point, for it shows that the circles F, H, and I are associated by being made up of two concentric rings of stones. The fact that stones were obviously taken from the periphery of E when D was built shows that E, too, was an earlier circle than D, our results associate E with the May-year and D with the solstitial group. The incompleteness of B suggests partial demolition prior to Borlase's survey, whilst its relatively smaller size suggests that what remains may have formed the interior ring of a double circle.

Conclusions

The cluster of circles at Botallek, St. Just, Cornwall, was erected for astronomical observations, and forms an epitome of the principal alignments to sun and stars previously found in Cornwall and other parts of the British Isles. The results justify the azimuths obtained from Borlase's plan and show that his orientation of the plan is not more than 3° in error.

It appears that in this cluster we have two distinct groups of alignments, one associated with the May-year worship, the other associated with the later solstitial-year ritual.

As a clock-star alignment occurs in each group, we are able to determine that the May-year worship preceded the solstitial-year by something like

300 years, the approximate dates being 1700 B.C. and 1400 B.C. respectively. This sequence is confirmed by the structure of the circles themselves as plotted by Borlase.

I have to thank Mr. Thomas for his local observations, and Mr. Rolston, of the Solar Physics Observatory, for assistance in the discussion and computing the various declinations.

On the Passage of Rontgen Rays through Gases and Vapours.

By J A Crowther, B A, Fellow of St. John's College, Cambridge, Mackinnon Student of the Royal Society.

(Communicated by Prof. Sir J J Thomson, F R S. Received December 22, 1908, —Read January 14, 1909)

Introduction.

The present work is a continuation of a previous research on the Secondary Rontgen Radiation from Gases and Vapours* It was there shown that while for gases and vapours containing only elements of small atomic weight the secondary radiation was simply proportional to the density of the gas, those containing elements of higher atomic weight, and notably compounds of arsenic and bromine, gave off quantities of secondary radiation greater out of all proportion than what would be expected from their density. It was also shown that while the secondary radiation from the first class of substances had approximately the same penetrating power as the primary rays producing it, the secondary radiation from the second class was generally of a considerably softer character A third class of substances, including stannic chloride and methyl iodide, gave off secondary rays, the hardness of which was equal to that of the primary, while their intensity, which, however, varied with the hardness of the primary rays, was intermediate between that of the first and second classes

It was thought that a further investigation of the phenomena attending the passage of Rontgen rays through these different classes of gases and vapours might possibly lead to some interesting results.

* 'Phil. Mag.,' [6], vol. 14, p. 653, 1907.

The phenomena which it was proposed to measure were :—

(i) the absorption of the primary rays by the gas,
(ii) the ionisation produced in the gas by the passage of the rays;
(iii) the secondary radiation given out by the gas.

Of the first two quantities, determinations have already been made by various experimenters, for some of the commoner gases and a few vapours. Thus Rutherford,* in the early days of X-ray work, measured both the coefficients of absorption and the relative ionisations of some dozen different substances In addition to this, determinations of the relative ionisation in different gases and vapours have been made by Perrin,† J J Thomson,‡ Strutt,§ McClung,‖ and Eve¶ Many of these measurements, however, were made without any care being taken to prevent the Rontgen rays from falling on the sides of the ionisation chamber and on the electrodes themselves. Under such circumstances, as we shall show later, a large proportion of the ionisation in the gas is due, not to the X-rays themselves, but to soft secondary β-radiation from the material of the walls and the electrodes The amount of this secondary ionisation will depend upon the material and construction of the vessels used Since the soft β-radiation is practically totally absorbed in the gas, its effect is in every case to add a nearly constant amount to the ionisation produced by the Rontgen rays, and thus to reduce the apparent relative ionisation, in gases which are more ionised than air.

The experiments of Prof Thomson, and of McClung, which were free from this objection, unfortunately did not include the gases whose behaviour it was most desired to study Moreover, as the experiments of the latter conclusively showed, the relative ionisation of any gas compared with air is not a constant, but depends upon the nature of the X-rays employed As there is at present no very satisfactory way of standardising Rontgen rays, or of comparing results obtained by different observers, it was felt that the only possible way of obtaining a set of comparable results was to determine the whole of the quantities concerned at the same time and with the same apparatus.

Apparatus

The apparatus employed was, in the main, the same as that used in the previous experiments, and described in detail in the previous paper, certain

* Rutherford, 'Phil Mag,' [V], 43, p 241, 1897.
† 'Ann de Chimie et de Phys.,' [7], vol 11, p. 496, 1897
‡ 'Camb Phil. Soc Proc,' vol. 10, p. 10, 1900
§ 'Roy. Soc. Proc.,' vol. 72, p. 209, 1903.
‖ 'Phil Mag,' [6], vol 8, p 357, 1904.
¶ 'Phil Mag,' [6], vol 8, p 610, 1904

alterations and additions being made to allow of the measurement of the ionisation and absorption in the gas, in addition to the secondary radiation

The gases, as before, were contained in two similar brass boxes A, A′ (fig 1) arranged symmetrically with respect to the focus tube F The rays enter by the thin aluminium windows *c*, *c′*, and a portion of the secondary rays pass upwards through aluminium windows *d*, *d′* into two cylindrical ionisation chambers B, B′

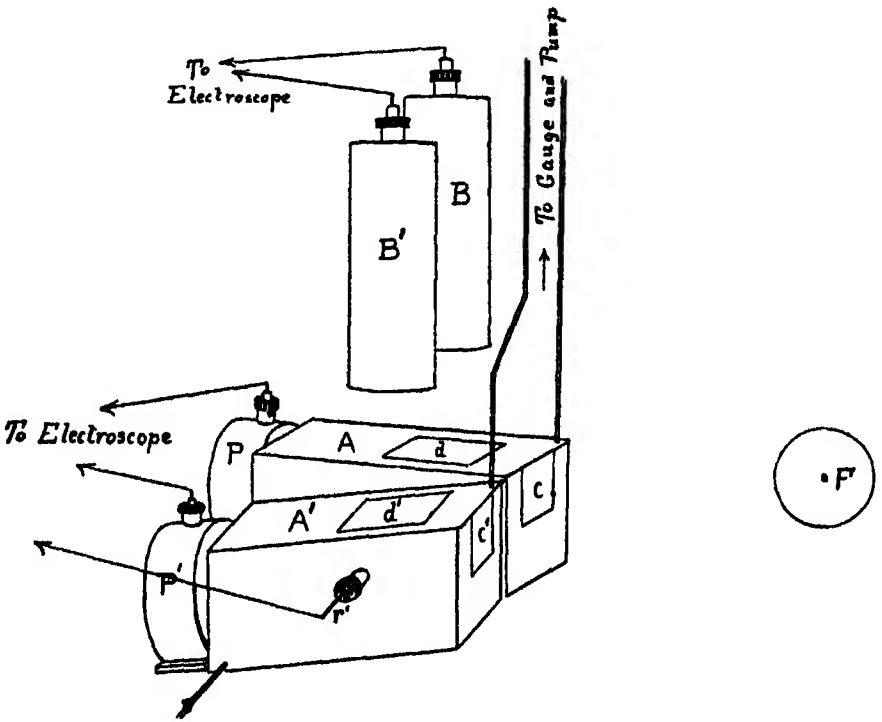

Fig 1

A portion of the primary beam, after traversing the length of the box, passes out through a third aluminium window *e*, as is shown in the plan of the apparatus given in fig 2, and is measured in the ionisation chamber P The amount of absorption of the rays in the gas can thus be determined To determine the ionisation produced in the gas by the passage of the rays, electrodes were inserted in the gas chambers A, A′ themselves In order to avoid intercepting any primary rays, these took the form of parallel plates of aluminium *r*, *s*, *r′*, *s′* placed at opposite sides of the boxes, and outside the path of the primary rays The electrodes were insulated from the walls of the boxes (which were earthed) by quartz tubes, the joints being made

air-tight with sealing wax, and, except in the case of one or two vapours when near their saturation pressure, no trouble was experienced from insulation leaks The plates s, s' were charged to a high potential by a battery of storage cells; the others, r, r' which were surrounded by earthed guard rings, were connected to a Wilson electroscope. The whole of the

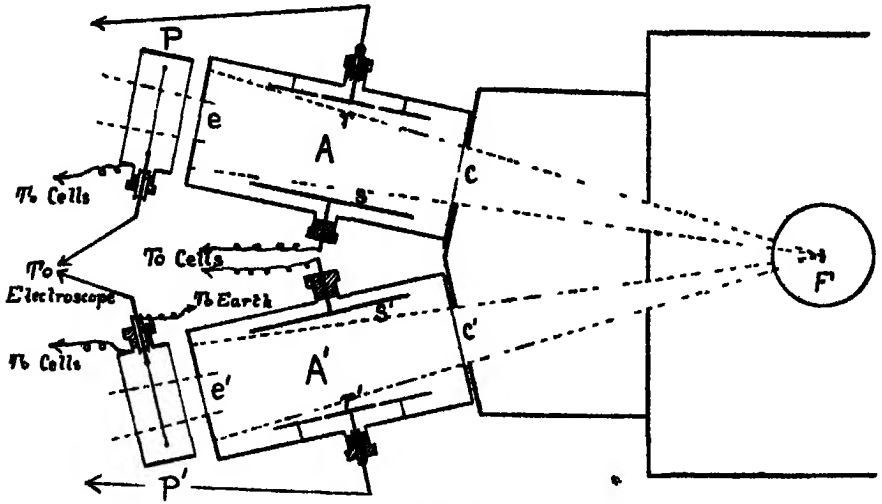

Fig 2

apparatus was made in duplicate, one of the boxes being filled with air and kept as a standard The different ionisation chambers were connected in pairs to three Wilson inclined electroscopes, each chamber in the one set being connected, together with its duplicate in the other, to a separate instrument. By means of highly insulating keys, which could be operated from a distance, it was arranged that either the test chamber, or the corresponding standard, could be connected at will with the electroscope By measuring the current through first one and then the other, the ratio of the ionisation in the test chamber to that in the standard could be accurately obtained For a Wilson electroscope, where the zero and the sensitiveness are both liable to fluctuation, this method of comparing two ionisation currents is both more convenient and more accurate than the use of two separate instruments.

The appliances for measuring the pressures in the two chambers, and for introducing the various gases and vapours, were the same as those employed in the previous paper, and fully described there. The various connections and keys were shielded from induction effects in the usual way, by earthed metal tubes.

Experiments

The actual experiments were performed in various ways, according to the nature of the observations which it was desired to make At least one set of experiments was made with each substance in which simultaneous readings were made of the relative ionisation, coefficient of absorption and secondary radiation, so as to avoid all possibility of any difference in quality of the rays employed In other cases, when some particular point was to be investigated, as, for example, the variation of the relative ionisation with pressure, it was found more convenient to make the desired observations by themselves, the quality of the rays being regulated by the length of the spark gap, which was just sufficient to extinguish the bulb

In practice it was found that, using the same coil and the same Rontgen bulb, this control was quite sufficient, and readings could always be repeated by bringing the bulb back to the same equivalent spark gap It was not found possible, however, to compare the hardness of different bulbs by this method, as two bulbs having the same equivalent spark gap would give out rays of apparently very different character In order to afford some idea of the meaning of the different values given in this paper for equivalent spark lengths, it may be mentioned that X-rays began to be given out in appreciable quantity when the equivalent spark gap was about 0 6 cm At this stage the whole bulb was filled with a bluish glow This glow disappeared when the equivalent spark gap had increased to about 1 2 or 1 3 cm , and the green phosphorescence of the walls was then alone visible At an equivalent spark gap of about 2 6 to 2 8 cm the coil (a large Rudge coil working with a hammer break) ceased to send any discharge through the bulb

Ionisation and Pressure

The ionisation produced in a gas by the passage of Rontgen rays, in the absence of any secondary radiation, should be proportional to the mass of the gas present, that is, if the temperature is constant, to the pressure of the gas. On the other hand, as was shown in the previous paper in the cases of air and carbon dioxide (and has since been confirmed for the more powerful radiators), the energy of the penetrating secondary radiation is also simply proportional to the pressure of the gas , and thus the ionisation produced by it in the radiating gas itself should be proportional to the square of the pressure.

We should expect that any soft secondary radiation which might be emitted would follow the same law, so long as it was sufficiently penetrating to reach the walls of the gas chamber. If, however, owing to the softness

of the radiation, the distance of the boundaries, or the pressure of the gas, the radiation was totally absorbed in the gas before reaching the boundaries, the ionisation produced by it would depend solely on its intensity, *i e*, it would be simply proportional to the pressure, and the whole ionisation in the gas would again follow a simple pressure law Neglecting, for the moment, the penetrating secondary radiation, we should expect that if any soft secondary radiation were present the ionisation in the gas would, at high pressures, be simply proportional to the pressure Then, as the pressure was reduced, and the absorption of the rays in the gas became less and less, a point would be reached when the secondary rays began to reach the electrodes, and the ionisation would then vary more rapidly than the pressure Experiments were made with various gases to test these conclusions.

The electrodes in the gas chambers were placed some distance from the front window in order to avoid the soft secondary radiation from the aluminium The rays had thus to pass through a certain thickness of gas before reaching the electrodes, and a certain amount of absorption occurred Knowing, however, from other measurements, the coefficient of absorption of the gas for the rays, it was easy to calculate the amount of this absorption at any given pressure, and to apply the necessary correction This has been done in all cases, in the results given below

The first observations, made with the electrodes at opposite sides of the gas chamber, failed to reveal any departure from a simple pressure law It was thought, however, that this might be due to the secondary rays being so soft as to be unable to penetrate to the boundaries of the gas, even at the lowest pressures used. The distance between the plates was, therefore, reduced to 5 mm, the primary beam being limited by a lead slit 1·2 mm wide, so that the rays still passed between the plates without striking them With this arrangement, a penetration of only 2 mm. would have sufficed to allow some of the soft secondary rays to reach the boundaries of the gas

Some of the curves obtained are shown in fig 3 For convenience in representation, the ordinates, in the case of methyl iodide, have been considerably reduced The curves obtained for ethyl bromide and methyl iodide at low pressures are shown on an enlarged scale in fig 3A. It will be seen that although experiments were made with air down to a pressure of 10 mm., and with methyl iodide and ethyl bromide with pressures as low as 0·7 mm and 0 2 mm respectively, the curves obtained are perfectly straight (except in so far as the curve for ethyl bromide is influenced at high pressures by the ionisation due to the penetrating secondary Röntgen

radiation), and there is no evidence of any departure from a simple pressure law The experiments were repeated with ethyl chloride and methyl bromide with similar results. We must assume, therefore, that the soft secondary radiation from a gas is either too absorbable to penetrate 2 mm of the gas even at these low pressures, or as on the whole seems more probable, that the ionisation produced by it, compared with that due to the direct action of the primary beam, is too small to have any appreciable effect on

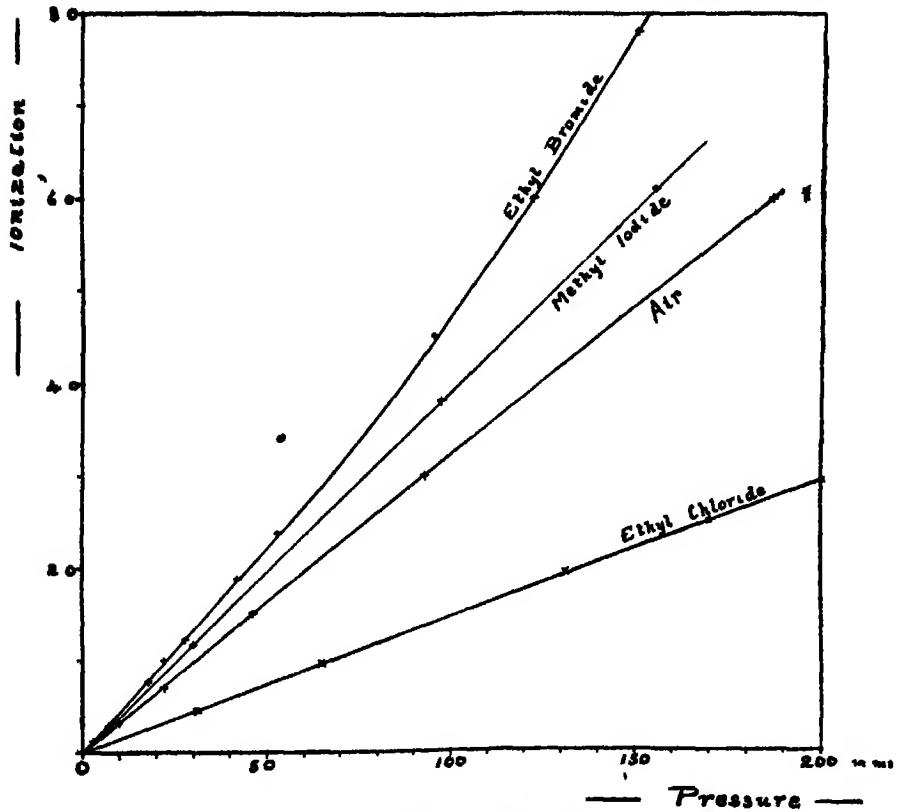

Fig 3.—Ionisation—Pressure Curves

the shape of the curves In this case the bulk of the ionisation in a gas must be due to the direct action of the primary rays

It is well known that when Rontgen rays fall upon a solid body they cause it to emit soft secondary rays in very appreciable amounts. Unless the action of the Rontgen rays upon the gaseous molecule differs very materially from its action upon the molecule in the solid state, the above result would appear to show that the soft secondary ionising radiation emitted by solids only represents a very small part of the whole effect of the primary rays, the

bulk of the secondary rays, that is to say the portion which in a gas manifests itself as direct ionisation, being too soft to ionise.

The penetrating secondary radiation is in most cases too small in amount to make any appreciable alteration in the ionisation pressure curves. Barkla* has shown that the energy of the secondary radiation from a cubic centimetre of air at atmospheric pressure is only about 0·00024 that of the primary beam. Taking this value as approximately correct, we see that even

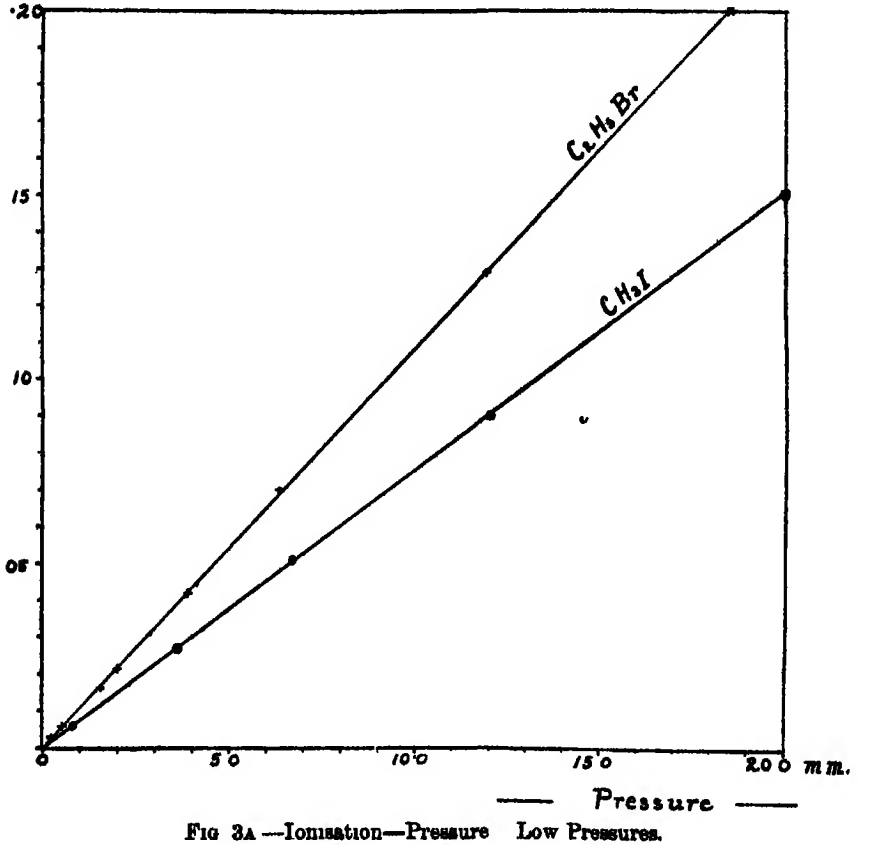

Fig 3A.—Ionisation—Pressure Low Pressures.

in the case of ethyl bromide, which gives the maximum amount of radiation of any of the gases employed, the energy of the secondary radiation at a pressure of 160 mm of mercury is only about 3 per cent. of that of the primary beam. Since the secondary is in this case about three times as absorbable as the primary beam, the amount of secondary ionisation should be about 9 per cent. of that produced by the primary rays at the pressure

* 'Phil. Mag.,' [6], vol. 7, page 543, 1904.

named It is evident, therefore, that for air, carbon tetrachloride, or even methyl iodide, it would be quite inappreciable

On turning to fig 3, it will be seen that the upward tendency in the ionisation-pressure curve for ethyl bromide is quite marked, and from the magnitude of its departure from a straight line, it is easy to deduce that at a pressure of 160 mm of mercury, the ionisation due to the secondary rays is about 16 per cent of that due to the primary As the secondary rays, being scattered in all directions, traversed the whole of the gas between the electrodes, while the primary beam, in order to avoid any possibility of its striking the electrodes, was only allowed to pass through the central portion of the gas, the agreement is sufficiently satisfactory

Absorption of the Primary Rays

When Rontgen rays pass through a gas they are more or less absorbed by it In order to measure this absorption it is necessary and sufficient to find the ratio of the ionisation current through P′ to that through P when the gas

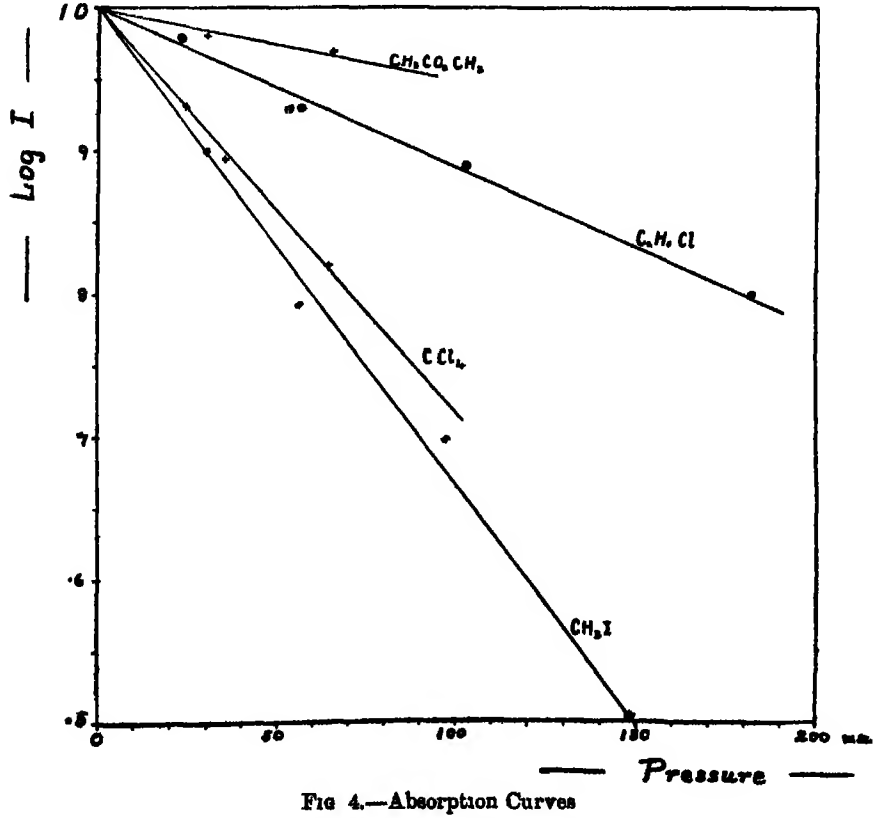

Fig 4.—Absorption Curves

chamber A′ is, first, evacuated, and, second, filled with the gas under observation The first gives a measure of the initial intensity I_0 of the beam; the second the intensity I after passing through the gas

Experiments were made with the different gases at different pressures, and the values thus obtained for $\log_{10} I/I_0$ are plotted against the pressures in fig 4.

The amount of absorption was found to vary considerably with the hardness of the rays, and a certain amount of difficulty was experienced in keeping the bulb quite constant during a long series of readings It will be seen, however, that within the limits of experimental error, the curve obtained is in every case a straight line The law of absorption is, therefore, exponential and may be written

$$I/I_0 = e^{-\lambda \frac{p}{\pi} d},$$

where p is the pressure of the gas,

π is the normal atmospheric pressure,

d the distance traversed by the rays in the gas, and

λ a constant which we may call the coefficient of absorption of the rays for the gas

The values of λ for the different gases employed are given for soft and hard rays, in the third and fifth columns of Table I

With the apparatus employed, it was not found possible to measure the coefficients of absorption of air, hydrogen, or carbon dioxide, as the amount of absorption of the rays in the length of the box (20 cm) was too small to be appreciable It certainly was not more than 2 or 3 per cent., and this was practically within the limits of experimental error In view of the very

Table I

	Soft rays		Hard rays		Secondary radiation	Relative density
	Relative ionisation	λ	Relative ionisation	λ		
Air	1 00	—	1 00	—	1 00	1·00
H_2	0·01	—	0 18	—	0 12	0·07
CO_2	1 57	—	1 49	—	1 54	1 53
$CH_3.CO_2.CH_3$	4·95	0 02	3·90	0·005	2 72	2 57
C_2H_5Cl	18·0	0·044	17 3	0·022	3 2	2·24
CCl_4	67 3	0 24	71	0·06	8·6	5 35
$Ni(CO)_4$	89	0 20	97	0 136	8 1	5·90
C_2H_5Br	72	0 30	118	0 14	217	3 78
CH_2Br_2	71	0 29	—	—	215	3·80
CH_3I	145	0 30	125	0 18	41 5	4·96
$Hg(CH_3)_2$	425	1 16	—	—	—	7 98

peculiar behaviour of hydrogen with regard to ionisation by Rontgen rays, which will be described later, it would be very interesting to have a measure of the absorption of hydrogen for rays of varying quality, and it is hoped to make some experiments on this point in the near future

Relative Ionisation in Different Gases

In calculating the relative ionisations for different gases, air was taken as the standard, and all other gases and vapours were compared with air at the same pressure Writing A′ and A for the ionisation currents across the two gas chambers, the ratio of the values of A′/A when A′ is filled first with the gas under observation, and then with air at the same pressure, gives the value of the relative ionisation for the given gas compared with air

As pointed out in a previous part of the paper, the values thus obtained have to be corrected for the absorption of the rays in the gas before reaching the electrodes When the ionisation due to the penetrating secondary radiation from the gas is appreciable, as in the case of ethyl bromide, this has also been corrected for

It may be mentioned, in passing, that it was found very difficult to "saturate" the current through the more ionisable gases, 120 to 150 volts per centimetre being required to produce even approximate saturation for ethyl bromide or methyl iodide at pressures of 250 mm In order to attain saturation without applying too high a potential, the electrodes in the vessel A′ were approached to about 2 5 cm apart, the primary beam being limited as before by a lead slit to prevent the rays from falling on either electrode.

The values obtained for the relative ionisation, for soft and hard rays, are given in the second and fourth columns respectively of Table I. For purposes of comparison, the amount of secondary radiation, relative to air, is given in the sixth column of the same table, while the seventh contains the relative densities of the different substances

It is at once evident that there is no close connection between the ionisation and the secondary radiation The former, for example, appears to increase more or less uniformly as we pass to compounds containing elements of higher and higher atomic weight, the maximum ionisation occurring in mercury methyl, the ionisation in which is over 400 times that in air The secondary radiation, on the other hand, reaches a maximum in the neighbourhood of bromine, ethyl bromide giving off the largest amount of secondary rays of the gases included in Table I For ethyl bromide and methyl iodide, the values obtained depend upon the quality of the rays employed, and we shall consider them in more detail further on.

Prof Thomson has suggested that the relative ionisation in a gas is an additive property, depending only on the number and nature of the different atoms present The values given in column 2 of Table I enable us to test this suggestion Using carbon dioxide, air, ethyl chloride, and methyl bromide, together with the value for hydrogen, we obtain, on this assumption, the following values for the ionisation per atom in the different elements concerned —[H] = 0 005, [O] = 0 55, [C] = 0 46, [Cl] = 17 0, [Br] = 70·5 Using these values, we can now calculate the relative ionisation of methyl acetate, carbon tetrachloride, and ethyl bromide The results are contained in the following table —

Table II

	Relative ionisation	
	Calculated	Observed.
$CH_3CO_2CH_3$	2 52	4 95
CCl_4	68 4	67 3
C_2H_5Br	71 5	72·5

It will be seen that the agreement is good for carbon tetrachloride and ethyl bromide, but not very satisfactory for methyl acetate It appears, therefore, that the ionisation is not altogether independent of the state of combination of the element and the complexity of the molecule Assuming, however, that the law is approximately correct, we obtain from the results in Table I the further values [Ni] = 85, [I] = 144, [Hg] = 424 It will be noticed that, with the exception of nickel (which for soft rays gives a somewhat larger value than bromine), the relative ionisation increases steadily with the atomic weight.

If we repeat the calculations, using the values obtained with the harder rays, we obtain similar results The discrepancy in the case of methyl acetate is, however, in this case considerably less, the calculated value of the ionisation being 2 81, the observed value 3 90, and possibly with still harder rays the effect of the complexity of the molecule might disappear altogether

Variation of the Ionisation with the Hardness of the Rays

The variation in the relative ionisation due to change in the quality of the ionising Röntgen rays has been previously investigated by McClung* for hydrogen, oxygen, carbon dioxide, and sulphur dioxide

His results are contained in the following table —

* McClung, 'Phil Mag,' [6], vol 8, p. 357, 1904.

Table III

	Relative ionisation	
	Soft rays	Hard rays.
Air	1 0	1 0
H_2	0 10	0 18
O_2	1 80	1 17
CO_2	1 46	1 33
SO_2	11·0	4 8

Comparing these figures with those given in Table I, it will be seen that in the cases of the two gases common to the two tables the results are in general agreement The relative ionisation of hydrogen increases with an increase in the hardness of the rays, while that of carbon dioxide diminishes The value obtained for the relative ionisation of hydrogen for hard rays in the present experiments coincides with that obtained by McClung The value for soft rays is only about one-tenth of the smallest value he obtained, but it is comparable with the value 0 026 obtained by Perrin * The rays in McClung's experiment entered his apparatus through a recessed aluminium plate, which must have been considerably thicker than the thin foil (0 05 mm. thick) used for the windows in the present experiments It is probable, therefore, that the very soft rays which give these very small values for the ionisation of hydrogen were unable to penetrate into his ionisation chamber, being practically all absorbed by the aluminium of the window

Table IV has been drawn up to show in more detail how the relative ionisation of the hydrogen varies with the hardness of the rays, the hardness being represented, as usual, by the length of the spark gap, which is just sufficient to extinguish the bulb

Table IV —Hydrogen

Equivalent spark gap	Relative ionisation
mm	
8	0·010
12	0·013
14	0·021
16	0·068
18	0 107
20	0 135
24	0 152
28	0 180

* Perrin, 'Ann de Chimie et de Phys,' vol 11, p. 496, 1897.

These figures are represented graphically by the curve drawn in fig 5 It will be noticed that the rate of increase is particularly rapid as the equivalent spark gap increases from 14 to 18 mm

Although not so noticeable as in the case of hydrogen, it will be seen from Table I that the relative ionisation of ethyl bromide also increases as the rays get harder Ethyl chloride and carbon tetrachloride remain practically

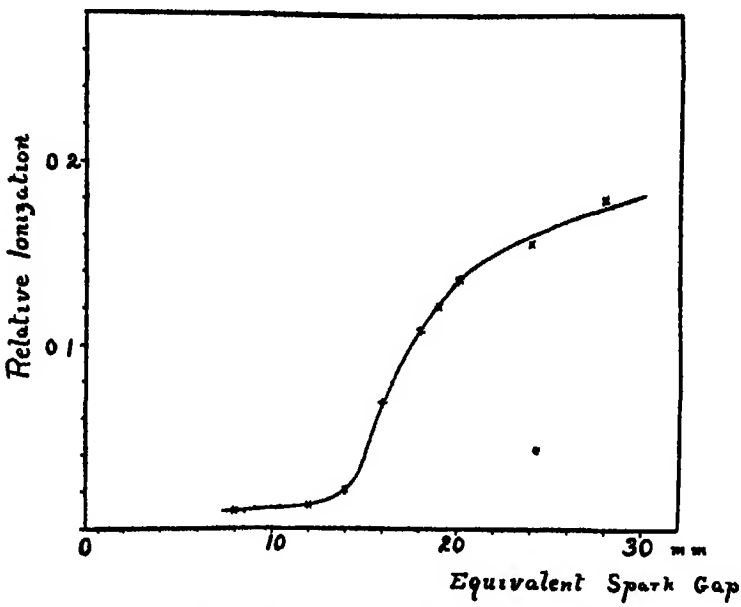

Fig 5 —Ionisation—Hardness. Hydrogen.

constant, while methyl iodide and methyl acetate show a decided decrease as the hardness of the rays increases.

During the progress of the above experiments my attention was somewhat forcibly directed to the necessity for avoiding any contact between the Röntgen rays and the electrodes in all measurements of the relative ionisation produced by these rays in different gases. Having had occasion to remove the gas chamber A′ to make some necessary repairs, I found on replacing it that the values now obtained for the relative ionisation were considerably smaller than before For example, the value for ethyl bromide was reduced from 72 to 29, and that for methyl iodide from 145 to 54. This discrepancy was finally found to be due to the fact that, owing to the vessel A′ having been replaced slightly out of adjustment with respect to the focus tube, the Röntgen rays no longer passed centrally down the vessel, but just grazed one of the electrodes.

Townsend* and other observers have shown that when Rontgen rays fall on a metal plate they cause it to emit soft secondary radiation, consisting principally of soft β-radiation, which from its very absorbable nature, produces in the neighbourhood of the plate an amount of ionisation far exceeding that due to the primary rays themselves The effect of the secondary radiation on the relative ionisation in different gases is not difficult to see. If the pressure of the gas is so high that the secondary rays are totally absorbed before reaching the boundaries of the vessel, then, since the total ionisation produced by these rays is not very different in different gases, the effect will be in every case to add a practically constant amount to the ionisation produced by the primary beam, and thus to reduce the apparent relative ionisation in every gas for which the relative ionisation is greater than unity On the other hand, since the relative ionisation produced in different gases by β-radiation is very nearly proportional to the density, if the rays are not totally absorbed by the gas, the result will be in this case to add an amount, proportional to the density, and thus to bring the apparent relative ionisations more nearly into accord with a density law In either case there would be a large reduction in the values obtained for the relative ionisation of such vapours as methyl iodide, ethyl bromide, or carbon tetrachloride, compared with air

By turning the vessel A′ with respect to the focus tube, so as to allow a larger proportion of the primary beam to impinge on the aluminium electrode, the values obtained for the relative ionisation of ethyl bromide compared with air was further reduced, first to 10 and finally to 3 62 This is very nearly equal to the relative density of the gas Under these circumstances, the pressure-ionisation curve is no longer a straight line, but bends over towards the axis of pressure, as shown in the curve in fig 6, which represents the results obtained for air, with the primary beam grazing one of the aluminium electrodes

These results throw considerable doubt upon some results of Eve† on the relative ionisation produced by very penetrating Rontgen rays, in which he obtained values for the relative ionisations in various gases and vapours agreeing very nearly with a density law. In these experiments the gases were contained in a cylindrical ionisation chamber of brass with an insulated central wire connected to a gold-leaf electroscope In this system the central wire forms one electrode, the walls of the cylinder the other. As the rays were passed into the chamber straight through the walls of the vessel, it will be seen that they thus impinged directly on both electrodes

* 'Camb. Phil Soc Proc.,' vol. 10, p. 217, 1899

† Eve, 'Phil Mag.,' [6], vol. 8, p. 610, 1904.

The conditions were thus favourable for the production of a large amount of secondary ionisation, and its presence is clearly shown by the shape of the curves connecting ionisation and pressure, which are given in Eve's paper These are similar to the curve given in fig 6, which represents the relation between ionisation and pressure for air when the rays are allowed to fall upon one of the electrodes Eve further states that "the portion of the curve between 18 and 50 mm [pressure] was found to be a straight

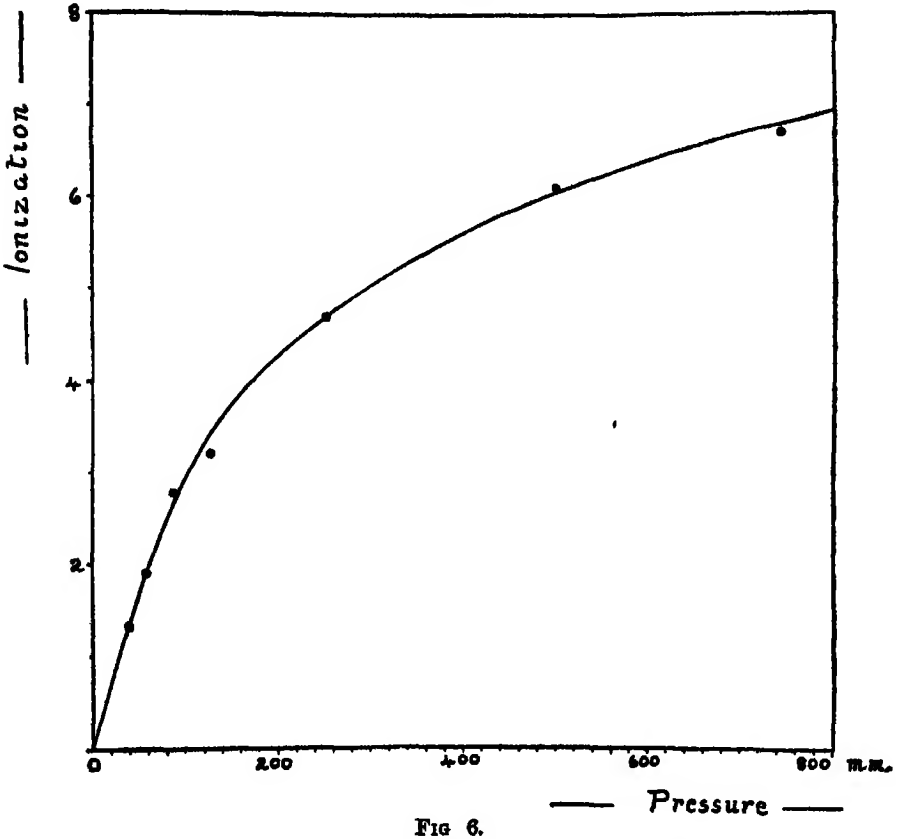

Fig 6.

line, and the readings for the gases used were taken between these limits." A little consideration will show that it is just in these earlier portions of the curve where the amount of the secondary ionisation compared with that of the primary has its largest value It is only when the pressure is high enough to produce a total absorption of the secondary rays that the slope of the ionisation-pressure curve assumes the value proper to the primary Rontgen rays. The curve then has the form

$$I = I_s + I_0 . p,$$

where I is the ionisation in the gas, I_s the ionisation produced by the total absorption of the secondary rays from the walls of the chamber, and $I_0\ p$ represents the ionisation produced by the direct action of the primary rays, which, as has been shown above, is proportional to the pressure p. From the shape of his curves there can be little doubt that, at the lower pressures used by Eve, the ionisation was very largely due to secondary radiation from the walls of his vessel, and that consequently the values given by him in his table refer not to penetrating Rontgen rays, as he supposes, but to the soft secondary rays excited by them in the material of his chamber, that is, in the main, to soft β-rays.

There is no evidence that the ionisation produced by penetrating Rontgen rays in different gases follows a density law. Drawing tangents to Eve's curves for H_2S and air, at the highest pressures given, where, as shown above, we get an approximation to the true slope for the primary rays, we find the value 4 for the relative ionisation of these two gases, as against 0·9 given in Eve's table.

Eve was not unaware of the presence of much secondary radiation in his experiments. He states that "with Rontgen rays there was abundant evidence of secondary and tertiary radiation," and found that lining the brass cylinder with sheet aluminium 0·85 mm. thick reduced the ionisation current from 100 to 22. With the chamber thus lined he repeated his original experiments, and obtained values agreeing with his previous ones, and concludes from this that "there cannot be any wide divergence between the relative conductivities from primary and secondary rays."

The results obtained during the present experiments with the rays striking on the aluminium electrodes show, however, that even aluminium gives off amply sufficient soft secondary rays to entirely mask the effect of the primary ionisation, and that the similarity between the results obtained with the brass, and with the aluminium plated walls, was due not to any similarity between the primary and the secondary rays, but owing to the ionisation produced by the direct action of the primary rays being so small compared with that due to the secondary as to be practically negligible in both cases.

Laby and Kaye* have recently shown that when γ-rays are passed into a gas through the walls of the containing chamber the ionisation in the gas is due almost entirely to the β-radiation from the metal of the walls. The similarity between the results obtained by Eve for penetrating Rontgen rays and for γ-rays would thus appear to be explained by the fact that in

* 'Phil. Mag.,' December, 1908, p. 879.

both cases the bulk of the ionisation was due, not to Rontgen rays or γ-rays, but to secondary β-rays from the walls of the containing chamber.

The results of the present experiments on the variation of relative ionisation with the hardness of the rays certainly do not seem to indicate any approximation to a "density law," such as Eve suggests, as the rays become harder. Among the lighter gases, as McClung's results also showed, the value for hydrogen increases with the hardness of the rays to a value considerably above that given by the relative density, while, on the other hand, the value for carbon dioxide, which for soft rays is nearly normal, falls below that to be expected on a density law as the hardness of the rays increases. Again, among the more ionisable gases and vapours, although the relative ionisation of methyl iodide, compared with air, decreases as the rays become harder, that of ethyl bromide shows, on the other hand, a very perceptible increase, while the values for ethyl chloride and carbon tetrachloride remain practically constant.

Secondary Radiation

Variation of Secondary Radiation with the Hardness of the Rays

The amount of the secondary Rontgen radiation from different gases and vapours was investigated in the previous paper,* and the results there described have been verified in the course of the present experiments. Some additional observations have been made, which it may be of interest to record.

It was shown that for most gases and vapours the amount of secondary radiation given off when compared with that from air remained sensibly constant over the range of primary rays employed. For stannic chloride and methyl iodide, however, there was a distinct increase, with increasing hardness of the primary rays. It was thought of some interest to investigate this matter further.

Accordingly a comparison was made between the relative amounts of secondary radiation emitted by air, ethyl bromide, and methyl iodide for different degrees of hardness of the primary rays. The results obtained are given in Table V. The first column of this table gives the hardness of the rays used, as measured by the equivalent spark gap of the focus tube, the second column gives the amount of secondary radiation from ethyl bromide compared with that from air, the ratio for the softest rays being reduced to unity to facilitate comparison, the third column gives, in the same way, the ratios for methyl iodide and air, and the fourth the ratio for methyl iodide

* 'Phil. Mag.,' [6], vol. 14, p. 653, 1907.

compared with ethyl bromide, calculated in the same way In the fifth column the relative ionisation of methyl iodide compared with that of ethyl bromide is given for the sake of comparison, the ratio for the soft rays having been reduced to unity, as in the other columns

Table V

Equivalent spark gap	Secondary radiation			Relative ionisation CH_3I/C_2H_5Br
	C_2H_5Br/Air	CH_3I/Air	CH_3I/C_2H_5Br	
mm				
8	1 00	1 00	1 00	1 00
14	1 00	1 27	1 27	—
18	0 98	1 60	1 63	—
22	0 91	2·08	2 28	—
26	0 80	2 42	3·03	0 52

All the values given have been corrected for the absorption of the primary rays in the gas, and for the absorption of the secondary rays by the aluminium of the window *d* The latter correction was not necessary in the case of methyl iodide and air, which give out rays of approximately the same penetrating power It was, however, necessary in the case of ethyl bromide, where the secondary radiation is considerably softer than that given out by either air or methyl iodide

A reference to Table V will show the general nature of the results obtained It will be seen that the amount of secondary radiation given off by ethyl bromide compared with air is practically independent of the hardness for soft and moderately hard rays, that is for rays with an equivalent spark gap of between 8 and 18 mm. For very hard rays, however, there is a distinct diminution in the values obtained, the ratio for the hardest rays used being 20 per cent. less than that for softer radiation

For methyl iodide, on the other hand, as was noted before, the amount of secondary radiation, compared with that from air, shows a rapid increase, with an increase in the hardness of the rays, the ratio for hard rays being nearly 2½ times that for the softer radiation

Comparing, now, methyl iodide with ethyl bromide, we see that the relative amount of secondary radiation given out by the former compared with that given out by the latter is trebled as the equivalent spark gap increases from 8 mm to 26 mm. On the other hand, the ratio of the ionisations in the two gases decreases to about half Since the radiation from ethyl bromide is greater than that from methyl iodide, while the relative ionisation is less, the effect of this double change is to bring these two vapours more nearly into

agreement with each other as the hardness of the rays increases. They give, however, no evidence of approximating to the lighter gases, such, for example, as air or carbon dioxide

Absorbability of the Secondary Radiation.

The absorbability of the secondary radiation from gases containing only elements of low atomic weight is similar to that of the primary producing it. Beatty,* who has made some careful experiments on the secondary radiation from air, concludes that even for air there is evidence of a certain amount of transformation of the very penetrating rays into a slightly softer type of radiation The effect is, however, very small, and the general statement made above certainly holds to within a few per cent The rays from methyl iodide and stannous chloride are also of the same type as those from air. From ethyl bromide, on the other hand, we get radiation of a much softer character, the coefficient of absorption for tin foil being nearly three times as great for the secondary as for the primary. It was thought interesting to see if the ratio of the two coefficients had the same value for the absorption of the rays by the gas itself, as for absorption outside the gas, by tin foil

No direct experiments have as yet been made on this point, but some experiments, which were made on the variation of the amount of secondary radiation emitted, with variation in the pressure, allow at any rate a rough calculation to be made of the coefficient of absorption of the secondary rays in the gas itself

The experimental curve connecting the amount of secondary radiation from ethyl bromide with the pressure is given in curve I, fig 7 It will be seen that it bends over considerably towards the axis of pressure. Now it has been shown in the previous paper that where there is no appreciable absorption, the amount of secondary radiation emitted is simply proportional to the pressure The departure of the experimental curve from the linear law at higher pressures is due—

(i) to the absorption of the primary rays in the gas,
(ii) to the absorption of the secondary rays in the gas

Now the absorption of the primary rays can be easily measured, as has been described above We can therefore correct the experimental curve for the absorption of the primary rays. The curve so obtained is marked II, in fig. 7 This curve still bends over towards the axis of pressures.

Since the actual amount of secondary radiation emitted is represented by the straight line III, tangential to curve II at its origin, while the amount

* 'Phil. Mag.,' [6], vol 14, p. 604, 1907

emerging from the gas after partial absorption is represented by curve II, the ratio of the ordinate of the latter to that of the former for any given pressure gives the ratio I/I_0 for that pressure

Plotting the logarithm of I/I_0 for different pressures against the pressure, we obtain curve IV This, as will be seen, is a straight line within the limits of experimental error, and represents the absorption of the secondary rays from the gas by the gas itself Taking 3 4 cm as the mean distance in

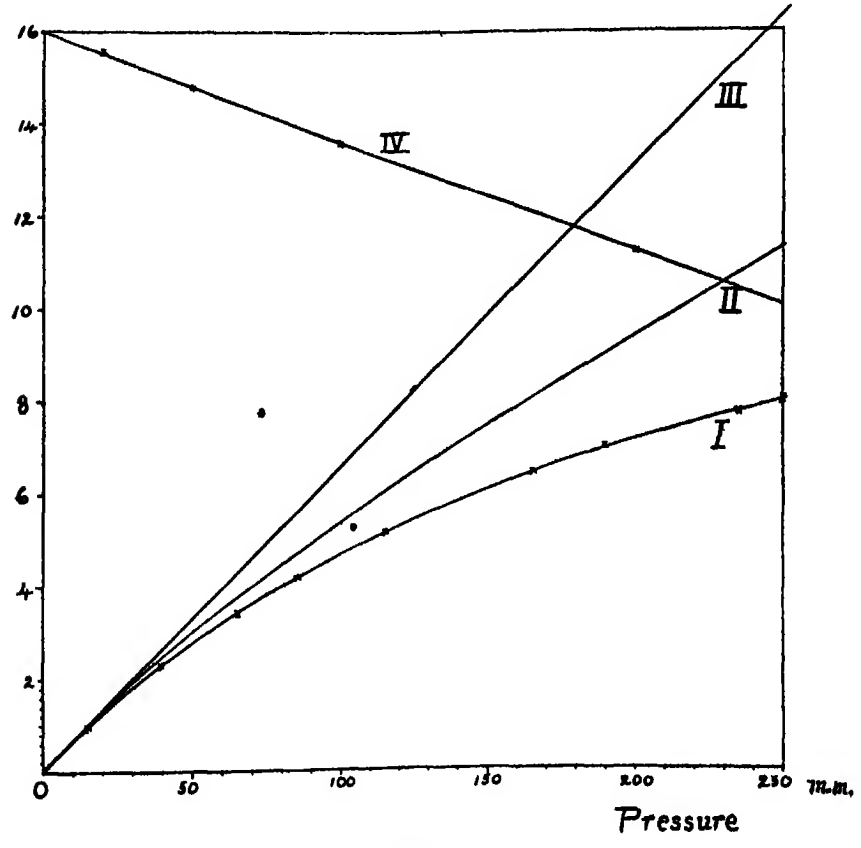

FIG 7

the gas traversed by the secondary rays before reaching the window, we obtain the value 0·40 as the coefficient of absorption of the secondary rays from ethyl bromide, in ethyl bromide itself The coefficient of absorption of the primary rays in the gas for the rays used (which were hard) was found to be 0 16 The ratio of the two coefficients is therefore 2 5 On measuring the absorption of tin foil, first for the primary beam and then for the

secondary, the ratio of the coefficients of absorption for tin foil was found o be 2 7.

A repetition of the experiments, with methyl bromide as the radiating gas, gave very similar results

The calculations are only approximate, but as far as they go they would seem to indicate that a gas or vapour is neither exceptionally transparent, nor exceptionally opaque, to the radiations it emits

Total Ionisation.

The total ionisation produced by the complete absorption of Rontgen rays of the ordinary type in different gases has not yet been directly measured. If q is the relative amount of ionisation produced in a given gas, and λ the coefficient of absorption of the rays for the gas, then it can easily be shown that the relative total ionisation T is given by the equation

$$T = q\int_0^\infty e^{-\lambda x}dx,^*$$

i.e.,

$$T = q/\lambda$$

Both q and λ are already known We can, therefore, calculate the value of T for the different gases employed If q/λ is the same for all gases, the total number of ions produced in different gases by rays of given intensity will be the same

The values obtained for the ratio q/λ for the different gases are given both for soft and hard rays in Table VI As it was not possible to measure the

Table VI

	Total ionisation (calculated)		Total ionisation (experimental) α-rays *
	Soft rays	Hard rays.	
Air	—	—	1 00
$CH_3CO_2CH_3$ ...	1 00	1 ·00	—
C_2H_3Cl	1 64	1 ·0	1 32
CCl_4	1 12	1 5	1 32
$Ni(CO)_4$	1 ·76	0 ·91	—
C_2H_3Br	0 96	1 ·06	—
CH_3Br	0 98	—	1 32
CH_3I	1 ·92	0 88	1 33
$Hg(CH_3)_2$	1 46	—	—

* Kleeman, 'Roy Soc Proc.,' A, vol 79, p 222, 1907.

coefficient of absorption for air, the value of q/λ for methyl acetate has been taken as the standard and reduced to unity. For the sake of comparison,

* J J Thomson, 'Conduction through Gases,' p. 303, 1906

some experimental values obtained by Kleeman for the relative ionisation produced in different gases by the total absorption of α-rays are given in the last column of the table

It will be seen that the ratio q/λ for different gases is not a constant, the variations being considerably greater than can be accounted for by experimental error The relative ionisation q is easy to measure with accuracy, and in the case of such gases as methyl iodide, mercury methyl, ethyl bromide, and carbon tetrachloride, where the absorption is high, the coefficient of absorption λ can also be measured with a fair degree of accuracy The values of λ for hard rays are less reliable than those for soft, as not only is the absorption of its rays in the gas much less, but also it is more difficult to keep the rays constant during a series of observations Even with hard rays the values for methyl iodide and ethyl bromide are probably not more than a few per cent. out

It will also be noticed that the relative values of q/λ for the different gases vary with the hardness of the rays, especially in the case of methyl iodide and nickel carbonyl, the properties of which seem very sensitive to fluctuations in the quality of the rays Thus for soft rays methyl iodide gives a value almost exactly twice as large as ethyl bromide, while for hard rays its value is only about four-fifths that of the latter gas

The values obtained for the hard rays are more nearly constant than those for the softer radiations Carbon tetrachloride appears out of place, but the absorption of the hard rays in this vapour is small, and too much importance should not be attached to this result

It may be mentioned that the measurements of q and λ for any given gas were made simultaneously, in order to ensure their being strictly comparable As both q and λ vary with the hardness of the rays, this precaution is of considerable importance

Energy spent per Ion

If the whole of the energy absorbed by a gas were spent in ionising the gas, then the work done in making one ion in the gas would be inversely proportional to the total number of ions made—that is to say, it would be inversely proportional to the total ionisation. Some of the energy absorbed, however, is given out again as secondary radiation In the case of ethyl bromide, for example, as much as one-third of the energy absorbed may be given out as secondary rays. Since the relative ionisation has been corrected for any ionisation due to the secondary rays, we must regard this energy as adding nothing to the total ionisation in the gas.

The amount of secondary radiation from the different gases and vapours is known, it is therefore quite easy to make correction for the energy lost in

this way The values, thus corrected, for the energy spent per ion in the different vapours are given in Table VII, methyl acetate being again taken as the standard in the absence of any reliable data for air

Table VII

	Energy spent per ion.	
	Soft rays	Hard rays
$CH_3CO_2CH_3$	1 00	1 00
C_2H_5Cl	0 61	1 0
CCl_4	0 89	0 7
$Ni(CO)_4$	0 57	1 1
C_2H_5Br	0 86	0 7
CH_3I	0 52	1 0
$Hg(CH_3)_2$	0 69	—

If there is no further loss of energy in addition to secondary radiation, these figures give the relative amount of energy required to produce an ion in the different vapours

It will be seen that the results obtained, though all of the same order, are somewhat different for different gases Ethyl chloride, carbon tetrachloride, and ethyl bromide, give figures distinctly smaller than those for methyl acetate The values for nickel carbonyl and methyl iodide vary rapidly with the hardness of the rays Unless some of the energy absorbed is dissipated in other ways than ionisation and secondary radiation, it would seem that in these gases soft rays produce an ion with the expenditure of only about half the energy required in the case of hard rays

Summary and Conclusion

In the present work a series of experiments has been made, under comparable conditions, on the behaviour of different gases and vapours with respect to the passage of Rontgen rays through them The results obtained may be briefly summarised as follows —

(1) The amount of ionisation produced by the direct action of the primary Rontgen rays on a gas is simply proportional to the pressure of the gas. No evidence was obtained of the emission of any appreciable amount of soft secondary radiation by the gas, the ionisation being apparently due to the direct action of the primary rays

(2) The relative ionisation in the different gases compared with air as the standard varies considerably with the hardness of the rays. Hydrogen and ethyl bromide show an increase as the hardness of the rays increases.

Other gases remain constant or give a diminution There is no indication of any approximation to a "density law" as the hardness of the rays is increased

(iii) The relative ionisation in a gas follows approximately an additive law It does depend somewhat, however, on the state of combination, especially for soft rays

(iv) The absorption varies with the pressure according to an exponential law

(v) The amount of secondary radiation emitted by different gases relative to air is, generally, approximately independent of the hardness of the primary rays For very hard rays ethyl bromide shows a slight decrease On the other hand, the values for methyl iodide increase fairly rapidly as the hardness of the rays is increased

(vi) The coefficient of absorption of the secondary rays emitted by a gas, in the gas itself, is not abnormal

(vii) The total ionisation in different gases is not a constant, and the relative values obtained differ with the hardness of the rays

(viii) The amount of energy required to produce an ion in different gases is different, and also varies with the hardness of the rays

No relationship has been found between the relative ionisation and the secondary radiation, or between either, and any other known property of the gases and vapours, and the explanation of the relatively large amounts of secondary radiation emitted by ethyl bromide and its class compared with air, and of the large relative ionisations in methyl iodide, ethyl bromide, etc., still remains to be sought

It appears that on the whole less energy is required to produce an ion in the more ionisable gases, but the values obtained do not differ very largely, and are totally inadequate to explain the very large amounts of ionisation in these gases and vapours.

Both the ionisation and the secondary Rontgen radiation follow, at any rate approximately, an additive law It appears, therefore, that these properties are properties of the atoms themselves, and that an explanation must be sought in their atomic structure

In conclusion, I wish to express my warmest thanks to Sir J. J. Thomson, for his helpful advice and stimulating interest during the course of the present experiments.

On the Velocity of the Cathode Rays ejected by Substances exposed to the γ-Rays of Radium

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(Communicated by Prof Sir J J Thomson, F R S Received December 22, 1908, —Read January 14, 1909)

§ 1

The cathode radiation from a substance exposed to the γ-rays of radium consists, as is well known, partly of rays of a very penetrating character Thus Eve found that the secondary rays from a lead plate, which in his experiments were deflected by a strong magnet into an ionisation chamber, possessed approximately the same penetrating power as the β-rays of uranium He tested the penetrating power of these deflected rays by placing successively different thicknesses of aluminium foil over the opening of the chamber and measuring the changes produced in the leak

This paper gives an account of some experiments of a more elaborate character by the writer on the subject Two different methods were used in these experiments One of the methods consisted in measuring the scattering of the cathode particles produced by different thicknesses of metal foil placed in the path of the rays. In the other method the cathode rays were deflected into an ionisation chamber by means of a strong electro-magnet, and the ionisation in the chamber measured for magnetic fields of different strengths

It will be then seen that each method has its particular advantages in bringing out the different properties of the secondary radiation.

The form of apparatus used in the first method is shown in fig 1 A is an aluminium ionisation chamber 12 5 cm long, 10 cm high, and 7 cm. deep, supported by a wooden frame arrangement from which it was insulated The metal plate B resting on the adjustable table C formed the lower side of the chamber The plate could be removed and replaced by another after lowering the table a convenient distance from the chamber Thirty milli-grammes of radium bromide (enclosed in a glass tube) were placed in the cavity of the lead cylinder D, and a piece of sheet lead 3 mm thick placed over the opening of the cavity to cut off the β-rays E is another ionisation chamber whose electrode was connected with that of the chamber A One of the chambers was connected to a positive potential of 200 volts and the

other chamber to a negative potential of 200 volts. The ionisation in both chambers was produced by the γ-rays from the radium, the amount of ionisation in the chamber E being regulated by screens of different thicknesses placed between the radium and the chamber The object of the chamber E was to partially compensate by means of its leak that part of the leak in the chamber A which is not due to the cathode rays from the plate B.

The penetrating power of the secondary cathode rays emitted by the plate B was investigated by measuring the ionisation in the chamber for different thicknesses of aluminium or paper foil placed upon the plate. If the radiation from the foil is small in comparison with the radiation from

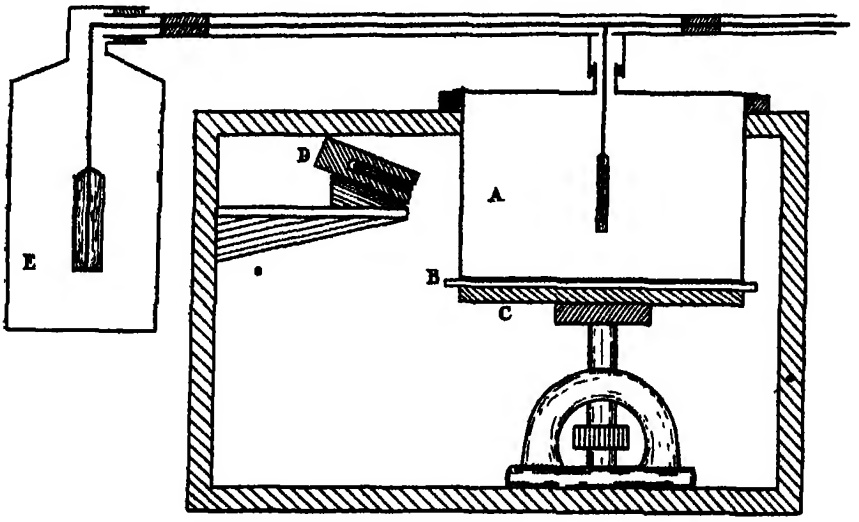

Fig 1

another sheet of foil of the same mass per cm^2, but of the material of the plate B, the secondary radiation from a combination of the plate B and a sheet of foil is the radiation from the plate modified by the scattering of the foil. The coefficient of absorption of the foil is in that case easily obtained. Thus, let M denote the leak in the chamber A produced by the secondary cathode radiation from the plate B, and N the remaining leak in the chamber minus the leak in the chamber E. The leak obtained when there is no foil on the plate is then $(M+N)$ When a sheet of foil is placed on the plate the leak obtained is $(\phi M+N)$, where ϕ denotes the fraction of the secondary radiation from the plate which penetrates the foil. If an additional sheet of foil of the same thickness is placed on the plate, the leak obtained is $(\phi^2 M+N)$. The difference between the first and third

leak divided by the difference between the first and second then gives the value of $(1-\phi^2)/(1-\phi)$, that is of $(\phi+1)$. And if x denotes the thickness and λ the coefficient of absorption of the foil, we have $e^{-x\lambda} = \phi$, from which λ can be obtained

The value of ϕ, we have seen, gives the fraction of the amount of radiation from the plate which penetrates a given thickness of foil But this applies only to the whole radiation from the plate if it is homogeneous If the radiation is heterogeneous, the value of ϕ applies only to that part of the radiation which is easily absorbed in comparison with the remaining part This follows from the particular way in which ϕ is obtained Thus, let the radiation from the plate be divided into two parts, A and B, and let the values of ϕ for the two parts be ϕ_1 and ϕ_2 respectively, The difference between the leak with a sheet of foil on the radiating plate and the leak without the foil is then $A(1-\phi_1)+B(1-\phi_2)$ Now, we may suppose the radiation divided into two parts, such that $(1-\phi_2)$ is so small that $B(1-\phi_2)$ is negligible in comparison with $A(1-\phi_1)$ The difference between the leak with two sheets of foil and the leak without foil is then $A(1-\phi_1^2)$, and the ratio of this difference to the other difference equal to $(1+\phi_1)$ Therefore if the rays are heterogeneous, the value of ϕ found by the above method and the coefficient of absorption deduced therefrom applies to the set of rays A. This method thus brings the existence of the soft rays in a set of rays into evidence, and furnishes also a measure of their penetrating power

The value obtained for ϕ, it will be observed, is influenced by the secondary radiation from the foil itself, if this is not of negligible magnitude. For the radiation from the foil, when added to the part of the radiation from the plate which penetrates the foil, has the effect of making the fraction of the radiation which penetrates the foil appear larger than it really is. A smaller value for λ than the true value will consequently be obtained

But by using a proper combination of plate and foil, the effect of the radiation from the foil becomes negligible Thus, if the radiating plate is of aluminium, or of a metal of higher atomic weight than aluminium, and paper foil used to absorb the secondary rays, the effect of the radiation from the paper on the value of ϕ is small. This was proved in the following manner — The radiations from a sheet of paper and a sheet of aluminium foil were compared with one another, each sheet radiating under the same conditions as the radiating plate. The details of this experiment will be described further on. The sheets of foil were two of those used to absorb the cathode rays in these experiments, the paper foil weighed $6\,4\times10^{-3}$ and the aluminium foil $9\,27\times10^{-4}$ gramme per cm^2 It was found that each of these two sheets of foil produced practically the same amount of secondary

radiation The radiations from aluminium and paper for equal masses per cm.^{2} are therefore in the ratio of 1 to 0 13 Consider the effect of placing a sheet of aluminium foil on an aluminium radiating plate. It is evident that the decrease of secondary radiation from the plate due to the scattering of the foil is exactly counterbalanced by the radiation from the foil But if a sheet of paper foil of the same mass per cm^2 as the aluminium foil be placed on the plate, the radiation from the paper, according to the foregoing, would be only about 0 13 of that from the aluminium foil. But the amount of scattering produced would be the same in each case This follows from the experiments on the scattering of cathode rays Thus it has been shown that the amounts of scattering of aluminium and paper for equal masses per cm^2 are equal to one another over a wide range of velocities of the cathode rays produced in a vacuum tube, and in the case of the β-rays of uranium We may suppose, therefore, as a first approximation, that a sheet of paper foil placed on a plate of aluminium produces only scattering of the radiation from the plate

But this must also be true if a radiator of higher atomic weight than aluminium is used For in that case, since the radiation from equal masses increases with the atomic weight of the material, the difference between the amounts of radiation of the paper foil and an equivalent piece of foil of the same material as the radiating plate is even greater than in the foregoing case

Further, since the ratio of the atomic weight of lead to that of aluminium is greater than the ratio of the atomic weight of aluminium to the average atomic weight of paper, this will obviously be also true in the case of aluminium foil and a lead plate

The relative amounts of radiation from a sheet of paper and a sheet of aluminium foil were found in the following manner The table C was removed, and a wire gauze fixed to the chamber so as to form its lower side The leak in the chamber was then measured A sheet of paper foil fixed to a metal frame was then placed underneath the gauze so as to be in contact with it, and the leak again measured In the same manner a leak was obtained with a sheet of aluminium foil fixed to the frame The first leak was then subtracted from the second and third, the result giving the amounts of radiation from the paper and aluminium foil respectively

Table I gives the values of ϕ for paper foil obtained with a number of different metals in the manner described. Each of the measurements required for calculating ϕ is the mean of at least eight readings The readings were taken in the same manner as described in a previous paper.*

* 'Phil. Mag.,' November, 1907, p. 619.

The *apparent* leak was usually decreased by about 50 per cent when a sheet of foil was placed on the plate *

Table I.

Radiator and its thickness		ϕ for paper weighing 6.4×10^{-3} gramme per cm.^{2}
	mm	
Pb	2	0.53
Pb	0.25	0.50
Sn	3	0.54
Zn	3	0.54
Cu	2.5	0.55
Fe	2.5	0.52
Ni	2.5	0.53
S	4	0.52
Al	3.5	0.53

It will be seen that the value of ϕ is approximately the same for the different metals, the mean value being about 0.53. Thus part of the secondary cathode radiation from each of these plates is reduced to about one-half by a sheet of paper (weighing 6.4×10^{-3} gramme per cm^2) placed close to the radiating plate. If all the radiation from a plate possessed the same penetrating power as the β-rays from uranium, the value of ϕ would be 0.99, and a single sheet of paper foil would in that case stop about 1 per cent of the radiation. It appears, therefore, that the secondary cathode radiation from a plate on the side where the γ-rays enter consists partly of very absorbable rays.

The penetrating power and heterogeneity of this absorbable radiation is best brought out by considering the values of ϕ and λ found for aluminium foil. Aluminium foil is more homogeneous than paper, and it is therefore better to use aluminium foil when possible. Table II gives the values obtained for ϕ and λ when a radiator of lead was used. The coefficient of absorption of aluminium for the cathode rays in a vacuum tube, and for the β-rays of uranium oxide, are placed in the table for comparison. The first column in the table gives the nature of the source of cathode radiation. The second column contains values of ϕ. The first value of ϕ given corresponds to the soft secondary rays from a lead plate exposed to γ-rays, and the second to those rays which penetrated a layer of aluminium leaf weighing 2.72×10^{-3} gramme per cm^2 placed upon the plate. The third column gives the values of λ for the different cathode rays.

It will be seen that the coefficient of absorption of aluminium for the soft

* The paper was blackened with graphite in order to make it a good conductor of electricity.

cathode rays from the lead plate is very much greater than that found for the β-rays of uranium, but smaller than that found by Lenard for the cathode rays in a vacuum tube A comparison of these coefficients of absorption with some coefficients of absorption and corresponding velocities

Table II

Nature of the source of cathode radiation	ϕ for aluminium leaf weighing $9\cdot27 \times 10^{-4}$ gramme per cm^2 Calculated thickness of leaf, $3\cdot4 \times 10^{-4}$ cm Equivalent in mass to a thickness 0·72 cm of air	λ
Secondary cathode rays from a plate exposed to the γ rays of radium	0 52	1898
The secondary cathode rays from the lead plate passed through aluminium leaf weighing $2\cdot72 \times 10^{-3}$ gramme per cm^2 Equivalent in mass to a thickness of 2 cm of air	0 804	640
Cathode rays in a vacuum tube Velocity about 6×10^9 cm /sec (Lenard)	—	7150
β rays from uranium (Rutherford)	—	14

given by Lenard,* suggests that the velocity of these soft secondary rays, on the supposition that they are homogeneous, is about twice that of the cathode rays in a vacuum tube and one-third that of the β-rays of uranium. But the rays are not homogeneous, as will be shown presently, and the velocities therefore range above and below this value

Next let us consider the values of ϕ, which, it will be remembered, denote the fractions of the soft cathode radiations penetrating a piece of foil of a given thickness Thus aluminium foil $3\cdot4 \times 10^{-4}$ cm thick reduces the soft radiation from a lead plate to 0 5 of its original value. The sheet of foil is equivalent in mass to a layer of air 0 72 cm. thick. Therefore, if we assume that the absorption of the rays by air and aluminium is the same for equal masses per cm^2, half of this soft radiation is absorbed in the 0 72 cm. of air adjacent to the plate.

When the cathode rays were first sifted through an aluminium layer weighing $2\cdot72 \times 10^{-3}$ gramme per cm., the value obtained for ϕ was 0 82, which is a larger value than that obtained without previous sifting. The radiation which penetrated the aluminium layer did not, therefore, contain as large an amount of soft radiation as the whole radiation from the plate.

* See 'Conduction of Electricity through Gases,' by Prof. J. J Thomson, second edition, p. 381.

Some of the soft rays were, therefore, completely absorbed by the aluminium layer And since the layer is equivalent in mass to about 2 cm. of air, some of the cathode rays from the plate were *completely* absorbed in the 2 cm. of air adjacent to the plate

The radiation which penetrated the aluminium layer contained much less soft radiation than the whole radiation from the plate Thus about 0 5 of the whole radiation from the plate was absorbed by a sheet of aluminium foil $3{\cdot}4 \times 10^{-4}$ cm. thick, while only (1—0 82) or 0 18 of the radiation which penetrated the aluminium layer was absorbed by the same sheet of foil

The foregoing considerations show that the soft rays from a plate are heterogeneous They also show that the coefficients of absorption found apply principally to those cathode rays which have a range of 1 or 2 cm only The properties of these soft rays were brought into prominence not only by the particular method of investigation adopted, but also by using very thin sheets of absorbing material For if a heterogeneous beam of cathode rays is gradually cut down by successive pieces of thin foil, the decrease will be more rapid at the beginning on account of the absorbable radiation being more rapidly absorbed than the more penetrating The coefficient of absorption obtained with thin foil will therefore be larger than that obtained with foil which is much thicker, and thin foil therefore brings out the properties of the soft radiation to a greater extent than thick foil

Table I shows that the value of ϕ for a given thickness of paper foil is independent of the nature of the radiating plate Thus the nature or penetrating power of the soft radiation in question is independent of the nature of the radiating material It does not follow, however, that this result must also be true for the penetrating radiation emitted by the plate It is probable that the penetrating cathode radiation set free in a plate is successively transformed into other radiations of less and less penetrating power, the radiation ultimately produced from all materials being δ-rays, which, in their turn, do not produce any secondary radiation The soft secondary radiation from a plate produced in that way would be practically independent of the nature of the radiating plate It seems, therefore, that the soft radiation is largely produced by the penetrating β-rays ejected by the γ-rays. Other experiments seem to point to the same conclusion.

Thus it was found that the nature of the cathode rays is practically independent of previous sifting of the γ-rays through a thick screen of metal. This result is shown by Table III. The soft radiation is, therefore, not produced by very absorbable rays in the beam of γ-rays, for previous sifting of the beam would diminish the very absorbable rays in a greater

proportion than the more penetrating rays, and the penetrating power of the secondary cathode radiation would thereby be increased.

Table III

Lead screen 1 3 cm thick	$\phi = 0\ 50$
Zinc screen 2 0 cm thick	$\phi = 0\ 50$
Lead screen 0 3 cm thick	$\phi = 0\ 53$

Very little of the soft cathode radiation seems to be produced by the soft γ-rays generated in the plate, for the penetrating power of the radiation is practically independent of the thickness of the radiating plate Thus the values of ϕ in Table I for two lead radiators, one of which was 0 25 and the other 2 mm thick, were 0 50 and 0 53 respectively Some of the radiation must, however, be produced by the secondary γ-rays, since secondary γ-rays are produced in a substance exposed to γ-rays, which are so soft as to be almost entirely absorbed by the substance * It may be mentioned in passing that there is some indirect evidence that such soft γ-rays are likely to be produced Thus Barkla† found that the secondary X-radiation from carbon becomes softer in comparison with the primary X-rays as the hardness of the primary rays is increased

The soft cathode radiation possesses considerable ionising power, since the leaks were not appreciably altered by reversing the sign of all the potentials If the leaks consisted largely of cathode particles which originated in the plate, the magnitude of the leaks would have been considerably decreased when the potential of the chamber was changed from positive to negative

The foregoing method may also be used to investigate the penetrating power of the soft radiation on the side of the plate where the γ-rays emerge But in this case the radiations from two sheets of foil of equal mass but of different materials do not differ so much from one another as in the foregoing case This is due to the fact that the radiation from the side of a plate where the γ-rays emerge does not decrease so rapidly with decrease of atomic weight of the plate as the radiation from the other side This effect was explained by Prof Bragg and Dr Madsen by supposing that the secondary cathode rays are initially projected in the direction of propagation of the γ-rays ‡ The value of ϕ obtained by this method will therefore be larger than the true value; but still the results with this restriction will be seen to be of importance

* Kleeman, 'Phil. Mag,' May, 1908, p. 638.
† Barkla, 'Phil. Mag,' February, 1908, p. 288
‡ 'Phil. Mag.,' May, 1908, p. 663

The experiments in this case were carried out in the following manner. The adjustable table was moved to one side of the chamber, and the radiating plate held in position by two arms attached to the table The radium was placed underneath the plate at a distance of about 9 cm. from its centre. Readings with foil were then taken in the same way as before

The values found for ϕ for aluminium and paper foil when a lead radiator was used were 0·45 and 0·16 respectively These values are smaller than those found under previous conditions, viz, 0·52 and 0·53 respectively. The difference is greater with paper than with aluminium foil, this being probably due to the radiation from the paper foil affecting the value of ϕ to a less extent than the radiation from the aluminium foil The radiation from the foil, we have seen, affects the values of ϕ for the side where the γ-rays emerge to a greater extent than the values for the other side And since the effect of the radiation is to make ϕ greater than its true value, the difference between the values for the two sides of the plate must be even greater than those given by the above values It appears, therefore, that the soft radiation on the side of the plate where the γ-rays emerge is considerably softer than that on the other side

This is an interesting result, but the true explanation is not at all clear One way of explaining it is as follows.

The soft radiation, we have seen, is to a large extent produced by the cathode rays projected by the γ-rays Since the cathode rays produced by the γ-rays are projected in the direction of propagation of the rays, it is not improbable, on the æther-pulse theory, that a γ-ray deflects in the direction of its propagation the soft cathode radiation over which it may happen to pass. This may take place principally in the following way

A γ-pulse is probably followed by a number of pulses of much less intensity produced by the same electron as the principal pulse, and also by secondary pulses produced in the matter which it traverses These pulses will pass over the cathode ray produced by the γ-pulse, since they are all (for a time at least) moving in the same direction The soft radiation produced by the cathode ray might, therefore, be to some extent deflected in the direction of propagation of the pulses

It is not improbable, however, that this is not the true explanation of the effect. Further investigations on secondary radiation and scattering of cathode rays will probably make this clear

Some experiments were also made on the soft radiation produced by the β- and γ-rays together on the emergent side of a plate. The radiating plate was a sheet of lead 0·25 mm. thick, stretched over a frame. The radium was placed underneath the plate in the same position as in the experiments just

described. It was not covered by a screen, and some of the β-rays which penetrated the glass tube therefore penetrated in part the lead sheet. The soft cathode radiation produced was therefore due to the action of the β- and γ-rays.

The values of ϕ for aluminium and paper foil obtained with this arrangement were 0 57 and 0 29 respectively These values are larger than those obtained with a lead plate 2 mm thick, and with the β-rays cut off by a screen, viz, 0 45 and 0 16 respectively It appears, therefore, that the soft radiation produced by the β- and γ-rays together is of a more penetrating character than that produced by the γ-rays alone. This is probably due, considering the way ϕ was obtained, to the β-radiation produced by the γ-rays being more heterogeneous than the β-rays of radium after having passed through the walls of a glass tube and a sheet of lead. And the less penetrating radiation was therefore smaller in comparison with the remaining radiation when both the β- and γ-rays were used, than when the γ-rays alone were used It would appear afterwards, from the magnetic deflection experiments, that the β-rays ejected by the γ-rays are heterogeneous—that is, have different velocities.

Table IV gives some values of ϕ obtained for different distances of the radium from the radiating plate The values refer to the side of the plate where the γ-rays entered It will be seen that there is practically no difference in the values obtained The nature of the soft radiation is, therefore, as we would expect, independent of the distance of the radium from the radiating plate

Table IV.

Distance of radium from lead radiator	ϕ
cm	
11·0	0 50
12 5	0 53
17·0	0 54

§ II

The experiments in which the velocity of the cathode rays was investigated by deflecting them by a magnetic field will now be described The form of apparatus used in these experiments is shown in fig. 2 A is an ionisation chamber; it consisted of an oblong lead box 8 cm. long, 5 5 cm. high, and 6 cm. deep. The side *ab* of the chamber consisted of tightly stretched tissue-paper. B and C are lead blocks which served to screen the chamber

from the direct action of the γ-rays from the radium contained in the lead cylinder D, they were 5 cm and 5·5 cm. thick respectively But since it is impossible to obtain perfect screening from γ-rays, there was a constant small leak in the chamber due to the rays which penetrated the lead blocks. F is an aperture of square section, 3 × 3 cm.^{2}, in the lead block B, its cross-section was made smaller when required by tightly packing its sides with suitable lead strips A metal plate a few millimetres thick was placed at *c*, on top of the aperture, or at *d*, at the bottom of the aperture This plate was the source of the secondary cathode radiation under investigation The rays were deflected by means of a magnetic field, whose direction was at

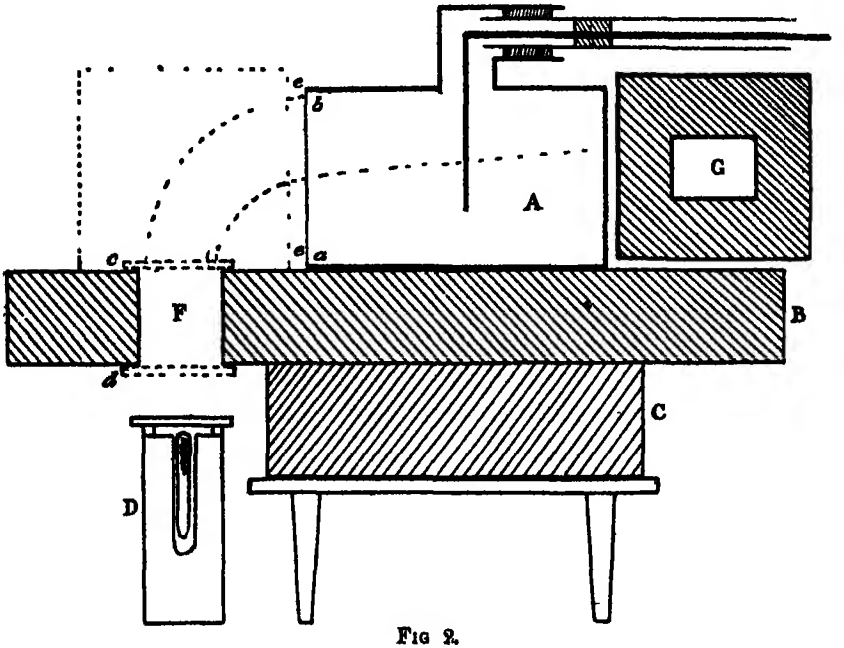

Fig 2.

right angles to the plane of the paper, towards the tissue-paper window of the ionisation chamber A. The increase of leak in the chamber caused by the β-particles which penetrated into the chamber was measured for magnetic fields of different strengths The position of the pole-pieces of the magnet (which were of square section) with respect to the other parts of the apparatus is indicated in the figure by a dotted square. The beam of cathode rays entering the chamber could be limited to any required size by placing at *e* a suitable lead stop of the form G.

First some experiments were made in order to see if previous screening of the γ-rays affects the velocity of the secondary rays. In one set of

experiments the radiating plate was placed at *c*, no stop of any kind being placed at *e*. Readings were taken with successively increased magnetic fields. The reading obtained with no magnetic field was subtracted from each of these readings, and the resulting values plotted against the current through the magnet. Fig. 3 shows some curves obtained in this way. They are the result of combining a large number of observations taken at different times. The sharp bends in the curves indicate the stage when the rays of

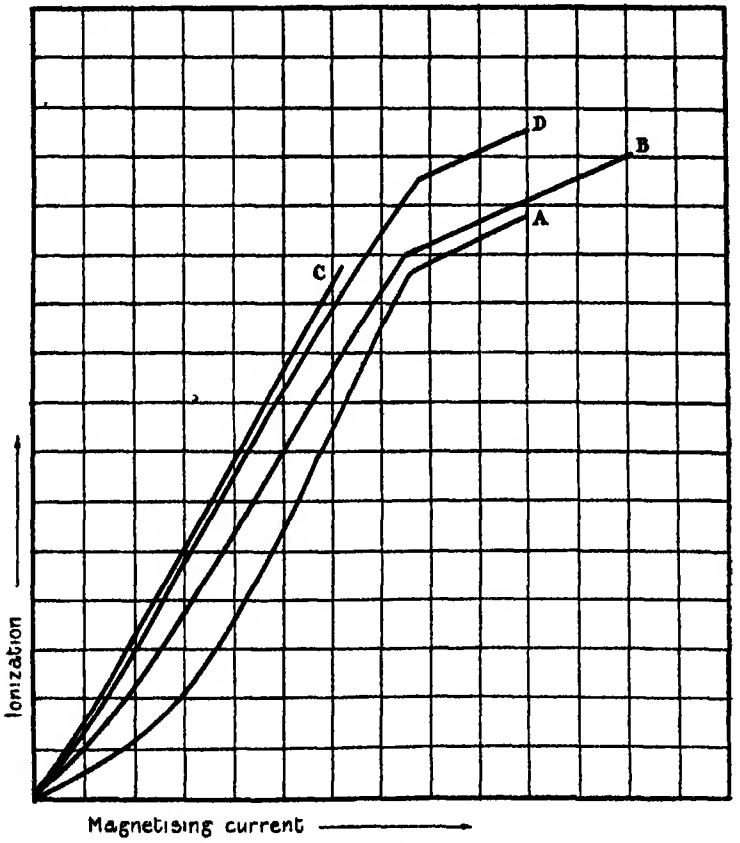

FIG. 3.

the outer edge of the principal stream of rays are deflected just sufficiently to enter the chamber. The curves have been so plotted that they are slightly separated from one another at the bends.

The curves A and B were obtained with a lead radiator. The γ-rays were sifted through lead screens of different thicknesses in the two cases, the curve A being obtained with a screen 1·5 cm. thick, and the curve B with

a screen 3 mm thick The difference in the form of the curves indicates that the proportion of electrons moving with a velocity less than that corresponding to the bends in the curves, is decreased when the thickness of the lead screen is increased A thick screen thus absorbs a greater proportion of the γ-rays which produce these slow electrons than a thin one. It appears, therefore, that the velocity of the secondary cathode radiation produced in a lead plate decreases with the absorbability of the γ-rays producing it

The curves C and D were obtained with an aluminium radiator, using the same lead screens as before The curve C corresponds to the thick screen, and the curve D to the thin screen being used It will be seen that now there is scarcely any difference between the curves obtained The previous passage of the γ-rays through a thick lead screen thus affects the more absorbable radiation from an aluminium plate to a less extent than that from a lead plate

If part of the soft radiation from a plate is produced by the γ-rays that are selectively absorbed by the plate we would expect such a result The substitution of a thick screen for a thin one would then affect the radiation from a plate of the same material as the screen to a greater extent than the radiation from a plate of some other material, for the γ-rays would be robbed in the first case to a greater extent of the rays most easily absorbed by the plate than in any other case The rays that are most easily absorbed by a substance, it should be observed, are also the rays that are selectively absorbed by the substance, if selective absorption exists.

Two sets of readings were also taken in succession with a zinc screen 2 cm. thick and the thick screen of lead used previously, the radiator being of lead. Curves of the same form as A and B were obtained, the curve obtained with the lead screen being more convex towards the current axis than the curve obtained with the zinc screen The lead screen thus robbed the beam of γ rays of a greater proportion of the rays which produce soft cathode radiation from a lead plate than the zinc screen This is in accordance with what we would expect according to the explanation given of the effect

It should be observed that the soft rays dealt with in these experiments are of a much more penetrating character than those discussed in the previous section. The average range of the latter rays is so small that scarcely any of them could have entered the ionisation chamber in the experiments just described.

A large number of experiments were made under different conditions to make sure that the effects just described were not due to a disturbing

influence of the radiation from other parts of the apparatus than that from the plate at *c*. Thus, readings were taken with the aperture F of different dimensions, and the pole-pieces at different distances apart, readings were also taken with the radiating plate placed at *d*. But in all cases a difference of the same nature as described was obtained between the curves for an aluminium and lead radiator when a thick and thin screen of lead were used. The difference in the curvature of the curves was, however, for two sets of experiments carried out under the same conditions, not always exactly of the same magnitude. This was probably due to hysteresis effects of the electro-magnet. The most consistent results were obtained when the radiating plate was placed at *d*. The reason for this appears to be that less of the very soft radiation, which would be greatly affected by the hysteresis of the magnet for small magnetic fields, entered the chamber in this case.

The conclusion, then, is that the velocity of the cathode radiation from a plate due to γ-rays decreases with the absorbability of the rays producing it.* But the set of rays most easily absorbed by a substance need not necessarily be the same for all substances.

Some experiments by Innes† on the velocity of the secondary cathode rays produced by X-rays may be mentioned in this connection. This observer found that the velocity of the cathode radiation from a metal plate decreased with the softness of the X-rays producing it. He also found that, keeping the penetrating power of his X-rays constant, the velocity of the cathode rays was influenced by the nature of the radiating metal. The effect in the latter case is easily explained by selective absorption of the metals used. The set of rays best absorbed by a substance, not necessarily the same set for each substance, varied probably in degree of absorbability from substance to substance, and cathode rays of corresponding slowness were produced.

The bends in the curves in fig. 3 indicate, as already pointed out, that there is a principal stream of cathode rays from the radiating plate. If the rays were ejected from a plate in all directions we would scarcely obtain any indication of bends of any sharpness. The sharpness of the bends was found to depend somewhat, as we would expect, on the dimension of the aperture F, and the distance of the ionisation chamber from the aperture. This result confirms the deduction which Prof. Bragg and Dr. Madsen‡

* Prof. Bragg and Dr. Madsen, according to a paper that has just appeared in the 'Phil. Mag.' of December, 1908, p. 918, have obtained the same result, but by a different method.

† 'Roy. Soc. Proc.,' A, vol. 79 (1907), p. 442. ‡ *Loc. cit.*

made from some of their experiments on secondary radiation, namely, that the secondary cathode rays produced by the γ-rays are ejected in the direction of propagation of the rays

An attempt was also made to determine absolutely the velocity of the fast-moving rays from a plate But it had to be given up on account of the great heterogeneousness of the rays The maximum velocity of a pencil of rays was found to depend on the size of the aperture F, and on the size of the aperture of the stop G placed at c, as we should expect if the rays are heterogeneous. And to obtain the relation between the velocity and amount of radiation, as has been done for the β-rays of radium, the apparatus was altogether unsuitable The rays were therefore compared as a whole with the β-rays from radium The size of the aperture F in these experiments was 1 1 cm^2 and the lead plate used as radiator was placed at c A stop of the form G was placed at c, the size of its aperture being 1 3 cm^2 A set of readings was first taken with the lead radiator for magnetic fields of different strengths in the same way as before A set of readings was next taken with the screen over the radium and the lead radiator removed A stream of β-rays from the radium now passed through the aperture F, and was deflected by the magnetic field towards the ionisation chamber It should be observed that, if the secondary rays from the lead plate radiator were ejected in the direction of propagation of the γ-rays, the beams in the two cases were similar in respect to the direction of motion of the electrons

The readings obtained in the two cases are plotted against the current through magnet in fig 4 The curve E refers to the cathode rays from the lead plate, and the curve F to the β-rays from the radium It may be mentioned in passing that a comparison of the magnetic field with current through magnet by means of an exploring coil and current indicator showed them to be approximately proportional to one another over the whole range of magnetic fields used in these experiments The curves have been so plotted that they are only slightly separated from one another actually the maximum leak for the β-rays was much greater than that for the secondary cathode rays. It will be seen that in both cases the maximum leak is obtained for approximately the same current through the maguet The velocity of the secondary cathode rays from a lead plate exposed to the γ-rays is therefore, as a whole, the same as that of the β-rays of radium

It will be interesting, without committing ourselves to any particular theory of the γ-rays, to see what this result means in the light of the two theories of the γ-rays at present put forward

According to the theory of Prof Bragg and Dr Madsen, the γ-ray consists

of a negative electron associated with a particle possessing an equal charge but of opposite sign, the combination forming a neutral pair of particles In traversing matter, the pair is liable to get broken up, with the result that the negative particle proceeds onwards, the γ-ray now appears as secondary cathode radiation A γ-ray is supposed to arise by an electron picking up a positive particle which it encounters in traversing matter

Now, if the positive particle has an appreciable mass, the velocity of the pair of particles must be less than that of the negative electron before the pair was formed Consequently, when the pair is stripped of its positive particle, the velocity of the cathode ray proceeding onwards should be

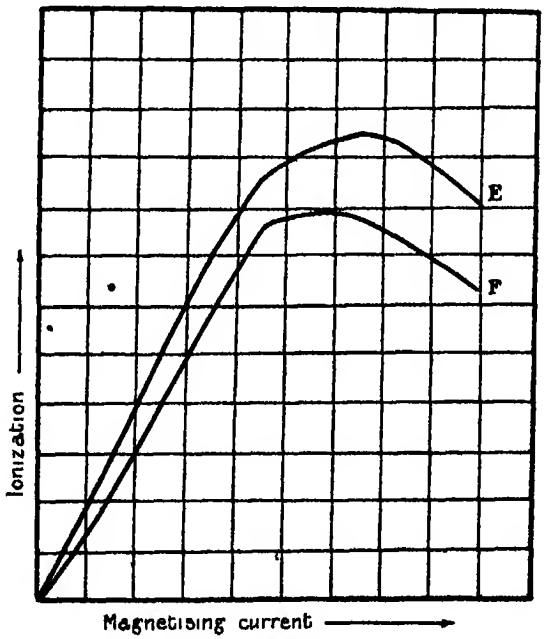

FIG. 4

smaller than its velocity before association with the positive particle If, for example, the mass of the positive particle is equal to that of the negative, the velocity of the secondary cathode radiation should be about $1/\sqrt{2}$, or about 0·7 of that of the β-rays of radium In that case the maximum for the curve for the secondary cathode rays in fig 4 would be almost half-way between the zero and the maximum for the β-rays of radium But since the curves show that the velocities are practically the same, it follows that the positive particle (if the neutral pairs are formed in the way described) must be small in comparison with that of the negative.

If the mass of the positive particle is entirely electrical, it follows from the

well-known equation $m = \frac{2}{3}\frac{e^2}{a}$, which connects the electrical mass with the charge and radius of particle, that the radius of the positive particle must be large in comparison with that of an electron This condition would be approximately fulfilled by a positive particle the diameter of which is equal to that of a molecule, but whose mass is entirely electrical

On the æther-pulse theory of the γ-rays in its usual accepted form, the pulse is supposed to spread out in the form of a spherical shell with the generating electron as centre If the energy of an electron ejected by a γ-pulse is derived entirely from that of the pulse, its energy can only be a small fraction of that of the pulse, and therefore of that of the electron which generated the pulse But this is contrary to experience

To overcome this difficulty, Prof J J Thomson proposed a modified æther-pulse theory* According to this theory, the pulse is supposed to proceed in one direction only from the radiating electron, and that the area of the wave front of the pulse remains constant as the distance of the pulse from the electron increases The electron ejected by a pulse is therefore able to absorb the whole energy of the pulse, and if the whole energy of the radiating electron is transformed into pulse energy, the velocity of the secondary electron may be equal, or at least very nearly equal, to that of the electron which produced the pulse Prof Thomson also suggested that the wave front of a ray of light may be similarly restricted

It will be interesting in this connection to obtain an estimate of the thickness of a γ-ray pulse If a ray of light and a γ-pulse are identical in structure the interesting experiments of Ladenburg† on the velocity of the cathode rays ejected by ultra-violet light furnish data from which an estimate of the thickness of a γ-pulse may be made This experimenter found that the product of the velocity of a beam of secondary cathode rays into the wave-length of the ultra-violet light producing it is constant An approximate value of this constant for our purpose is obtained by taking the velocity 2×10^7 cm /sec. for the cathode rays (a value found by Lenard) to correspond to the wave-length 288 $\mu\mu$ of ultra-violet light Therefore, if λ denotes the thickness of a γ-pulse, and we take the velocity of the β-rays as $2{\cdot}5 \times 10^{10}$ cm /sec, we have $\lambda 2{\cdot}5 \times 10^{10} = 288\,\mu\mu \times 2 \times 10^7$, or $\lambda = 174\,\mu\mu \times 10^{-3}$ From the last equation it will be seen that the thickness of a γ-pulse is about one-thousandth of the wave-length of ultra-violet light. And since the wave-length of ultra-violet light is about one

* 'Camb. Phil Soc. Proc.,' vol. 14, part 4, p 417

† 'Verh. der Deut. Phys. Gesellschaft,' 1907, p. 507.

thousand times the diameter of a molecule, the thickness of a γ-pulse is about equal to the diameter of a molecule

Summary

Part of the cathode radiation from a plate exposed to the γ-rays of radium consists of very soft rays which are absorbed in 1 or 2 cm of air

The softness of the radiation is practically independent of the thickness of the radiator, and previous sifting of the γ-rays through a thick screen

The radiation appears to be considerably softer on the side of the radiating plate where the γ-rays emerge than on the side where they enter

Measurements of the softness of the radiation for radiators of different materials on the side where the γ-rays entered showed that it is practically independent of the nature of the material of the radiator

The soft radiation produced by the β- and γ-rays of radium together is of a more penetrating character than that produced by the γ-rays alone

The penetrating cathode rays produced directly by the γ-rays have been shown to possess different velocities It was found that the penetrating power of the cathode radiation from a plate decreases with the increase of absorbability of the γ-radiation which produces it.

The velocity of these secondary rays as a whole is, as a first approximation, equal to that of the β-rays of radium

It gives me great pleasure to thank Prof Thomson for his interest and advice during this research

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Electrolytes and Colloids.—The Physical State of Gluten.

By Prof T B Wood and W B Hardy, F R.S

(Received October 24,—Read December 10, 1908)

[This paper is printed in Series B (No. 545), vol. 81, pp. 38—43.]

Determination of the Surface-tension of Water by the Method of Jet-vibration.

By Prof N Bohr (Copenhagen)

(Communicated by Sir W Ramsay, K.C B, F R S Received Jan. 12,—Read Jan 21, 1909)

(Abstract)

In the present determination of the surface-tension of water the method of jet-vibration proposed by Lord Rayleigh is used, this method has the fundamental advantage that a perfectly fresh new-formed surface can be examined.

In the theoretical part of this investigation it is shown how Lord Rayleigh's theory of infinitely small vibrations of a jet of a non-viscid liquid can be supplemented by corrections for the influence of the finite amplitudes as well as for the viscosity

In the experimental part it is shown how it seems to be possible, in a simple manner, to secure that the jet-piece used for the measurements satisfies the assumptions on which the theoretical development rests

As the final result of the experiments, the author finds the surface-tension of water at 12° to be 73·23 dyne/cm

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The Origin of Osmotic Effects II.—*Differential Septa*

By Henry E Armstrong, F R S

(Received January 23,—Read January 28, 1909)

[This paper is printed in Series B (No 546), vol 81]

The Specific Heats of Air and Carbon Dioxide at Atmospheric Pressure, by the Continuous Electrical Method, at 20° C and at 100° C.

By W F G Swann, A R.C S, B Sc

(Communicated by Prof H L Callendar, F R.S. Received October 8,—Read December 10, 1908)

(Abstract)

A steady stream of gas was passed through a jacketed tube (the calorimeter proper), in which it was heated by a current of electricity passing through a platinum coil of 1 ohm resistance, the rise in temperature being measured by two 12-ohm platinum thermometers used differentially

If C is the electric current, E the potential difference between the ends of the heating coil, $\delta\theta$ the rise in temperature of the gas, Q the rate of flow of the gas in grammes per second, J the mechanical equivalent of heat, and S the specific heat of the gas at constant pressure, the elementary theory of the experiment gives

$$CE = JSQ\,\delta\theta + h\,\delta\theta,$$

where $h\,\delta\theta$ is a term representing the heat loss by radiation, etc

A similar experiment with a rate of flow about half the above value, and with the electric current adjusted so that the rise in temperature was about the same as before, gave a second equation, so that h could be eliminated and S determined

Before passing into the calorimeter the gas passed through a brass tube packed with copper gauze, the tube, together with the calorimeter jacket, being surrounded by another jacket through which steam or water could be passed, so that the gas entered the calorimeter proper at a constant temperature.

The largest currents of gas through the apparatus were of the order of 0 5 litre per second The rate of flow was kept constant by an automatic pressure regulator It was measured by passing the gas through 16 fine metal tubes arranged in parallel, and observing the pressure difference between their ends, the mean pressure, and the temperature The expression giving the rate of flow in terms of these quantities was found by a series of experiments in which the gas was pumped into a reservoir of about 50 litres capacity, and then allowed to discharge through the apparatus By means of a special device, the times taken for certain quantities of gas to pass

through the apparatus were recorded automatically while the gas was actually flowing, so that initial fluctuations were avoided

The value of the electric current was obtained by measuring the potential difference set up at between the ends of a standard resistance coil, in terms of cadmium cells The heating effects of the leads of the heating coil were determined by experiments made under the exact conditions of the main experiments.

The rise in temperature in the main experiments was about 5° C, and it was measured to 0 001° C Thus the specific heats were measured practically at single temperatures instead of over large ranges

In the paper the results of experiments made to test the validity of assuming the heat loss for a given rise in temperature to be independent of the rate of flow of the gas are recorded. The matter is also examined from a theoretical standpoint, and corrections are calculated and applied where the assumptions made in the elementary theory are such as to lead to errors of more than about one part in 10,000 The corrections are small, only amounting to one or two parts in 1000 The full details of the various other precautions and corrections are given in the paper, and the mean of a large number of observations gave the following results —

Air	Carbon dioxide
0 24173 cal per gramme degree at 20° C	0 20202 cal per gramme degree at 20° C
0 24301 „ „ 100° C	0 22121 „ „ 100° C

The several determinations agree in each case to about 1 5 parts per 1000, and the mean results are probably correct to one part in 1000

It is possible to make a comparison of the results with the values deduced on theoretical considerations from Joly's measurements of the specific heats at constant volume This comparison is made in the paper, and it is shown that the results agree nearer than to one part in 1000 in the case of air, while in the case of carbon dioxide the results agree to 1 per cent., which is as near as can be expected, in view of the fact that, in order to compare the results, extrapolations and interpolations have to be made over rather wide ranges of pressure and temperature, which is hardly justifiable in the case of carbon dioxide

The values of the specific heats obtained by the author are greater by about 2 per cent than the corresponding values found by Regnault, and later investigators who have employed methods similar in principle to that of Regnault An explanation of this fact is suggested in the paper It is shown that in Regnault's experiments, in virtue of the fact that the corrections for the heat conducted through the pipe connecting the heating bath

to the calorimeter were determined from experiments in which no gas was flowing through the pipe, the results obtained by him must be too low by a quantity which is uncertain to the extent of 5 per cent of the value of the specific heat In virtue of the form of connecting pipe used, the error probably amounts to something of the order of half this amount, which would bring the results into close agreement with the author's

The work was carried out in Prof Callendar's laboratory at the Imperial College of Science, and the method adopted was, in its main features, the same as that employed by Prof Callendar for the determination of the specific heat of steam

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On the Depression of the Filament of Maximum Velocity in a Stream flowing through an Open Channel

By A H Gibson, M Sc (Viet)

(Communicated by Prof J E Petavel, F R S Received December 18, 1908,—Read January 14, 1909)

When water flows with sinuous motion through a circular pipe the resistance introduced by the solid boundaries reduces the velocity of axial flow as the sides are approached, this velocity being greatest at the centre and least at the sides, as indicated by the transverse velocity curve of fig 1 When flow takes place through a closed rectangular pipe, the same effect is

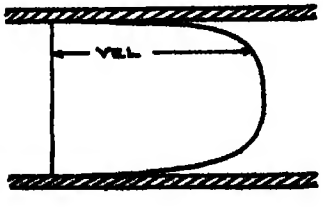

Fig 1.

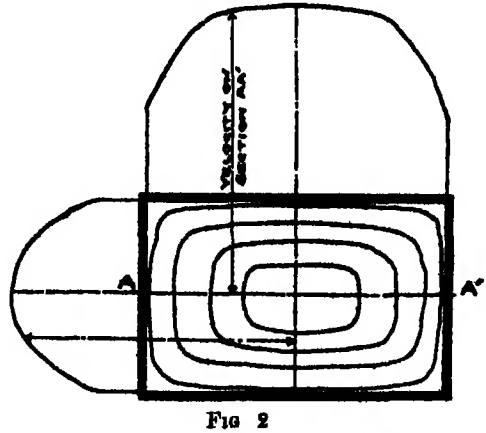

Fig 2

noticed, the transverse velocity curves and the curves joining points of equal velocity, or the contours of equal velocity in a cross-section, being much as shown in fig 2 Here, again, the maximum velocity is found at the centre of the pipe

From analogy with this latter case it might be expected that when flow takes place through an open rectangular flume such as would be obtained by taking the portion of the rectangular pipe of fig 2 below the level AA', or indeed through an open channel of any ordinary section, the filament of maximum velocity would be found in the water surface and in the centre of the stream

In the majority of cases the latter assumption is fairly well justified by the results of experiment, although in some instances two points of maximum velocity have been noticed, one on each side of and at some distance from the centre. Further reference to this point will be made at a later stage of the paper

Except, however, in the case of a broad, rapid, and shallow stream, it is found that this filament occurs at some depth below the surface Its depth varies somewhat with the direction of the wind and with the physical characteristics of the stream, and, on a calm day, usually ranges from about one-tenth to four-tenths of the depth of the stream

This phenomenon of the depression of the filament of maximum velocity has been much discussed, and three theories have been propounded for its explanation

In the first of these the surface film is supposed to act in much the same way as a solid boundary in producing retardation of the surface layers This theory is, however, discounted by the fact that, even with a downstream wind of considerably greater velocity than the stream, the filament remains below the surface

In a second theory it is suggested that "eddies of water, stilled by contact with the bed, are thrown off and wander through all parts of the stream, but finally accumulate and spread out at the surface, forming a layer of slowly moving water" As regards this there would appear to be no special reason why eddies formed near the bottom of a stream should tend to wander upwards and finally stay on the surface And if this did happen, since according to the theory any given eddy would need to pass through all intermediate sections of the stream before reaching the surface, it would be impressed with the mean axial velocity of each section in succession, and would finally reach the surface with a velocity greater than that of any intermediate section.

The third theory suggests that, as the water is less constrained at the

surface than at any other point, irregular movements of all kinds are set up here and energy is therefore utilised in giving motions, not of translation, to the water *

This suggestion is directly opposed to the fact brought out by Osborne Reynolds in his researches on the causes of instability of flow in water, viz, that unconstrained boundaries tend to stability, not to instability, of flow †

The unsatisfactory nature of these theories led the author to an investigation of the question, and as a result of this the following explanation of the phenomenon is offered

In any channel, however smooth its wetted perimeter may be, the velocity of forward flow is greatest near the centre and least near the sides and bottom, and if it were possible to obtain a state of affairs in which motion might take place in stream lines parallel to the axis of the stream, we should have, with steady flow along a straight reach of the channel, the velocity greatest in the surface and at the centre of the stream, and the water surface level from side to side

In practice this is modified by the eddy formation which always takes place at the sides of the stream, and the phenomena in question would appear to be due almost entirely to the modification thus introduced

A consideration of the process of eddy formation as it usually occurs at the sides of a stream shows that this involves the temporary existence of a region of less than normal pressure on the downstream side of the projection causing the eddy, and, as a result of this, where eddy formation is proceeding continuously in a uniform stream with a bed which is horizontal from side to side, it is to be expected that the depth of water will in consequence be slightly less at the sides than at the centre The cross-sectional elevation of the surface thus becomes concave to the bed of the stream

This curvature of the surface profile has been noticed by several observers, and was commented upon by Messrs. Humphreys and Abbot in their report on the gauging of the Mississippi ‡

The superelevation of the surface near the centre creates a tendency to a general outward flow from centre to sides of the channel, this, for permanence of *régime*, being accompanied by an inward flow consisting of water projected in the form of eddies from the sides

Now supposing an eddy, extending from the surface to the bottom, to break away from the side of a stream. Its forward velocity is somewhat less than that of the current in which it finds itself, the difference being greater the

* Flamant, 'Hydraulique'

† 'Roy Inst. Proc.,' 1884, also 'Scientific Papers,' vol 2, p. 153.

‡ Humphreys and Abbot, 'Report on the Mississippi River'

nearer the surface From a consideration of its direction of rotation and of the external forces acting on its mass in virtue of its rotation in a stream moving more rapidly than its mass centre, it appears that these tend to drive it towards the centre of the stream * This effect becomes greater as the relative velocity of the mass of water forming the eddy and of the passing current becomes greater, and will therefore increase from the bottom to the surface It follows that the drift of the "eddy current" will be greatest near the surface and least near the bottom, and, as a nett result, that a system of transverse currents will be set up consisting of an inward surface drift from each side towards the centre, an outward drift over the lower portions of the stream, and, accompanying these, a downward current near the centre and an upward current near each side

Since the inward surface drifts consist of water which has travelled up the sides and has come from the region of minimum velocity, they will evidently have the effect of reducing the surface velocity and of depressing the filament of maximum velocity

The sketches in figs 3 and 4 show respectively the directions of the transverse currents, and of the resultant motion of the stream, the full lines

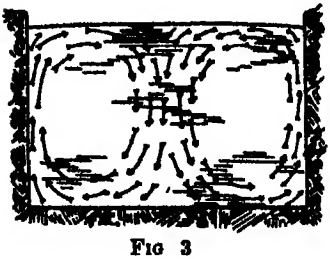

Fig 3

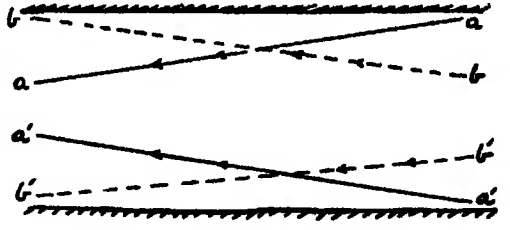

Fig 4.

$a'a'$, aa, in fig 4 representing the direction of the surface currents, and the dotted lines $b'b'$, bb, those of the bottom currents

With a view of confirming these deductions, experiments were carried out on a small experimental channel in the hydraulic laboratories of the Manchester University, on a straight reach of the Mersey, a few hundred yards below the county bridge at Northenden, and on two straight reaches of the Derbyshire and Peak Forest Canal near Marple

In the case of the model channel, which has wooden sides and bottom, the

* Observation shows that whenever two eddies are moving down stream with different velocities of drift, and at approximately the same distance from the side, the slower tends to move centre-wards relative to the faster Also that an eddy having greater velocity of drift than that of the current in which it finds itself always tends to move towards that side of the stream at which its direction of rotation would presume it to have been formed

current formation was examined by means of aniline dye introduced to various parts of the stream, and also by means of threads fixed to pins in the bottom and sides

While the latter method did not prove very satisfactory owing to the weight of the threads when wetted, it did show a very definite surface drift from side towards centre, floating threads attached to the sides making an angle of about 5° with these The examination of the filaments of coloured water, however, showed the whole process very clearly and definitely proved the existence of the transverse currents

The Mersey on the reach examined is practically straight, and has an almost uniform width of approximately 36 feet and a mean depth of about 6 feet The flow throughout the reach appeared to be as nearly as possible uniform, with a mean surface velocity of about 3 feet per second In this case the side-to-centre surface drift was very apparent. Of 10 float rods, 6 inches in length, thrown into the stream at about 2 feet from either bank, eight arrived within 2 feet of the centre of the stream before having traversed more than 100 yards of the reach The observed behaviour of portions of water-logged leaves suspended in the stream offered substantial evidence of an upward current near the sides and of a downward current within a few feet of the centre on each side, but the dirty state of the water prevented the behaviour of such bodies being noted for a depth of more than about 6 inches

The two reaches of the Peak Canal are each about 7 feet 6 inches wide and 5 feet deep, and were explored by means of weighted wax pellets These gave distinct evidence of an upward current at the sides, a current commencing about 1 inch below the surface towards the centre, and a downward current commencing about 2 5 feet from the sides Owing to the muddy character of the water it was impossible to follow the pellets to any considerable depth, and to note the depth of the return current, although their behaviour showed this to be present

In addition to this direct experimental verification of the theory, much indirect evidence of its validity is available in that it explains several interesting phenomena which have been noted and are of importance in stream gauging, and for which the reason has not hitherto been clear

Thus it is well known that the depth of the filament of maximum velocity—

(*a*) Depends on the physical condition of the channel, and increases as the roughness of the sides increases

(*b*) Depends on the depth of the stream, and especially on the ratio of depth to width, its depth increasing as the latter ratio increases In a wide shallow stream the filament is found in the surface.

(*c*) Is greater for any given vertical in a rectangular channel as that vertical approaches the side, and is at about mid depth near the side of such a channel

(*d*) Depends on the velocity of flow, increasing as the latter diminishes

Now it is evident that since an increase in the roughness of the sides will tend to increase the surface depression at the sides, this, by increasing the head available for producing a transverse current, will increase the magnitude and the effect of this current and will tend to depress the filament of maximum velocity As the depth of the stream increases relatively to its width, the influence of the sides will increase, so that the effect will be the same as an increase in their roughness On the other hand, the roughness of the bottom tends to retard the transverse current without having any compensating effect, so that an increase in the roughness of the bottom as opposed to that of the sides of the channel tends to raise the filament of maximum velocity An increase in the width relatively to the depth of the stream will have the same effect, and with a very shallow wide stream the influence of the sides will be quite negligible.

Also since the effect of the current will diminish as its distance from the sides increases, this explains the greater relative depression of the filament near the sides

In a given channel with a given depth of water, an increase in the mean velocity of flow might reasonably be expected, as is found to be the case, to diminish the relative importance of the transverse current and hence to elevate the filament of maximum velocity.

These points are well brought out in the published records of the gaugings of a very large number of rivers and channels by members of the U.S Geological Survey.* From these the results of a number of gaugings of the experimental canal of the Cornell University have been chosen in illustration These may be divided into two groups, denoted by A and B The experiments in series A were carried out with high velocities of flow and small depths of water, the mean velocity ranging from 2 06 to 3 16 feet per second, and the depth from 0 46 to 1 88 feet.

In series B the depths were greater and the velocities less, the depth ranging from 6 0 to 9·5 feet and the velocity from 0 23 to 2 feet per second.

The canal is of rectangular section, with concrete sides and bottom having a slope of 1 in 500, and has a width of 16 feet Velocity measurements were made in eight verticals in a cross-section by means of carefully

* U S. Geological Survey, 'Water Supply and Irrigation Paper,' No 95, p 111.

calibrated current meters The meter station in series A was 234 feet from the head of the canal and in series B was 280 feet from the same point

The curves in figs 5 and 6 show the variations of velocity in a vertical plane in typical of these experiments, each plotted point giving the mean of all eight observations at that depth in the cross-section Here the curves of fig 5 refer to series A and those of fig 6 to series B

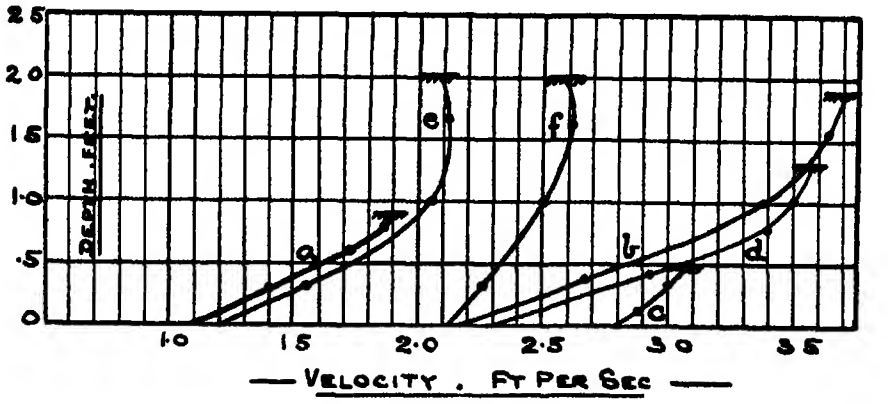

FIG 5

The effect of a large ratio of width to depth in raising the filament of maximum velocity is evident from a comparison of the curves of fig 5 and of fig 6, while the effect of an increased velocity of flow in raising the filament is evident from a comparison of the several curves of fig 6. The

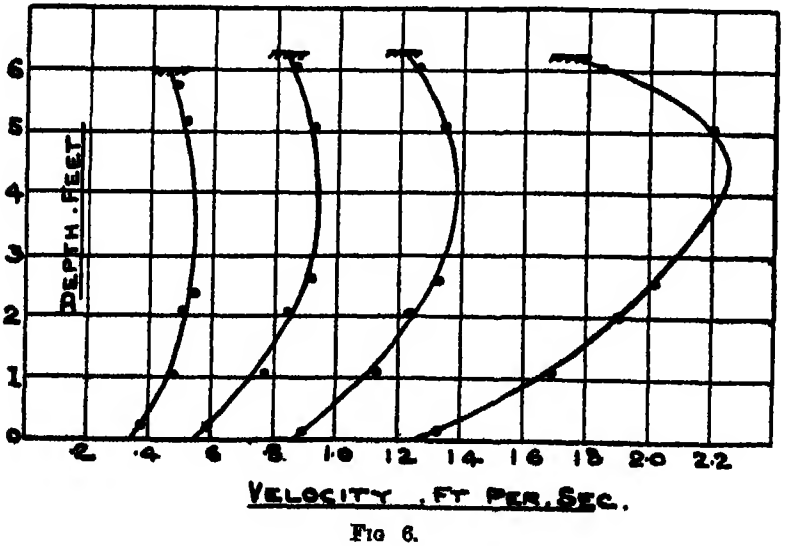

FIG 6.

relationship between the velocity of flow and depth of filament, as obtained from the whole of the experiments of series B, is as follows —

Mean velocity of flow, feet per second	0 45	0 80	1 4	1·9
Depth of filament of maximum velocity	0 44h	0 42h	0 34h	0 29h

where h is the depth of the stream

The great difference between the several curves of fig 5 is probably accounted for by the fact that while in some cases discharge at the lower end of the canal was restricted by the partial closure of the outlet gates, in other cases it was free In experiments (e) and (f) a definite backwater was caused by the throttling of the outflow, and in these two cases it is definitely known that the flow at the meter section was being retarded In the other cases discharge is said to have been free, and judging by the low velocities of flow it is certain that the flow was still being accelerated at the meter section, except possibly in experiment (C)

Before considering these curves further it may be well to see whether an acceleration or retardation of the flow, such as might occur in the one case in a contracted reach of a stream, or, in the other case, above a weir or dam, is likely to have any effect on the position of the filament of maximum velocity

This is evident if it be considered that since any acceleration in the mean flow will be most strongly marked in the central portion of the stream, the surface level at any section which undergoes acceleration will fall to a greater extent at the centre than at the sides The converse holds if the flow is being retarded Thus any acceleration of the stream will tend to diminish, and any retardation to increase, the formation of the transverse currents already described This effect is very marked in the results of experiments (e) and (f) of series A, where, as compared with experiment (b), with about the same depth of water, the depth of the filament of maximum velocity is depressed 0 15h

To test whether, after all, the transverse currents, when found, are not mainly due to a retardation of the current, further experiments were carried out on the model channel at the Manchester University with both accelerated and retarded flow These showed conclusively that in this channel at all events the influence of the sides was the predominating factor producing the currents, these being substantially as already described, with either type of flow

Indeed, unless this is so, practically all deep-river gaugings, showing as they do a depression of the filament of maximum velocity, must have been inadvertently carried out under a retarded flow The improbability of this having been the case does not need any emphasis

A further verification of this conclusion is afforded by the gauging of the Farad flume of the Truckee River General Electric Company* This flume has timber sides and bottom, and was gauged at two cross-sections 200 feet apart The width was 10 09 feet throughout The depth at the upper section was 5·98 feet, and at the lower section was 5 96 feet, while the filament of maximum velocity, this being the mean of the maxima in six verticals, had a depth of $0\,50h$ at the upper and $0\,42h$ at the lower section, in spite of the slightly accelerated flow

It would appear probable that in the comparatively few instances in which the filament of maximum velocity has been found to be in the surface of a river of any considerable depth this is due to the gauging station having been fixed at a section undergoing a strongly accelerated flow

The increasing depth of the filament as the sides of the channel are approached is well shown in fig 7, which is an example of a gauging by Darcy of a rectangular channel 0 25 metre deep and 0 8 metre wide

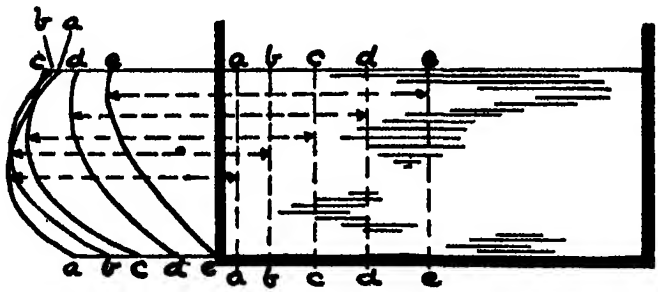

Fig 7

There is still another fact which strengthens the theory, and which will be made clearer by an examination of figs 8 and 9 These show velocity curves obtained in horizontal sections of the Cornell Canal, the curves in fig. 8 being examples of those obtained in the accelerated flow experiments of series A, and those of fig 9 being samples of those obtained in series B†

From these it will be noted that while in series A (fig 8) the point of maximum velocity in a horizontal plane is approximately in the centre of the stream, the departure from this point being probably due to a greater roughness of one side of the channel,‡ in each of the curves of series B there are two points in each section, one on each side of the centre, at which the velocity is greater than at the centre This point was brought out in each experiment of the series

* U S Geological Survey, 'Water Supply and Irrigation Paper,' No 95, p 111

† 'Water Supply and Irrigation Paper,' No 95, pp. 73 and 74

‡ This fact is mentioned in the report

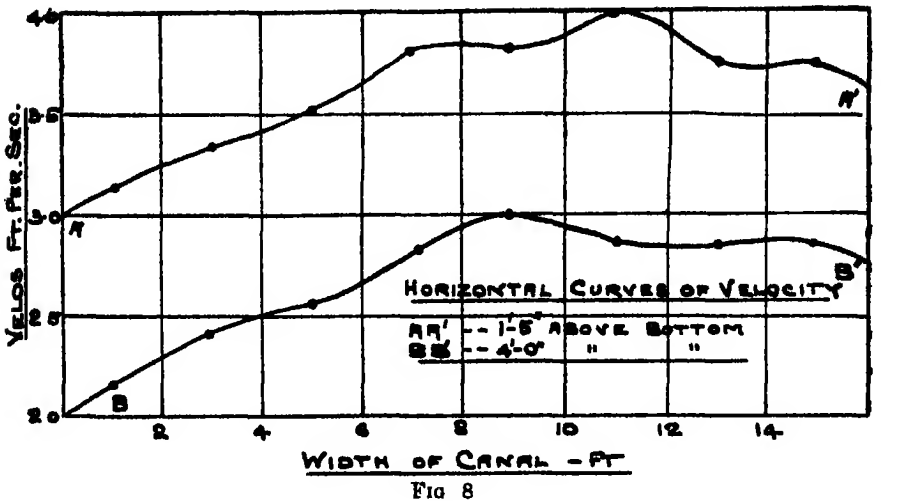

Fig 8

But this is precisely the effect which a vigorous system of cross currents, such as is shown in fig 3, would tend to produce, the central downward current of slowly moving water causing the depression of the centre of the curves of fig 9 On the other hand, in series A the depth of stream was small and the action of the sides negligible, so that this phenomenon would not be expected

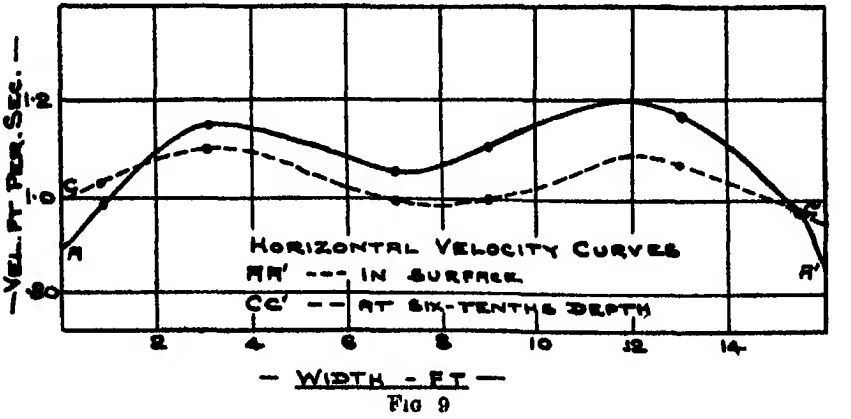

Fig 9

So far the channel has been assumed to be straight If it is curved, the state of affairs is further complicated by a current which, as shown by James Thomson,* sweeps up the inner bank, and then, as a surface current, extends from the inner to the outer bank of the bend, thus spreading a layer of slowly moving water over the surface of the stream

* 'Roy Soc. Proc.,' 1877, p. 356

This will have the effect of depressing the filament of maximum velocity by an amount which will be greatest at the inner bank, and will probably only be slightly felt at the outer bank In fact, it is probable that, owing to the sweeping of high-velocity water from the centre to the outer side of the stream, the filament of maximum velocity at all points between the centre and outer bank will be in the surface This conclusion is confirmed by the published results of gaugings of the Oswego River, at Battle Island, New York,* these gaugings being taken in close proximity to a bend

In the majority of rivers this action will sensibly modify the action of the sides and the distribution of the transverse currents

Conclusions

As a result of the investigation, the following points would appear to be well established —

(1) The depression of the filament of maximum velocity in a straight reach of a river or canal is due to the action of the sides of the channel in producing transverse currents inwards along the surface and outwards along the bed of the stream, thus distributing a layer of slowly moving water over the central portions of the stream

(2) This effect is increased by a retardation and diminished by an acceleration of the flow, but, in the majority of cases in a stream of any considerable depth, is never so greatly diminished by an acceleration as not to be felt

(3) In a curved channel this effect is modified by the formation of a transverse current, due to the centrifugal action of the water and extending over the surface from the inner to the outer bank, this current tending, on the whole, to depress the filament of maximum velocity at all points between the inner bank and the centre of the stream, and to elevate it at points between the centre and the outer bank

* 'Water Supply and Irrigation Paper,' No 95, p 129

The Photo-electric Fatigue of Zinc.—II

By H Stanley Allen, M A, B Sc, Senior Lecturer in Physics at King's College, London

(Communicated by Prof H A Wilson, F R S Received November 25, 1908,—Read January 21, 1909)

1 *Object and Method of the Experiments*

In a former paper* an account was given of the way in which the photo-electric activity of zinc diminishes when the metal is exposed to light from a Nernst lamp It was shown that the photo-electric activity at a time t from the commencement of an experiment could be expressed by the formula

$$I = K_1 e^{-\lambda_1 t} + K_2 e^{-\lambda_2 t}$$

In a typical experiment with a polished zinc plate the first exponential term fell to half value in 4 9 minutes, the second in 94 minutes, the constants of change being $\lambda_1 = 0\,141$ and $\lambda_2 = 0\,00737$, K_1 and K_2 were of the same order of magnitude

In an experiment with amalgated zinc the first term fell to half value in 4 2 minutes, corresponding to $\lambda_1 = 0\,164$, while the second term fell to half value in 167 minutes, corresponding to $\lambda_2 = 0\,00414$, the value of K_1 was about six times K_2

The experiments described in the present paper were carried out to determine whether the results were similar when using a source of light giving far more ultra-violet radiation than the Nernst lamp A mercury-vapour lamp of fused quartz by W C Heraeus, supplied with current by a battery of 50 accumulators, was employed†

Photometric measurements with lamps of this kind have been made by R. Küch and T Retschinsky in the laboratory of the makers‡ These experimenters examined the relation between the radiation from the lamp and the energy supplied both for visible and for ultra-violet radiation The economy curve was found to be of the same general character in the two cases

The spectrum has been photographed and measured as far as wave-

* 'Roy Soc Proc,' A, vol 78, p 483, 1907

† I am indebted to the Council of King's College for a Treasury grant for the purchase of the lamp, and to the Government Grant Committee of the Royal Society for supplying the accumulators

‡ 'Annalen der Physik' (4), vol. 20, pp 563—583, 1906

length 3340 A U by J Stark,* using a three-prism spectroscope constructed of Jena ultra-violet glass

[*December* 23.—Dr T M Lowry has kindly photographed the spectrum of my lamp, using lenses and prism of quartz With a short exposure a well-marked line spectrum was obtained, extending to about 2400 A U]

The emission in considerable quantity of waves of still shorter wave-length is shown by the production of large amounts of ozone, rendering it necessary to place the lamp outside the room in which work is being carried on Lenard† has shown that the ozonising action is due to very short waves, having a wave-length less than 2000 A U Regener‡ has shown that there is also a deozonising action for certain waves, whose wave-length falls between 3000 A U. (absorption by glass) and 1850 A U (absorption by quartz)

As regards the radiation from the Nernst lamp, I am informed by Mr W A Scoble, who has photographed the spectrum, that in no case was any effect produced beyond wave-length 2100 A U, and the results were comparatively faint from at least 2500 A U By far the greatest photographic effect was obtained in the visible violet and blue

The method of experimenting was similar to that described in the previous paper, but the testing cell, consisting of the zinc plate and a positively charged sheet of wire gauze, was in the open air instead of being enclosed in a brass case An electrometer of the Dolezalek type, giving about 500 divisions per volt, was used to measure the leak across the gap between the gauze and the zinc plate As the leak in some of these experiments was much larger than in those made with the Nernst lamp as a source of light, a parallel plate air condenser of adjustable capacity was constructed and connected with the electrometer so as to secure a convenient rate of deflexion of the needle The capacity of the electrometer and its connections was about 100 electrostatic units, by means of the condenser the capacity of the system could be increased up to about 1000 units

The mercury lamp was started 20 minutes before the commencement of a series of observations, so that it might have time to assume a steady state Readings of the current and voltage for the lamp were taken at intervals during the experiments, but the variations observed were inconsiderable (See, however, § 4) Observations of the rate of leak were commenced one minute after the preparation of the plate and were continued at intervals of two minutes

* 'Annalen der Physik' (4), vol 16, pp 490—515, 1905
† 'Annalen der Physik' (4), vol 1, p 486, 1900
‡ 'Annalen der Physik' (4), vol 20, pp 1033—1046, 1906

2 *Results of Experiments with Polished Zinc*

The experiments with polished zinc showed that the fatigue took place in exactly the same manner when the plate was illuminated by the mercury vapour lamp as when a Nernst lamp was used as a source of light A typical experiment is represented by the lower curve in fig. 1, which is plotted on semi-logarithmic paper The curve can be represented by the sum of two exponential terms, the first term* falling to half value in eight

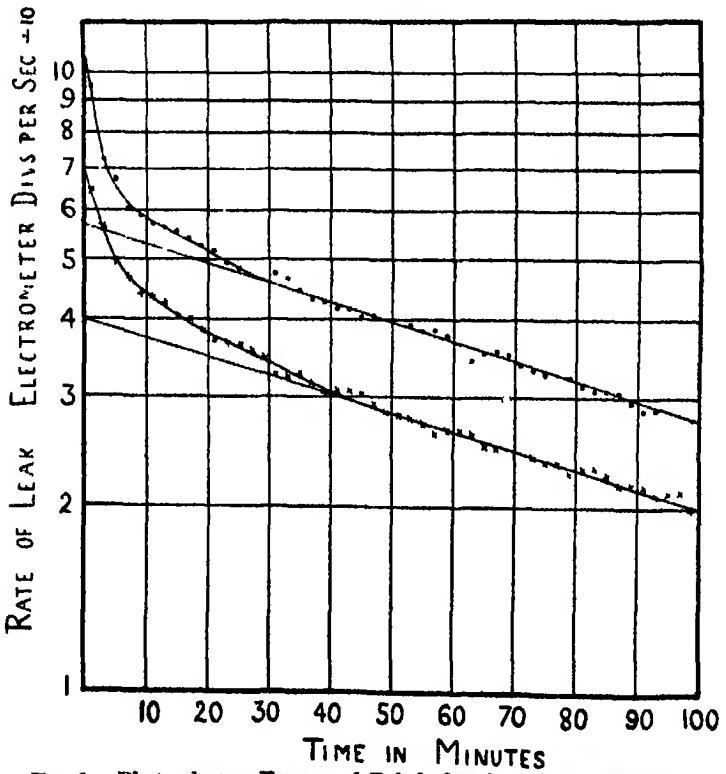

Fig 1.—Photo-electric Fatigue of Polished and Amalgamated Zinc

× Experiment made February 12, 1908, with polished zinc at 120 cm from mercury vapour lamp

• Experiment made April 15, 1908, with amalgamated zinc at 63 cm. from mercury vapour lamp.

minutes, the second in 100 minutes, the constants of change being $\lambda_1 = 0{\cdot}0867$ and $\lambda_2 = 0{\cdot}00693$ Expressing the photo-electric current in scale divisions per second, $K_1 = 30$, $K_2 = 40$ In this experiment one

* It should be noted that the constant of change for the first term, which is found by a difference method, cannot be determined with the accuracy possible in the case of the second term.

division per second corresponded to $0\cdot20 \times 10^{-12}$ ampere, so that if the current is in amperes, $K_1 = 6 \times 10^{-12}$ ampere, $K_2 = 8 \times 10^{-12}$ ampere The current is about twice as large as that in the experiment of August 7, 1905, described in the former paper The greater activity of the mercury vapour lamp as compared with the Nernst lamp is nearly compensated for by the increased distance (120 cm) from the source of light

In another experiment the zinc plate was only 6.3 cm from the mercury lamp In this case the fatigue was slightly more rapid, the first exponential term falling to half value in 6 8 minutes, the second in 76 minutes In scale divisions per second $K_1 = 26$ and $K_2 = 47$, but as a capacity of about 400 electrostatic units was employed with the electrometer, the current was much greater than in the former experiment, $K_1 = 26 \times 10^{-12}$ ampere, $K_2 = 47 \times 10^{-12}$ ampere

3 *Results of Experiments with Amalgamated Zinc*

A typical fatigue curve for amalgamated zinc is shown in the diagram It is seen at once that this is similar in character to the curve for the polished metal In the present case we find that the first term falls to half value in 3 2 minutes, the second in 96 minutes, giving $\lambda_1 = 0\cdot217$ and $\lambda_2 = 0\cdot00722$. The value of K_1 is 47×10^{-12} ampere, and of K_2 is 57×10^{-12} ampere

Comparing these results with those obtained previously, when a Nernst lamp was employed instead of the mercury lamp, we notice that the second term changes a good deal more rapidly in this case, and that K_1 and K_2 are now of the same order of magnitude The slower rate of fatigue with the Nernst lamp may, perhaps, be attributed to the action of the waves of greater wave-length, described in the former paper This suggestion receives support from experiments made with the plate illuminated by the two sources of light at the same time In these the fatigue took place more slowly than with the mercury lamp alone

4 *Minor Variations in the Observations*

A close examination of the original curves here reproduced suggests the presence of small undulations superposed on the smooth exponential curves These variations have a period of about 10 minutes Similar periodic changes in the rate of decay were observed in curves obtained when using a Nernst lamp as the source of light (see figs 3 and 4 of the former paper) It does not appear probable that the fatigue really proceeds through a series of small maximum and minimum values, though such periodic changes are not unknown For example, Ostwald has shown that the change involved in the

solution of certain varieties of chromium in dilute hydrochloric and sulphuric acids is oscillatory in character *

The explanation of the small variations observed in my experiments is probably to be found in the character of the source of light In both the Nernst lamp and the mercury vapour lamp we may suppose that continual adjustment is taking place as regards the resistance and the potential difference between the terminals, causing a more or less regular fluctuation in the intensity of the emitted light

5 *Conclusion*

The photo-electric activity of a zinc plate decays in such a way that it can be represented as the sum of two exponential terms The constants of change are but little altered by considerable variations in the character and intensity of the illumination employed, though the value of the photo-electric current is changed considerably The rate at which the surface is altered is not greatly affected by using a mercury vapour lamp in place of a Nernst lamp

Experiments with other metals are still in progress

In conclusion, I wish to express my thanks to Prof H A Wilson for advice during the course of the work

* Mellor, 'Chemical Statics and Dynamics,' pp 348–352

The Mobilities of the Ions produced by Rontgen Rays in Gases and Vapours.

By E M Wellisch, M.A (Sydney), Emmanuel College, Cambridge, Barker Graduate Scholar of the University of Sydney

(Communicated by Prof Sir J J Thomson, F R S Received December 19, 1908, —Read January 21, 1909)

(Abstract)

The velocities of the positive and negative ions produced by Rontgen rays in 4 gases and 15 vapours have been measured at normal temperatures over a wide range of pressures and under different electric intensities Langevin's direct null method was employed throughout

For a constant pressure the velocity of the ion was found to vary as the electric intensity

It was found that, in general, the mobility (k) of the ion varied inversely as the pressure (p) In the case of nitrous oxide and carbon dioxide there was a slight tendency for the product pk to increase both for the positive and negative ions as the pressure was reduced below about 7 cm of mercury In the case of ethyl chloride there was a marked tendency for the product pk to decrease as the vapour approached the saturated state, there is reason to believe that this tendency for pk to decrease would appear in the case of all the vapours in the neighbourhood of the saturated state

In the case of vapours there was, in general, little difference in the values of the positive and negative mobilities The mobility of the positive ion was found greater than that of the negative for aldehyde, ethyl alcohol, aceton, sulphur dioxide, ethyl chloride, pentane, ethyl acetate, methyl bromide, and ethyl iodide

There appeared to be no direct relation between mobilities and molecular weights, the smaller mobilities invariably belonged to gases possessing high critical temperatures (the vapours), the larger mobilities to gases with low critical temperatures.

From the kinetic theory of gases an expression has been deduced for the mobility of an ion moving through a gaseous medium under the influence of an electric field This expression takes into account the effect of the charge carried by the ion on its mean free path and involves only known physical constants of the gas.

As a result of the theoretical considerations, it appears that the

experimental values of the mobilities in the different gases at various pressures, as well as certain observed deviations from the law connecting the mobility and gaseous pressure, can be explained approximately on the supposition that the ion consists of a single molecule of the gas with which is associated a charge equal to that carried by the monovalent ion in electrolysis

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The Leakage of Helium from Radio-active Minerals.

By the Hon R J Strutt, F R.S, Professor of Physics, Imperial College of Science, South Kensington

(Received December 30, 1908,—Read January 21, 1909)

In a paper published in 'Roy Soc Proc,' A, vol 81 (1908), p 272, I showed that phosphatised bones and similar materials were notably radio-active, and that helium could be detected in them The quantity of helium found was not, however, uniformly greater in the geologically older materials than in younger ones of equal activity This was hypothetically attributed to escape of helium in certain cases I desired if possible to obtain direct experimental confirmation of this conjecture

It would clearly be impossible to detect leakage of helium from materials such as the mineralised bones, even in a lifetime For any chance of success it was necessary to have recourse to the ores of uranium and thorium, in which the quantity of helium is something like 100,000 times greater

The method of experimenting was to place a considerable quantity of the ore (usually a kilo or more) in a bottle provided with an exit tube and stop-cock and connected to a mercury pump The bottle was exhausted and the stopcock closed After the lapse of a definite interval of time, usually a day or more, a small quantity of oxygen was admitted to the bottle and then collected through the pump, carrying with it any helium which had come off from the mineral. The oxygen was absorbed with melted phosphorus, leaving a small residue of helium, together with impurities. Any hydrogen present in the original gas, which may have been liberated by radio-active decomposition of traces of moisture, was burnt along with the phosphorus, and thus got rid of The residue was transferred to an apparatus consisting of a McLeod gauge in connection with a reservoir containing charcoal. On cooling the charcoal with liquid air, helium was isolated, and the quantity could be measured As a test of purity, the spectrum could be examined in the capillary measuring tube of the gauge, using external tinfoil electrodes.

I was astonished at the quantity of helium observed in the first experiments. It exceeded anticipation by hundreds, or even thousands, of times I shall not describe the experiments in the order in which they were made, but rather in that which seems to connect them best

It was found that after a radio-active mineral had been powdered, helium was evolved from it, rapidly at first, then at a diminishing rate The following observations illustrate this

A quantity (337 grammes) of monazite from the Transvaal was powdered and passed through a wire-gauze sieve of 120 threads to the inch. This took about one hour Immediately afterwards it was put in a bottle and the air pumped out The rate of evolution of helium in cubic millimetres per day per kilo of material was as follows:—

Time (days)	Rate
0 031	261
0 59	76 6
1·6	17·1
2 6	12 3
4 6	9 57
10 6	4 38
33 0	1 14

The first experiment was made as quickly as possible, helium being collected for one hour Times are measured from the moment when the powdering was half completed to half-way through the period of accumulation Naturally the first rapid variations can only be roughly investigated in this way.

Leakage of helium from this sample is still continuing, and it is intended to watch its future course

It will be observed that the whole quantity which has escaped while the mineral has been under observation is but an insignificant fraction (probably less than a 500th*) of the whole quantity present

Moss† has observed that quantities up to 1 per cent. of the helium contained in a mineral can be liberated by grinding in a vacuum. The present observations show that this is but the first rapid stage of a long-continued leakage of helium from the newly created surfaces. The view that heat generated in grinding is the important factor appears untenable, for in that case escape of helium should cease on cooling

It is uncertain how long this evolution of helium continues, in all

* This sample of monazite was very poor in helium, containing only $\frac{1}{10}$ c.c per gramme

† 'Roy Dub. Soc. Trans.,' vol. 8, p. 153.

probability, however, the period is prolonged and since the majority of radio-active materials can only be obtained in the form of pieces which have been broken off from their natural home a moderate number of years back, any observations made upon them are inconclusive as to the rate at which helium escapes when they are undisturbed in their original surroundings

It was found, in fact, that pieces from the same stock of monazite, about the size of a lump of sugar, which had not been fractured since they came into my possession two years ago, evolved helium at the rate of 0·002 c mm per kilo of material per diem

This rate, though quite insignificant in comparison with that exhibited by the powdered material, is much in excess of the probable rate of generation of helium by radio-active change It follows that the present stores of helium could never have been accumulated had the present rate of evolution prevailed throughout the life-history of the mineral

With a view to testing a mineral more nearly in its natural condition, experiments were made on thorianite, which occurs in gravels, in detached cubic crystals, washed out of their original matrix This, too, showed a considerable leakage of helium (0 069 c mm per kilo per diem) Tests were made at intervals over a considerable period The rate was found quite uniform, the volume of helium pumped out being proportional to the time of accumulation A second collection, made immediately after the first, yielded scarcely anything. It is difficult to account for this large evolution of helium from a mineral so nearly in its natural condition It was thought possible that an explanation might be found by supposing that the temperature of the laboratory (65° F) was somewhat higher than that of the natural surroundings of the mineral The latter must, however, be above the freezing point, and it was decided to test experimentally the rate of evolution at that temperature. The bottle containing the mineral was kept in ice for some days, under these conditions the helium leakage was reduced to about a quarter of its value at the higher temperature (0 018 c mm per kilo per day) This is still greatly in excess of the rate of accumulation

It seemed possible, though unlikely, that the mineral, when kept in a vacuum, lost helium which it would have retained at atmospheric pressure. To test this explanation, the bottle containing the mineral was left filled with oxygen up to atmospheric pressure At the close of several days the oxygen was pumped out, and absorbed with phosphorus The ordinary quantity of helium was obtained, the rate of escape being undiminished.

It was noticed that the surfaces of some of these crystals were somewhat weathered. With the idea that this might determine the escape of helium, another sample of thorianite much fresher in appearance was tested. The

rate of evolution in this case was only 0 0127, about one-fifth of that observed with the previous sample

The majority of minerals allow water to percolate through them The effective superficial area must therefore much exceed the external surface

It is probable that loss of helium occurs from the weathering of these interior surfaces as well as from the external faces of the crystals Abrasion of the external surfaces by comparatively recent rolling in water-courses may have produced some effect.

Under laboratory conditions the rate of escape of helium from minerals always far exceeds the rate of production by radio-active change Therefore the conditions under which the life of the minerals has been mainly passed, deep down in the earth, where atmospheric agencies have no place, must be supposed more favourable to retention of helium, for otherwise the present accumulation could never have been formed The observations here recorded leave little room for surprise that fossilised bones and other materials do not always contain as much helium as would be expected from their radio-activity and geological age

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On the Electricity of Rain and its Origin in Thunderstorms.

By GEORGE C SIMPSON, D Sc

(Communicated by Dr Gilbert T Walker, F R S Received January 6,—Read February 4, 1909)

(Abstract.)

During 1907–08 an investigation was undertaken at the Meteorological Office of the Government of India, Simla, into the electrical phenomena which accompany rain and thunderstorms.

Two lines of research were adopted .—

(*a*) A systematic record was obtained by means of self-registering instruments of the electricity brought down by the rain throughout one rainy season

(*b*) Laboratory experiments were made with the object of determining the source of the electricity of thunderstorms.

The chief results of the first part of the work may be briefly summarised as follows :—

(1) The aggregate amount of rain which fell during the periods of rainfall investigated was 76·3 cm

(2) The total quantity of positive electricity which fell on each square centimetre of surface was 22·3 electrostatic units, and of negative electricity 7·6 units, thus 75 per cent of the electricity brought down by the rain was positive.

(3) During 71 per cent of the time that charged rain fell the charge was positive

(4) Considering that falling rain carrying a positive charge is equivalent to a positive current, and rain with a negative charge to a negative current, then positive currents greater than 300×10^{-15} ampere per square centimetre were measured in six storms and negative currents greater than 300×10^{-15} ampere per square centimetre were measured in two storms

(5) In seven storms rain was recorded carrying greater positive charges than 6 electrostatic units per cubic centimetre of water, and in two storms a greater negative charge than this amount occurred

(6) The heavier the rainfall the more the positively charged rain preponderated over the negatively charged rain, and all rainfall which occurred at a greater rate than a millimetre in two minutes was positively charged

(7) Light rain was more highly charged than heavy rain

(8) The proportion of negative electricity brought down by the rain was slightly greater in the second than in the first half of the storms

(9) The potential gradient was more often negative than positive during rain

(10) No relationship between the sign of the potential gradient and the sign of the electricity of the rain could be detected

The laboratory experiments showed that when a large drop of water is broken up into small drops in the air the water becomes positively and the air negatively charged

In the first series of experiments drops of water, each having a volume of 0·24 c.c., fell on to a vertical jet of air which broke them up into small drops It was found that under these circumstances the water of each drop, after having been broken up on the jet, carried a charge of $5\cdot2 \times 10^{-3}$ electrostatic unit of positive electricity Further, it was found that the presence of an original charge on the drops did not alter the effect. Drops originally charged positively had their charges increased and drops charged negatively had their charges decreased

In the second series of experiments water was introduced through two small tubes into a vertical current of air which carried the water upwards. Part of the water which escaped from the air current was caught in an

insulated vessel and was found to be positively charged, the charge being 15×10^{-3} electrostatic unit per cubic centimetre of water

In the third series of experiments drops of water were broken up in a similar manner to that employed in the first series, but within a compartment from which the air could be drawn through an Ebert apparatus The result showed that the breaking of the drops caused an ionisation of the air The breaking of each drop released $3 3 \times 10^{-3}$ electrostatic unit of free negative ions and $1 1 \times 10^{-3}$ electrostatic unit of free positive ions, the excess of negative ions corresponds to the positive charge retained by the water

In 1904 Prof Lenard* showed that drops of water having a greater diameter than 5 5 mm are unstable when falling through air and rapidly break up into smaller drops He also showed that all drops having a smaller diameter than 5 5 mm have a final velocity when falling through still air of less than 8 metres a second Thus no water can fall through an ascending current of air having a velocity of 8 metres a second, for all drops less than 5 5 mm in diameter are carried upwards, and all drops having a larger diameter quickly break up into smaller drops These facts, together with the results of the observations and experiments described above, have led to the formation of the following theory for the origin of the electricity of thunderstorms

It is exceedingly probable that in all thunderstorms ascending currents greater than 8 metres a second occur Such currents are the source of large amounts of water which cannot fall through the ascending air Hence, at the top of the current, where the vertical velocity is reduced on account of the lateral motion of the air, there will be an accumulation of water This water will be in the form of drops which are continually going through the process of growing from small drops into drops large enough to be broken Every time a drop breaks a separation of electricity takes place, the water receives a positive charge, and the air a corresponding amount of negative ions The air carries away the negative ions, but leaves the positively charged water behind

A given mass of water may be broken up many times before it falls, and in consequence may obtain a high positive charge. When this water finally reaches the ground it is recognised as positively charged rain. The ions which travel along with the air are rapidly absorbed by the cloud particles, and in time the cloud itself may become highly charged with negative electricity. Now within a highly electrified cloud there must be

* Lenard, 'Met Zeit.,' vol. 21, p. 249, 1904

rapid combination of the water drops, and from it considerable rain will fall, this rain will be negatively charged, and under suitable conditions both the charges on the rain and the rate of rainfall could be large

A rough quantitative analysis shows that the order of magnitude of the electrical separation which accompanies the breaking of a drop is sufficient to account for the electrical effects observed in the most violent thunderstorms. All the results of the observations of the electricity of rain described above are capable of explanation by the theory, which also agrees well with the actual meteorological phenomena observed during thunderstorms.

The Tension of Metallic Films deposited by Electrolysis.

By G Gerald Stoney.

(Communicated by the Hon C A Parsons, C B, V-P R S Received January 16, —Read February 4, 1909)

It is well known that metallic films deposited electrolytically are in many cases liable to peel off if deposited to any considerable thickness This is the case with nickel which, when deposited over a certain thickness, will curl up into beautiful close rolls, especially if it does not adhere very tightly to the body on which it is deposited For example, if a piece of glass is silvered by any of the usual silvering solutions, and then nickel is deposited on the silver, it is found that the nickel and silver peel off the glass in close tight rolls almost at once In 'Practical Electro-Chemistry,' by Bertram Blount, reference is made on pp 114 and 272 to the tendency of nickel to peel off, and it is stated that it "will peel—spontaneously and without assignable cause" (p 272), but that a thick coating can be obtained by keeping the solution at between 50° and 90° C

The late Earl of Rosse* tried, about 1865, to make flat mirrors by coating glass with silver chemically, and then electroplating with copper, but he found that, owing to the "contraction" of the copper film, it became detached from the glass. I have had the same experience in protecting silver films in searchlight reflectors by a film of electro-deposited copper, it being found that if the film of copper is more than 0 01 mm thick peeling is apt to take place

Dr Gore, F R.S., in papers in the 'Phil Trans,' in 1858 and 1862, found

* 'Nature,' Aug 20, 1908, p. 366

that antimony, when deposited, was in a very unstable state, and that it was liable to break to pieces by vibration or the local application of heat, this breaking to pieces being accompanied by the evolution of heat, and crackling sounds and cracks were found due to alterations in the cohesive state of the metal, while "in common with electro-deposits generally the inner and outer surfaces of these deposits are in unequal states of cohesive tension", and he concludes by saying, "it would be interesting to enquire to what force of mode of physical action is the evolution of heat in antimony due" In his book, 'The Art of Electro-Metallurgy,' he says, "In common with electro-deposits generally the inner and outer surfaces are in unequal states of cohesive tension, frequently in so great a degree as to rend the deposit extensively and raise it from the cathode in the form of a curved sheet with its concave side towards the anode"

Dr E. J Mills, in a paper in the 'Proceedings of the Royal Society,' 1877, vol 26, on "Electrostriction," shows that when thermometer bulbs were coated electrolytically with metals, the mercury rose in the stem, proving that the metal was deposited in a state of strain, also that in the case of nickel and iron, when more than a certain thickness was deposited, the coat split He found that a pressure of 19 2 tons per square inch could be obtained with nickel, 18 2 with iron, 66 4 with silver, and 108 5 with copper

It seemed, therefore, that metals are deposited under tension, and if so that they should strain the material on which they were deposited so as to bend it, and that by the amount of this bending the tension under which they were deposited could be determined, and it was found that when nickel was deposited on one side of a thin sheet of metal very considerable bending took place. Thin steel rules, 102 mm long, 12 mm. wide, and 0 32 mm. thick, were varnished on one side and coated with nickel from the usual bath of ammonio-sulphate of nickel, and it was found easy to get these bent to the extent of 3 to 4 mm. The thickness of the film was determined by weighing before and after coating, and was checked in the case of some of the thicker films by micrometer measurements, the density of nickel being taken at 8 82, as given by Miller Owing to the evolution of hydrogen which always takes place, the amount deposited is considerably less than that due to the current used, generally only 50 to 60 per cent of the theoretical quantity, and therefore that method could not be used to determine the quantity of nickel deposited.

The amount of bending, combined with the thickness of the deposit, enabled the tension under which it is deposited to be calculated For let a thin steel rule of thickness d have deposited on it a layer of nickel of

small thickness t, and let it be curved by the film of nickel to a radius r, neglecting the thickness of the rule, which is small in comparison with r, and taking moments for the steel, we have, b being the depth from the surface of the rule to the neutral axis,

$$\int_a^0 \frac{E}{r}(b-x)\,x\,dx = 0, \quad \text{so that} \quad b = \tfrac{2}{3}d,$$

and if P is the tension per unit area of section on the film of nickel, resolving horizontally,

$$Pt = \int_a^0 \frac{E}{r}(b-x)\,dx$$

$$= \frac{E}{r}\left(bd - \frac{d^2}{2}\right) = \tfrac{1}{6}E\frac{d^2}{r}$$

The curvature was measured by the deflections z of the rule in a length l Thus $r = l^2/8z$ Putting in this value for r, we have

$$P = \tfrac{4}{3}\frac{Ed^2z}{tl^2} \qquad (1)$$

This equation is only true when the film of nickel is very thin, but since the properties of nickel closely resemble steel it is easy to allow for the thickness of the nickel film on the assumption that the modulus of elasticity of nickel is the same as steel, and since this is approximately true for forged nickel,* and is therefore probably true for deposited nickel, this assumption is allowable without serious error where the thickness of nickel deposited is moderate

Thus, for any thickness of nickel deposited y, we have for a further thickness deposited dy an increase of bending dz, and equation (1) becomes, the tension P being the same for the successive layers as they are laid on the curved surface,

$$P = \tfrac{4}{3}E\frac{(d+y)^2\,dz}{l^2dy},$$

and, integrating this differential equation,

$$P = \tfrac{4}{3}E\frac{(d^2+td)\,z}{tl^2} \qquad (2)$$

* Forged nickel I have found to have the same modulus of elasticity as steel, that is, 2,100,000 kilogrammes per sq cm Prof Ewing, in 'Strength of Materials,' gives 2,100,000 kilogrammes per sq cm as a mean value for carbon steel, which I have found correct for these rules, and this is independent of the quantity of carbon in the steel and whether it is tempered or not, i e, is the same approximately for mild steel and tool steel. Nickel steels with from 3 to 5 per cent. of nickel have about the same modulus of elasticity as carbon steels Forged nickel as above has a tensile strength of 25 tons per square inch and a yield point of 16 tons per square inch, with an elongation of 10 per cent in 2 inches, or very similar to wrought iron.

The results given in the table are obtained from depositing nickel on a rule 102 mm long by 12 mm. wide

Weight of nickel deposited	Thickness of nickel	Thickness of rule	Bending in 90 mm.	Tension of nickel	
				Kilos per sq cm	Tons per sq inch
gramme	mm	mm	mm		
0·061	0·0056	0 31	0 50	3020	19 2
0 498	0 0462	0 31	3 45	2850	18·2
0 186	0 0172	0·32	1 30	2820	18·0
0 285	0·0264	0 32	1 98	2860	18 2
0 083	0 0077	0 31	0 64	2820	18·0
0 175	0 0162	0 31	1 25	2700	17 2
0 156	0 0144*	0 32	0 48	1230	7·8
0 330	0·0305†	0 32	1 22	1550	9·9

* Deposit rough owing to large current

† Solution at 80—90° C

The first six were deposited at a temperature of 10° to 15° C, and with currents varying from 0·05 to 0 11 ampere No 7 was with a current of 0 22 ampere at 12° C, and was very rough and granulated, but until the deposit began to get rough the current density did not affect the tension It is thus seen that a good deposit has a tension of between 18 and 19 tons per square inch

The last was deposited with 0 11 ampere at 80° to 90° C, and the tension obtained was much lower, and this reduction in the tension of the deposit probably accounts for what Mr Blount found, as mentioned before, that thick deposits of nickel were easier to obtain at higher temperatures

I have not found the strength of the nickel-plating solution to affect the tension under which the nickel is deposited

It was also found on heating these rules to a red heat so as to anneal them that they straightened out to a considerable extent, the remaining deflection being only about one-third or one-half of the original

A Further Note on the Conversion of Diamond into Coke in High Vacuum by Cathode Rays

By Alan A Campbell Swinton

(Communicated by the Hon C A. Parsons, C B, V-P R S Received January 28, —Read February 4, 1909)

In a previous paper on this subject by the Hon Charles A Parsons and the writer,* experiments were described designed to ascertain whether any gas was emitted by diamond during its conversion into coke Two spectrum tubes were connected to the cathode ray furnace, in which the diamond was heated to destruction One of these was sealed off just before and the other just after the conversion, but when the spectra of these two tubes were photographed alongside one another the differences that existed did not appear sufficiently marked to determine with exactitude any variation in the nature of the gases present

The present note has reference to further and more detailed investigation made on the suggestion of Mr Parsons by the writer, with special regard to the possibility of diamonds containing neon, krypton, or other rare gas which would be emitted on the diamond being converted into coke

As before, spectrum tubes connected with the cathode ray furnace were sealed off so as to contain samples of the residual gas before and after the conversion The spectra of these were compared both photographically and also by direct visual examination in the spectroscope, with the result that though differences were observed in regard to the relative brightness of various individual lines in the two spectra, careful observation showed that in no single instance was there any line in one spectrum that could not be obtained in the other by suitably adjusting the strength of the electric discharge through the spectrum tube

From this it would appear that the conversion of diamond into coke, if it sets free any gas at all, at any rate does not liberate any other than one or more of the comparatively common gases that are generally found as residuals in cathode ray tubes exhausted from air in the ordinary way

Though this is a negative result, it has been thought well to put it on record.

* 'Roy Soc. Proc.,' A, vol 80, pp 184–5

The Statistical Form of the Curve of Oscillation for the Radiation emitted by a Black Body

By Prof HAROLD A. WILSON, F.R.S., King's College, London.

(Received January 4,—Read February 25, 1909)

The view adopted in the following paper is that the radiation from a black body is an irregular disturbance subject to statistical laws. This view is, I believe, that now generally held.* It is shown that these laws can be deduced from the observed distribution of energy in the spectrum and that they enable the general character of the disturbance to be described.

At any point in an actual spectrum of the radiation we have a large number of simple harmonic vibrations of arbitrary phases and various amplitudes. The radiation may be regarded as the sum of all the simple vibrations in the spectrum. The component vibrations may be regarded as continually varying, but the distribution of energy in the spectrum, when averaged over a short interval, remains constant.

Suppose that we compound a large number n of simple harmonic vibrations all along the same line and of the same amplitude, but having phases distributed at random. It is easy to calculate by the method given by Lord Rayleigh† the chance that the displacement at any instant selected at random will lie between given limits, say y and $y+dy$. It is $a^{-1}(\pi n)^{-\frac{1}{2}}e^{-y^2/na^2}dy$, where a denotes the amplitude of each of the components. If we compound a number of such sets of vibrations and in the nth set denote the amplitude by a_n, the number by n_n, and the wave-length by λ_n, the expression for the chance of a given displacement is

$$\frac{e^{-y^2/\Sigma na^2}}{\sqrt{\pi}\,(\Sigma na^2)^{\frac{1}{2}}}dy$$

Let the energy in a vibration be ka^2/λ^2, and let $E_n = kn_n a_n^2/\lambda_n^2$. Then in the spectrum of plane polarised radiation let $\phi(\lambda)\,d\lambda$ denote the energy between λ and $\lambda+d\lambda$, so that E_n corresponds with $\phi(\lambda)\,d\lambda$ and

$$\Sigma na^2 = \frac{1}{k}\int_0^\infty \lambda^2\phi(\lambda)\,d\lambda.$$

Hence

$$\frac{e^{-ky^2/\int_0^\infty \lambda^2\phi\,d\lambda}}{\left(\frac{\pi}{k}\int_0^\infty \lambda^2\phi\,d\lambda\right)^{\frac{1}{2}}}dy$$

* *Cf.* Larmor, 'Phil. Mag.,' vol. 10, p. 574, 1905; Rayleigh, 'Phil. Mag.,' vol. 11, p. 123, 1906.

† 'Theory of Sound,' vol. 1, p. 39.

is the chance that the displacement at any instant in the radiation lies between y and $y + dy$

If $y = a \sin\left(\frac{2\pi}{\lambda} vt + \epsilon\right)$ denotes one of the component vibrations, then the amplitude of $\frac{d^n y}{dt^n}$ is $a\left(\frac{2\pi v}{\lambda}\right)^n$ Hence the chance that $d^n y/dt^n$ lies between p_n and $p_n + dp_n$ is $\pi^{\frac{1}{2}} c_n^{-1} e^{-p_n^2/c_n^2} dp_n$, where $c_n^2 = \frac{(2\pi v)^{2n}}{k} \int_0^\infty \lambda^{2(1-n)} \phi d\lambda$ The corresponding expressions for unpolarised radiation can easily be obtained in the usual way, but they have no special advantage

It is more convenient for some purposes to consider y as a function of distance along the path of the radiation, which will be denoted by x Then $\frac{d^n y}{dt^n} = v^n \frac{d^n y}{dx^n}$, so that c_n^2 for $\frac{d^n y}{dx^n}$ is equal to $\frac{(2\pi)^{2n}}{k} \int_0^\infty \lambda^{2(1-n)} \phi d\lambda$.

Consider the average number (n_y) of times per centimetre that the curve $y = f(x)$, representing the radiation, cuts the line $y =$ constant Evidently n_y is given by the equation*

$$n_y \frac{\sqrt{\pi}}{c_1} = \frac{e^{-y^2/c_0^2}}{\sqrt{\pi} c_0},$$

for $n_y \sqrt{\pi}\, dy/c_1$ is the length per centimetre in dy occupied by the curve. Hence $n_0 = c_1/\pi c_0$ Here $2/n_0$ may be called the mean wave-length of the radiation

Let M_y denote the average number of maxima in $f(x)$ per square centimetre at a distance y from the x axis, and m_y denote the number of minima The number of times per centimetre that df/dx vanishes is independent of y, hence

$$M_y + m_y = A e^{-y^2/c_0^2}$$

Also $$\frac{dn_y}{dy} = -\frac{2c_1}{\pi c_0^3} y e^{-y^2/c_0^2} = -2(M_y - m_y)$$

Hence $M_0 = m_0$ and

$$M_y = \left(M_0 + \tfrac{1}{2} \frac{c_1}{\pi c_0^3} y\right) e^{-y^2/c_0^2},$$

$$m_y = \left(M_0 - \tfrac{1}{2} \frac{c_1}{\pi c_0^3} y\right) e^{-y^2/c_0^2}$$

The number of zeros per centimetre in df/dx can be shown, in the same way as for $f(x)$, to be $c_2/\pi c_1$, hence

$$\frac{c_2}{\pi c_1} = 2 \int_0^\infty 2 M_0 e^{-y^2/c_0^2} dy \quad \text{or} \quad M_0 = \frac{c_2}{2\pi^{\frac{1}{2}} c_1 c_0}$$

* This is assuming that there is no correlation between dy/dx and y, which can easily be verified.

Thus the distribution of maxima and minima can be determined from the c's. In the same way the distribution of maxima and minima and the number of zeros per centimetre for any of the derivatives of $f(x)$ can be found if desired

In order to calculate the relative values of the c's it will be convenient to take the distribution of energy in the spectrum to be represented by Planck's formula

$$\phi(\lambda) = A\lambda^{-5}(e^{-\alpha/\lambda\theta}-1)^{-1},$$

where θ denotes the absolute temperature and A and α are constants whose approximate values are known This gives, putting $\alpha/\lambda\theta = z$,

$$c_n^2 = (2\pi)^{2n}\frac{A}{k}\left(\frac{\theta}{\alpha}\right)^{2n+2}\int_0^\infty \frac{z^{2n+1}dz}{(e^z-1)}$$

The value of k is unknown, so we can only calculate relative values of the c's The following table gives the values got by taking $A/k = 1$ and $\alpha = 1\,46$ —

n	c_n	c_{n+1}/c_n
0	$0\ 876\,\theta$	$8\ 58\,\theta$
1	$7\ 52\,\theta^2$	$16\ 95\,\theta$
2	$127\ 3\,\theta^3$	$30\ 3\,\theta$
3	$3{,}860\,\theta^4$	$36\ 8\,\theta$
4	$142{,}000\,\theta^5$	$45\ 0\,\theta$
5	$6{,}390{,}000\,\theta^6$	$53\ 7\,\theta$
6	$343{,}000{,}000\,\theta^7$	$62\ 3\,\theta$

The larger values of c_{n+1}/c_n are given almost exactly by the formula $4\,303\,(2n+2\,5)$

The mean wave-length (λ') is equal to $2\pi c_0/c_1$, hence $\lambda'\theta = 0\,73$ The wave-length (λ_m), at which the energy in the spectrum is a maximum, is given by the well-known equation $\lambda_m\theta = 0\,294$. Hence $\lambda'/\lambda = 2\,5$.

The total number of maxima and minima in $f(x)$ is very nearly double the number of zero values, and the number of points of inflexion is a little less than double the number of maxima and minima.

Substituting the values found for c_0, c_1, and c_2 in the expression for M_y, we get

$$M_y = 1\,76\left(1+\frac{y}{\theta}\right)e^{-y^2/0\cdot876\theta^2}$$

The following table gives some values of M_y —

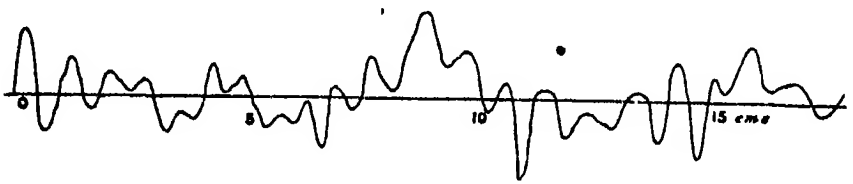
$\frac{y}{0{\cdot}876\,\theta}$	2	$\sqrt{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1
M_y	0 089	0 534	1 21	1 98	1 76	0 77	0 081

The corresponding values of m_y can be got by changing y into $-y$

The distribution of the zero values of $f(x)$ or its derivatives along x can easily be obtained If N is the mean number of zeros per centimetre, then the number of spaces between two zeros greater than x is ne^{-Nx}, where n is the whole number of spaces considered

Fig. 1 shows a curve drawn so as to have approximately the statistical properties determined above The spaces between the zeros maxima and minima and the amplitudes at the maxima and minima were selected at random from collections having the proper statistical distributions

It is important to consider under what circumstances such a curve will resemble the actual curve showing the displacements in a stream of radiation



Suppose the black body consists of a box having a small hole in one side and that an image of this hole is focussed on the slit of a spectrometer The disturbance at any point on the slit is the sum of the disturbances coming from an enormous number of radiating electrons on the walls of the inside of the box Suppose each electron emits a series of more or less separate impulses, then if the number of electrons concerned is big enough, the displacement at a point on the slit will be at any instant the sum of a large number of displacements each due to one electron In this case the displacement will practically never be zero, because no impulses have arrived, but only when the sum of the displacements in the impulses is zero This is the case to which the theory given above is intended to apply. In this case, if a large number of short lengths of the displacement curve were selected at random, then most likely a great many of them would resemble the curve shown in fig 1 In this sense this curve may be said to represent the character of the disturbance in the radiation

If the distribution of the energy in the spectrum of the radiation emitted

by a single electron could be examined it might be found, when averaged over a long enough period, to be the same as that actually found, but we have no evidence as to this If the electron were freely moving about it would probably go through all the possible states of the electrons, and so would probably give the usual distribution of energy on the average over a long enough period The theory given above is not intended to apply to this case

The curve shown does equally well for any temperature provided the scale of x is taken to be proportional to the temperature and the scale of y inversely proportional to the temperature

The theory given above applies of course equally well to the radiation emitted by any body or to radiation having any kind of energy distribution in its spectrum

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Best Conditions for Photographic Enlargement of Small Solid Objects

By A Mallock, F R S

(Received January 26,—Read February 18, 1909)

Having recently had to photograph some small solids on an enlarged scale, I was led to enquire into the conditions as to the angular aperture and focal length of the lens which would give the best definition in the picture obtained

If the actual distance apart of the closest points in the object which must be separated in the picture is greater than several wave-lengths of the light employed, the problem may be solved by geometrical optics

Let f_0 = the principal focal length of the lens

$f_1 = f_0 + x$ = the distance between some part of the object and the lens

$f_2 = f_0 + y$ = the focal length conjugate to f_1

D = diameter of lens.

d = its resolving power, *e g*, the distance between the closest points which it can separate.

a = the magnitude of the smallest detail in the objects which are to appear separated in the picture.

Let b = the greatest distance of any part of the object in front of or behind the focal plane through f_1 at which details of the magnitude a appear separated in the picture

a', b' = the quantities corresponding to a and b at conjugate focal plane through f_2.

To secure the best photographic result, a/b must be a minimum

Any point at f_1 is represented in its image at f_2 by a disc surrounded by faint rings, and any point which is b out of focus is represented at f_2 by a circular patch of light whose diameter is approximately the sum of the diameters of the disc and the base of the cone, whose length is b' and whose angle is $D/(f_2-b')$

The known relation between d, D, f_1, and the wave-length of the light employed is d/f_1 = constant λ/D There is, as pointed out by Lord Rayleigh, a certain personal element in the constant, different observers having different ideas as to what constitutes "separation," but in any case it is not far from unity, and therefore we may, for the present purpose, put $d = f_1\lambda/D$, corresponding to a disc of diameter $f_2\lambda/D$ at f_2. Hence the diameter of the patch of light on the focal plane at f_2, which represents a point b, out of focus is $b' \frac{D}{f_2-b'} + \frac{f_2\lambda}{D}$ Taking D as a variable, this expression is a minimum when $b' \frac{D}{f_2-b'} = f_2 \frac{\lambda}{D}$ or $\frac{f_2}{f_1} d$, and this minimum diameter is therefore $2f_2d/f_1$ and corresponds to magnitude $2d$ at the object In order that two such patches should not overlap in the photograph, their centres must not be within one diameter of one another, and hence a cannot be less than $2d$, thus showing that the lens should have twice the resolving power required to separate the detail which is to appear in the worst defined part of the picture

Putting, therefore, $a = 2d$ and (from the ordinary formulæ for lenses) $xy = f_0^2$, we have $b' = b \frac{f_0^2}{x(x+b)}$, whence $b' \frac{D}{f_2-b'} = b \frac{Df_0}{xf_1}$, and this must be equated to $\frac{f_2}{f_1} d$ or $\frac{a}{2} \frac{f_2}{f_1}$ From this it appears that $b = a \frac{f_1}{2D}$ and since $D = \frac{2\lambda f_1}{a}$, $b = \frac{a^2}{4\lambda}$ If $a = n$ wave-lengths, $b = \frac{1}{4}n^2\lambda$ These are the relations between a and b which make a/b a minimum.

If a is given, a lens should be used whose angular aperture $(D/f_1) = 2\lambda/a$ —a condition which can be satisfied either by varying the distance of the object from the lens or by the use of a stop. Thus the greatest "depth of

focus" which can possibly be attained if details of the magnitude a are to show in the picture is $\pm a^2/4\lambda$ or $n^2\lambda/4$

For example, suppose it is required to photograph a rough broken surface covered with markings as close as 10,000 per inch If these markings are to be distinct in every part of the enlargement, what is the greatest permissible depth of the roughnesses, and what lens should be used? Since λ is about 1/50000 inch and $a = 0.0001$, $n = 5$, and $b = \frac{2.5}{4}\lambda = 0.000125$ inch If the focal plane is adjusted to be at the half depth of the roughnesses, the total depth over which separation of 1/10000 inch can be secured is 0.00025 inch.

The choice of a lens will be settled merely by considerations of convenience, having regard to the magnification required and the total area to be photographed, but whatever the actual size of the lens, the ratio of aperture to focus should be 2/5 If definition sufficient to separate thousandths of an inch were all that was wanted, we should have $b = \frac{1}{4} . 2500\,\lambda$ (= 0.025 inch about) and D/f_1 should be 1/25

Although this note was written with reference to the enlargement of small objects, the formulæ apply to the ordinary photographs of objects at a distance

Suppose a number of chess boards with 1 inch squares are set up at different distances from the camera, over what range can the squares be distinguished in the picture? Here $a = 1$ inch, so that $n = 50{,}000$, b, therefore, $= \frac{1}{4} \times 2.5 \times 10^9\,\lambda = 6.25 \times 10^8\,\lambda = 1{\cdot}25 \times 10^4$ inches, or 1000 feet nearly The angular aperture required (using the phrase in the photographer's sense, viz, the angle subtended by the aperture at the plate)* will be found to be 2/25

* Microscopic objectives are named from the distance of the lens to the object The focal lengths of photographic lenses, on the other hand, are stated as the distance of the lens from the image

On the Spontaneous Crystallisation of Monochloracetic Acid and its Mixtures with Naphthalene.

By H A Miers, F R S, and Miss F Isaac

(Received January 26,—Read February 18, 1909)

(Abstract)

The object for which the present investigation was undertaken was to study the spontaneous crystallisation of mixtures of two substances which form mixed crystals and possess a minimum, or eutectic, freezing point According to Cady,* naphthalene and monochloracetic acid are such a pair of substances, and possess a minimum melting point of 53° 5 for the eutectic mixture of 29 4 per cent naphthalene, 70 6 per cent monochloracetic acid We have attempted to verify Cady's freezing and melting point curves, but, although our experiments confirm the former, we were quite unable to verify the latter Pickering mentions four modifications of monochloracetic acid,† and traces the freezing point curves for three of them Our study of mixtures of monochloracetic acid and naphthalene has not given us the information we expected concerning the composition of the "mixed crystals" which separate spontaneously from a solution, but it has led to interesting results concerning the spontaneous crystallisation of the different modifications of a substance dissolved in another substance which is not polymorphous

The Different Modifications of Monochloracetic Acid and their Mutual Transformations

Microscopic examination of crystals of monochloracetic acid obtained from fusion or solution show clearly that three different modifications α, β, and γ of the acid exist These modifications have melting points 61° 5, 55°, and 50°, they are formed on the microscope slide under different circumstances, each modification yielding rhombs quite distinct from those of the two other modifications If fused monochloracetic acid be cooled suddenly it crystallises as the γ-modification in rhombs having a plane angle of about 59° If these rhombs be touched they at once transform into the β-modification and give rhombs having a plane angle of about 72°, or occasionally the γ-rhombs are transformed at once into the stable α-modification of the acid, which exists as broad needles having an acute angle of

* 'Journ. Phys. Chem.,' 1899, vol 3, p 127

† 'Journ Chem Soc.,' 1895, vol 67, p. 664.

43° The transformation from γ to α takes place much more rapidly than the transformation from γ to β Similarly, if rhombs of β are formed on a microscope slide they may be at once transformed into the α-modification by inoculating with a fragment of α Each transformation is accompanied by a rise of temperature A remarkable feature of the change is that the more stable modification crystallises with sharp edges in the solid mass of the less stable substance, as though it were growing in a liquid

Aqueous Solutions of Monochloracetic Acid

The aqueous solutions were treated by the methods already employed for the various solutions and mixtures for which we have determined the supersolubility curves The changing constitution of the cooling stirred solution was traced by means of the refractive index measured by the method of total reflection within a prism immersed in the solution As the temperature fell the refractive index of each solution was found to rise until it reached a maximum, at this point a dense shower of crystals appeared, accompanied by a fall in index and a rise of temperature It was found that for a given solution, if the shower of crystals was of the γ-modification, the index attained a higher maximum, and the shower occurred at a lower temperature than for showers of the α- or β-modification When the shower was of the β-modification the maximum index was higher and the temperature lower than for a shower of α-crystals This indicates that the solutions become labile for the γ-modification at lower temperatures than for β and α, and at a lower temperature for the β- than for the α-modification Long series of experiments have enabled us to establish three supersolubility curves separating the labile and metastable regions corresponding to the modifications α, β, and γ of the acid These supersolubility curves are separated from each other by intervals of 5° to 6°

After a cooling solution has first given a shower of β-crystals on the β-supersolubility curve, a second shower of α-crystals may occur, and the whole of the β-crystals will be transformed into α-crystals with rise of temperature Similarly, with a solution first giving a shower of γ-crystals, on the γ-supersolubility curve, a shower of β-crystals usually occurs, with rise of temperature, and transformation into β, and then, further, a shower of α-crystals may occur, with another rise of temperature, and transformation into α

The three supersolubility curves have also been verified by an independent method. Aqueous solutions of varying concentrations were enclosed in sealed tubes and heated to about 80° until all the crystals had dissolved The tubes were then allowed to cool very gradually while they were shaken

continually in a water bath, and the temperature at which crystals first formed spontaneously in each tube was noted Sometimes α-crystals first separated, and sometimes β- or γ-crystals The temperatures at which the various modifications crystallised spontaneously in the tubes were found to agree very closely with the temperatures determined by the maximum index and spontaneous showers in the preceding experiments

The solubility curves for three modifications of monochloracetic acid have already been traced by Pickering these modifications he calls α, β, and δ, he also obtained one point on the solubility curve for a fourth modification which he called γ We have obtained the three solubility curves for the α-, β-, and γ-modifications by an independent method which has been used in our previous work Solutions of known concentrations were enclosed in sealed tubes and were heated until all the crystals had dissolved except one or two very small crystals which were preserved at the upper end of the tube The tubes were then allowed to cool gradually in a water bath while the enclosed crystals were watched with a lens The small crystals were seen to dissolve at first, but when the water bath had fallen to a certain temperature the crystals began to grow, their edges becoming quite sharp This is the temperature of saturation In this manner the solubility curves have been obtained for the three modifications α, β, and γ of the acid The results obtained for the α- and β-curves agree almost exactly with those of Pickering, also the point found by Pickering on the γ-solubility curve coincides with the point on the γ-solubility curve here determined, but no direct evidence has been obtained of the existence of the fourth solubility curve for the δ-modification of the acid obtained by him

Mixtures of Monochloracetic Acid and Naphthalene

The second part of this paper deals with mixtures of monochloracetic acid and naphthalene These substances Cady describes as forming mixed crystals and possessing a minimum eutectic freezing point. In a long series of experiments, however, we have never found any indication of mixed crystals The melting and freezing points of a large number of mixtures were carefully determined, but in no case was there found to be any appreciable difference between these temperatures, and microscopic experiments confirm this result The study of these mixtures, therefore, yields results similar to those obtained for mixtures of salol and betol,* a new feature being introduced by the existence of the three modifications of monochloracetic acid

The solubility curves for the mixtures of monochloracetic acid and

* 'Roy Soc Proc,' A, vol. 79, 1907

naphthalene were first determined by means of a precisely similar method to that described above

Mixtures of various concentrations were enclosed in sealed tubes and the temperature of saturation noted by observing the point at which a small immersed crystal first begins to grow in the liquid mixture In this way it has been found possible to trace four solubility curves, *viz*, the solubility curve for naphthalene in liquid monochloracetic acid, and the three solubility curves for the modifications α, β, and γ of the acid in liquid naphthalene Each of these latter meets the naphthalene solubility curve in a eutectic point, the four solubility curves thus giving three eutectic points

The supersolubility curves have also been determined The melting points of these mixtures were too high to render the use of the goniometer convenient Their refractive indices were, therefore, not investigated, but the temperatures of spontaneous crystallisation were observed in the manner mentioned above, by enclosing mixtures of known concentrations in sealed glass tubes and shaking the tubes continually as they cooled The tubes also contained fragments of glass or corundum to ensure friction

In this way four supersolubility curves for these mixtures have been determined, giving the highest temperatures at which naphthalene and the α-, β-, and γ-modifications of the acid form spontaneously in a liquid mixture It is thus possible for a given mixture to crystallise at four different temperatures on different occasions according as naphthalene or the α-, β-, or γ-modification of the acid forms spontaneously within it

The supersolubility curve obtained for naphthalene intersects the three monochloracetic acid curves in three hypertectic points These points give the highest temperatures at which naphthalene and each modification of monochloracetic acid can crystallise spontaneously together.

The four solubility and four supersolubility curves, when plotted on a diagram, show that in a mixture of two substances, one of which exists in three modifications, no less than eight freezing points may be exhibited by a single cooling mixture

This investigation has, therefore, shown that each modification of a polymorphous substance possesses a definite and different temperature of spontaneous crystallisation in its mixtures with another substance which is not polymorphous

Note on the Stability of Jacobi's Ellipsoid

By Sir G H Darwin, K C B, F R S

(Received February 22,—Read March 11, 1909)

It is known that Maclaurin's spheroid of rotating liquid becomes unstable when its eccentricity reaches the value sin 54° 21′ 27″ This is a form of bifurcation, and for increasing momentum the stability passes over to Jacobi's ellipsoid. It is possible to prove these results by the method applied by me to the discussion of the stability of the pear-shaped figure of equilibrium,* and it is worth while to do so, because we obtain thereby a verification of the complicated analysis used in the previous investigation, and because the series which arise are exactly similar to the former series

In vol 3 of my 'Scientific Papers' I shall give a few details about the present analysis It will here suffice to say that it gives the known results correctly, and a good approximation to the form of Jacobi's ellipsoid

M Liapounoff maintains that if, in considering the pear-shaped figure, I had taken more terms into account, the conclusion arrived at would have been reversed It is therefore worth while to examine the corresponding series in the present precisely similar problem, in which a few terms are found to give good results Although, of course, we cannot thus prove the correctness of my former conclusion, as against M Liapounoff's contention, yet I think that this investigation tends to indicate that I was right before

In the case of Jacobi's ellipsoid there are two series proceeding *pari passu*, viz, those depending on the zonal and on the fourth tesseral harmonics The reader is requested to refer to my former papers for the interpretation of the notation For these the results are as follows —

Zonal Harmonics			*Fourth Tesseral Harmonics*		
s	$(B_s)^2/C_s + [s, 0]$	Ratio of each term to the next	s	$(B_s^4)^2/C_s^4 + [s, 4]$	Ratio of each term to the next
2	37 2393	37	4	3 3307	10.
4	1 ·0077	6½	6	0 3352	13
6	0 1554	9	8	0 0253	—
8	0 0170	—			

In the case of the pear-shaped figure we need only consider the zonal terms, since the tesseral terms are clearly insignificant The result was

* 'Phil Trans,' A, vol 200 (1902), pp. 251—314, and vol 208 (1908), pp. 1—19

s	$(B_s)^2/C_s + [s, 0]$	Ratio of each term to the next
2	−0·000080868	—
4	0·000247274	3 9
6	0·000062350	3 2
8	0·000019576	3 1
10	0 000006328	—

We may conjecture that the uncomputed residue is about 0 000003 * The convergence is clearly slower than in the case of Jacobi's ellipsoid, but I cannot think that the series can converge so slowly as to justify M Liapounoff

It seems to me, then, that this present investigation should lead us to look with doubt on an argument adverse to the former conclusion

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On the Wave-lengths of Lines in the Secondary Spectrum of Hydrogen

By Herbert Edmeston Watson, B Sc (Lond)

(Communicated by Sir William Ramsay, K C B , F R.S Received February 12,— Read March 11, 1909)

The spectrum of hydrogen is one which has engaged the attention of numerous observers from the year 1851 onwards Historically it is of great interest, firstly owing to the part it played in the controversy between Ångstrom and Wüllner on the plurality of spectra, and secondly, because it was the first spectrum in which a relation between the wave-lengths of the lines composing it was discovered

It is not proposed to discuss the origin of the two hydrogen spectra in this communication, as it has recently been done in a very complete manner by Dufour,† the main points at issue, however, may perhaps be briefly stated

Wüllner,‡ who was the first to make detailed observations of the spectra of hydrogen under different conditions, described three spectra. The first consisted of three bright lines, the second of a number of bands, and the third of several bright lines mostly in the green This was disputed by

* The absolute magnitudes of the numbers in the series compared are very different, but this merely depends on the definitions chosen for the several harmonic functions.

† 'Ann. Chim Phys.,' (8), vol. 9, pp. 361—432, 1906.

‡ 'Phil Mag.,' (4), vol. 37, p 405, 1869, and vol 39, p. 365, 1870

Ångstrom,* who maintained that one substance could have only one spectrum Hasselberg,† however, showed that though Wüllner's third spectrum was due to impurity, yet the other two could always be obtained from one sample of hydrogen, their relative intensities alone varying The results of numerous investigations following this early work have given rise to two hypotheses firstly, that the primary spectrum is due to atomic, and the secondary to molecular, hydrogen, and secondly, that the primary spectrum arises from the presence of water vapour. Some observers also maintain that the secondary spectrum is due to impurities. The above-mentioned paper by Dufour, however, seems to point very definitely to the correctness of the first hypothesis mentioned, inasmuch as he found that hydrogen prepared by several different methods and purified by diffusion through a hot platinum membrane gave the same spectrum in vacuum tubes of glass, silica, and fused boron trioxide. In all cases, both the primary or stellar spectrum, and the secondary spectrum were seen Observations were also made at atmospheric pressure by means of an arc between electrodes of various carefully purified metals The results were the same as before, even when the tube was cooled in liquid air. The primary spectrum alone was produced in an atmosphere of water vapour, but was also found in a tube of dry hydrogen which had been standing in contact with phosphorus pentoxide for a long time, and hence the spectrum is not due solely to water vapour Other experiments upon the effects produced on the spectrum by a magnetic field, and the introduction of condensers into the circuit, led to the same conclusion, viz, that the primary spectrum is due to atomic, and the secondary to molecular, hydrogen

The above brief abstract will show the stage which has now been reached by experimental work on the origin of the two hydrogen spectra, and though we are not definitely able to assert that the conclusions drawn are correct, yet the data on the subject are extensive and sufficiently complete.

When, however, we look upon the spectrum of hydrogen from another point of view, namely, that of the wave-lengths of the lines comprising it, it will at once be seen that the same completeness does not exist The lines of the primary spectrum have, it is true, been measured with accuracy by Ames,‡ and Evershed,§ but it must be remembered that, in spite of their importance owing to the relation between their wave-lengths discovered by Balmer,‖ the number so far seen is only 29, and, of these, two-thirds occur

* 'Pogg Ann.,' vol 144, p 300, 1871, and 'Comptes Rendus,' vol 73, p 368, 1871

† 'Bull Acad Imp St Petersb.,' vol 27, p. 97, 1881

‡ 'Phil. Mag,' vol 30, p 33, 1890

§ 'Phil Trans.,' A, vol. 197, p 381, 1901, and vol 201, p. 457, 1903

‖ 'Wied Ann,' vol 25, p. 80, 1885.

only in stellar spectra. Moreover, the intensity of nearly all of them is very small when compared with that of many of the secondary lines seen in an ordinary hydrogen vacuum tube, so that in order to eliminate hydrogen from a spectrum, it is equally important to know the wave-lengths of the secondary lines.

The classical research of Hasselberg,* which has given us the most complete list of wave-lengths of the hydrogen lines we possess, is not sufficiently accurate for modern requirements, especially in the red portion of the spectrum In the first place, the scale upon which the wave-lengths are based is that of Ångstrom, and, in the second place, it is pointed out by Hasselberg himself that the effect of temperature changes was very great, and difficult to eliminate, two prisms of carbon disulphide having been used as the dispersive medium However, as will be seen from the following table, the results are remarkably near to those obtained by more refined apparatus

The only other available measurements are those of Ames,† who gives 61 of the chief secondary lines between the green and ultra-violet regions, and of Frost,‡ who determined the wave-lengths of 90 lines between λ 4725 and λ 4358, the source being a helium tube which developed the hydrogen spectrum These measurements could not be continued owing to the tube running out In addition, Dufour gives a list of the wave-lengths of 125 lines from λ 4935 to λ 3652, but these determinations were made by means of a grating without a slit, and are only accurate to half an Ångstrom unit

This want of data first attracted my attention when measuring the spectrum of the lighter constituents of the atmosphere,§ which consisted partly of hydrogen. When working with the rare atmospheric gases it is very difficult to remove hydrogen from the electrodes of vacuum tubes, and, moreover, there are cases in which it may be of advantage to have hydrogen present, in order to use its lines as standards Consequently, it is highly desirable that the wave-lengths of all the brightest lines in the secondary spectrum should be known as accurately as possible

With this end in view, the following experiments were made The gas to be examined was contained in a vacuum tube with aluminium electrodes, and having a quartz plate sealed on opposite the end of the capillary portion

* 'Mem. Acad. Imp. St Petersb.,' vol 31, p. 14, 1883, and 'Bull. Acad Imp.,' vol. 30, p. 14, 1886.

† *Loc cit*

‡ 'Astrophys. Journ.,' vol 16, p 100, 1902.

§ 'Roy. Soc. Proc.,' A, vol 80, p. 181, 1908.

In one case no hydrogen was put in the tube, but a new electrode was used, and the tube filled with moist air It was then pumped empty and a current passed A large quantity of hydrogen was produced, which was pumped off until the tube became phosphorescent The direction of the current was reversed, and thereby causing a further evolution of hydrogen After three exhaustions the tube was sealed off, and it was found that on running for a few hours the quantity of gas did not appreciably increase When a series of photographs had been taken with this tube it was evacuated and filled with dry hydrogen made from palladium hydrogen, the pressure being about 1 mm At greater pressures the brilliancy diminished The two series of spectra obtained in this way were very similar, but the first one contained a considerably larger number of lines than the other These additional lines were always very weak, and could not be attributed to impurities, so that it seems probable that they were merely made visible because of the slightly greater brilliancy of the first tube used, and no distinction is drawn between them and the others in the following table One plate, however, was obtained which showed a number of not very weak lines in the extreme ultra-violet, and several of these appear to be identical with the stronger lines occurring in the spectrum of the oxyhydrogen flame given by Liveing and Dewar * I was unable to obtain them again

The secondary spectrum of hydrogen consists of two parts, the first being composed of a large number of lines, which are nearly all sharply defined, and which are particularly intense in the orange and violet regions Although plates were used which were sensitive as far as λ 8000, no lines could be detected on the less refrangible side of the C line, but, on the other hand, a considerable number were measured the wave-lengths of which were less than λ 3646, the theoretical limit of the primary spectrum according to Balmer's formula. It is possible, however, that owing to the correspondence with Liveing and Dewar's values just mentioned, the majority of these arise from the presence of water vapour The second part of the spectrum appears to be continuous, and extends from about λ 4570 to the extreme ultra-violet

The lines of the primary spectrum were not especially brilliant, the intensity of the F line being about equal to, but certainly no greater than, that of the stronger secondary lines. Only eight primary lines could be seen at all, whereas Ames was able to measure 13.

The tubes always contained mercury vapour, which was only very slowly absorbed by the electrodes, so that mercury lines were obtained on all the photographs, and the colour of the incandescent gas was a pale pink.

* 'Phil. Trans.,' A, vol. 179, p. 27, 1888.

A current from an induction coil without a Leyden jar was used to illuminate the tubes, and exposures of 16 hours were given This was not really sufficient, as many of the weaker lines were hardly visible, and very difficult to measure, but it was found that not much advantage was to be gained by increasing the exposure beyond this time, as the temperature changes and vibration were apt to produce a blurred image All the measurements were made in the first order of the Rowland Concave Grating at University College, the focal length of which is 10 feet, and which is ruled with 14,438 lines to the inch. A quartz condensing lens was used to focus the image of the capillary part of the tube on to the slit, and an iron arc spectrum was photographed on each plate above the hydrogen

Wratten Panchromatic plates, 25 cm long, were used, except when photographing beyond the C line, in which case Spectrum plates were employed

Both the hydrogen and principal iron lines were measured twice on each plate, and the wave-lengths of the latter calculated The differences of the values obtained in this way from the known iron wave-lengths were then plotted against the micrometer readings, and a curve thus obtained by means of which the wave-lengths of the hydrogen lines could be corrected for errors arising from the screw, etc At first, the wave-lengths of the iron lines given by Kayser and Runge* were used as standards But it was found that they were not sufficiently accurate, and finally, only the standard iron lines, given by Kayser,† and some of Rowland's standards were employed In several cases where the correction was not quite certain both the spectra were remeasured on a different part of the screw.

No attempt was made to determine the absolute wave-lengths of the hydrogen lines from those of the iron lines because of the probability of a lateral shift between the two spectra, but lines in the gas spectrum were themselves used as standards, those chosen for the purpose being three primary hydrogen lines, and the yellow and green mercury lines From these, the wave-lengths of some second order mercury lines were determined, and used as standards in the ultra-violet

The values adopted for the wave-lengths of these standard lines were —

6563 05	3654 96
5790 86	3650 28
5769 80	3131 94
5460 98	3125 78
4861 50	2655 26
4358 49	2653 83
4101 90	

* Watts, 'Index of Spectra,' App C, p. 1, 1888

† 'Hand. der Spectroscopie,' vol. 1, p 726.

The C and F lines are Rowland's standard values, and the figure for H is the one recently given by Evershed.* Some difficulty was experienced in obtaining reliable values for the yellow mercury lines, as those given by Kayser and Runge† differed by more than 0·3 Å.U from the values actually obtained. Consequently, a redetermination of the wave-lengths of these lines was undertaken, a mercury lamp being used as the source of light, and the preliminary results confirmed those previously obtained with the hydrogen tubes. However, before the investigation was completed, the required values were found in a paper by Hartmann,‡ in which the determinations of Fabry and Perot of the wave-lengths of the four chief mercury lines were reduced to Rowland's scale, and these values were consequently adopted. The lines 3654 and 3650 are given by Ames,§ with whose values mine agree very closely, and the last four lines on the list were measured in the second order. It must be borne in mind, however, that these so-called standards were really more of the nature of checks upon the values obtained. Actually all the wave-lengths were calculated from those of the C and F lines by means of the iron standards, and it was only necessary to introduce a slight additional correction amounting at most to 0·04 Å.U. in order to obtain the most exact values.

The error in the measurements is difficult to estimate. The numbers given in the table are, for the most part, the mean of three values obtained from three different plates, weighted according to the distinctness of the line on the plate, although, as before mentioned, many of the very weak lines could only be seen on one plate, and in consequence, the liability to error of such is considerably greater than that of other lines. Each plate was measured twice, the readings of one line rarely differing by more than 0·01 mm., corresponding to 0·05 Å.U., and the agreement between numbers given by different plates being fairly good. This correspondence really gives the best idea of the errors. Their extent may be seen from the following table, which

6	55	56	57	2	19	22	22
2	60	59	55	1	92	98	98
2	52	56	49	1	88	91	89
10	01	01	01	1	55	56	52
3	44	48	46	0	26	32	?
4	22	22	20	0	46	48	54
3	18	15	15	1	56	57	61
9	08	05	01	0	06	11	13
0	26	26	36	3	26	26	26
1	61	68	69	1	27	32	22

* 'Astrophys. Journ.,' vol. 28, p. 162, 1908.

† Watts, 'Index of Spectra,' App. D, p. 17, 1891.

‡ 'Astrophys. Journ.,' vol. 18, pp. 167–190, 1903.

shows the decimal parts of 20 consecutive lines chosen at random. The numbers in the first column denote the relative intensities, and it will be seen that the very weak lines are much more liable to error than the others

The only line which caused difficulty owing to its strength was the C line, but its position was accurately determined by taking a number of readings.

Table I gives a list of the lines measured, together with their relative intensities For the sake of comparison, Hasselberg's values are also given, but as they were on Ångström's scale, the corrections given in Watts' 'Index of Spectra,' App B, p 1, have been applied to reduce them to Rowland's standard The second place of decimals has, however, not been altered owing to the uncertainty of the amount of such alteration Hasselberg's intensities have also been ranged from 0 to 10 instead of from 1 to 6.

It will be noticed that Hasselberg gives a considérable number of lines in the yellow-green region which I have not observed It seems probable that as his measurements in this part of the spectrum were visual, and the eye has a maximum of sensitiveness in this place, these additional lines were too weak to be recorded photographically, and, indeed, indications of several lines can be seen which are too weak to measure It may be, however, that, in view of the way in which the intensities of different parts of the secondary spectrum vary, Hasselberg's tubes were of such dimensions, and were filled at such pressures, that the yellow portion of the spectrum was brighter than in my experiments

In the table, the values of Frost and Ames are included, and it will be seen that the agreement is very close

Table II gives a list of the principal mercury lines which were measured, the values being deduced from photographs in both the first and second orders. The figures are very near those of Ames, and hence are probably rather more accurate than those of Kayser and Runge, in spite of the "limit of error" given by these observers

It will also be noticed that the intensities differ considerably from those previously published, a possible explanation being that my values were obtained for a mixture of mercury vapour and hydrogen.

In conclusion, I should like to take this opportunity of offering my best thanks to Sir William Ramsay for supplying me with a sample of pure hydrogen, and for the constant interest he has taken in this work.

Table I.

Wave-length	Intensity.	Hasselberg	Intensity	Wave-length	Intensity	Hasselberg	Intensity
*6563 05**	*50*†			6159 ·82	0	59 68	1
6527 ·68	1			6155 86	0	56 ·04	2
6517 ·99	1			6153 48	0	53 75	1
6506 82	1			6151 83	0	51 84	1
6475 70	0					46 80	1
6473 92	0			6144 29	0	44 43	1
6441 ·83	0			6143 80	0	41 78 P	0
6438 10	0			6139 72	0	39 ·90	0
6435 ·02	0			6135 56	6	35 45	10
6433 80	0			6134 58	2		
6429 57	0			6132 ·06	0		
6428 36	2	23 ·67	1	6127 53	2	27 51	6
6412 ·96	0			6122 ·01	10	21 88	10
6402 51	1					19 32	1
6399 ·71	1	95 32	1	6109 81	0	12 ·94	0
6383 20	0			6108 19	0	08 53	0
6380 ·30	0			6098 45	3	98 ·66	2
6377 59	0			6097 57	0		
6372 41	0			6096 ·21	4	96 20	6
6365 ·07	0					94 00	6
6362 ·67	1	59 ·54	0	6091 16	3	91 ·00	5
6340 ·75	2	38 ·80	1			84 85	0
6335 53	0			6080 ·03	9	81 10	8
6332 70	1			6079 38	0	79 51	0
6329 ·84	0*d*‡			6074 ·66	1	74 92	4
6327 ·25	8	24 87	6	6070 21	*2*	70 66	8
6320 ·64	0			6067 ·96	1	67 ·92	4
6303 68	0	01 75	1	6066 89	1		
6302 46	0			6063 54	1	63 ·98	4
6299 60	5	97 ·90	5	6056 ·28	0	56 67	1
6285 56	4	84 39	4	6053 ·49	0		
6277 30	1			6052 58	1	53 ·06	6
6275 10	2	74 ·00	0	6048 10	0	48 ·24	3
6271 55	2	70 ·68	0	6045 80	0	45 44	1
6268 ·20	0			6044 80	0	43 30	1
6249 42	0			6041 27	1	41 ·23	1
6240 42	0			6032 14	5*d*	32 ·07	10
6238 57	6	38 26	5	6030 ·30	0*d*		
6236 ·03	0			6028 21	6	28 ·21	6
6233 22	1	33 ·09	0	6024 ·00	4	23 87	5
6230 48	2			6021 53	4	21 ·43	6
6225 ·02	10	24 ·86	6	6018 53	9	18 46	10
6221 99	0			6012 82	0		
6201 88	2	01 66	1	6011 72	0	12 ·02	0
6199 ·58	4	99 57	6	6007 27	0	07 ·40	0
6196 11	0			6005 19	0	05 ·24	0
6197 32	2	97 ·04	4	6003 ·08	4	03 ·25	5
6183 19	4	83 ·09	6	5996 19	0	98 ·28	0
6176 40	0	76 ·57	2	5994 ·30	4	94 ·64	4
6175 14	0			5992 14	0*d*	92 ·65	4
6174 28	1	74 57	5	5990 ·79	1	90 ·81	3
6169 ·65	1	70 46	3	5980 47	3	80 ·32	4
6167 ·92	0	68 ·07	0	5982 79	3	83 ·07	6
6164 41	0	64 ·95	2	5975 ·68	8	75 77	8
6162 ·69	0			5974 45	1		
6161 81	1	62 22	5	5970 49	1	70 ·15	4

* Ames, 6563 ·04

† The lines of the primary spectrum are printed in italics

‡ *d* signifies that the line is diffuse

Table I—*continued*

Wave-length.	Intensity	Hasselberg	Intensity	Wave-length	Intensity	Hasselberg	Intensity
5967·55	1	67·57	5	5796·82	0	96·07	0
5963·70	1	63·62	4	5794·94	1	94·23	2
5959·96	3	60·00	5	5791·93	2	91·42	2
5956·64	0	56·47	0	5788·50	1	87·22	0
5950·15	6	50·15	6	5786·00	3	85·39	6
5947·56	2	47·80	5	5783·32	0		
5943·65	0	43·86	0	5780·24	0		
5942·24	0	42·15	0	5779·22	0	79·12	4
5936·87	6	38·91	8	5775·28	2	74·85	6
5936·29	1	36·36	0	5773·48	1	73·02	0
5931·62	6	31·77	8	5766·26	0	66·42	3
5926·14	0	28·48	0	5763·60	0		
5925·09	2	25·17	6	5763·01	0	62·94	0
5920·95	2*d*	21·09	6	5760·58	1	60·35	5
5918·37	0			5759·76	1		
5916·76	1	16·60	6	5757·57	1	57·42	5
5916·31	1			5755·96	0		
		12·32	0	5742·05	0		
5909·71	0	10·02	3	5740·15	0	40·55	0
		05·66	0	5738·48	0	38·90	0
5908·77	0	04·06	1	5737·06	3	35·77	6
5900·92	0	00·97	1	5735·37	1	34·30	1
		98·50	0	5732·11	2	30·85	6
5896·21	0	96·41	0	5728·76	2	27·56	6
5894·24	0	94·36	1	5723·63	0	22·63	0
5891·55	0	92·15	0	5716·31	0	15·17	1
5890·19	2			5713·59	1	12·83	2
5889·29	2			5709·93	0	09·14	0
5888·41	3	88·87	10	5703·50	0	03·25	4
5884·91	2	84·52	10	5700·84	0	00·34	1
5884·18	2			5697·76	0	97·09	1
5882·17	0			5694·35	0	93·97	1
5879·55	1	79·08	6	5689·43	2*d*	89·10	6
5878·73	2			5684·33	0	84·09	5
5876·35	0	76·45	0			82·64	5
5872·12	2*d*	72·38	6			76·36	0
5870·30	0					74·62	0
5869·47	2	69·76	6			73·88	2
5864·74	1	64·91	2	5671·10	0	70·70	2
5861·77	0	62·01	1			67·37	2
5859·96	1	60·32	0			63·46	0
5857·40	0	57·87	0	5661·97	0	61·80	3
5852·72	6	51·96	2			59·57	2
5849·56	2	49·61	2			57·36	2
		47·84	0	5655·98	1	55·61	4
5836·28	5	36·45	6			52·50	1
5833·18	2*d*	33·34	3	5647·87	0	47·41	0
5831·27	1	31·53	3			46·17	0
5824·64	0	25·00	0			42·54	4
5822·99	3	23·00	5	5635·05	0	34·43	4
5822·28	1					31·97	0
5819·58	1	19·32	4			30·30	3
5816·70	1	17·10	0			26·80	3
5815·18	2	15·48	4			25·39	0
5813·83	5	13·00	10			23·24	0
5811·74	1			5621·09	0	20·05	1
5806·35	2	05·40	1			16·38	0
		04·00	0	5612·77	1	11·80	6
5801·45	0	00·83	1			08·94	0
5799·25	0	98·70	0			03·56	2

Table I—*continued*

Wave-length	Intensity	Hasselberg	Intensity	Wave-length	Intensity	Hasselberg	Intensity.
5600 65	0	99 65	4	5319 32	0	20 ·50	0
5597 80	1	96 ·75	5	5316 ·06	0	18 18	2
		91 35	1			14 ·08	0
		79 ·43	1			09 28	2
		74 21	1	5303 35	2	03 54	6
		72 35	1	5291 79	1	91 78	4
		64 ·61	0	5284 65	0	84 ·64	3
		61 ·95	0			78 78	0
		55 14	1	5272 48	2	73 ·00	4
5552 ·60	0	52 55	3	5270 68	0		
		47 ·77	0	5266 22	2	66 78	4
5543 71	0	43 36	3			64 65	4
5537 67	3	37 40	6	5261 34	1	61 94	2
		33 84	0	5256 78	1	57 23	2
		30 ·04	0			38 36	2
		26 98	1			31 30	0
		24 ·04	0	5229 ·79	0	29 ·05	2
		21 52	0	5226 ·90	0	26 43	2
		18 24	3	5224 52	0		
		15 32	0			22 66	2
		07 78	0			20 ·78	0
5505 79	2	05 50	6			14 67	2
5499 ·84	1	99 45	6			05 39	0
5496 23	0	95 79	4			02 ·93	0
		94 07	0			99 ·93	2
5481 31	2	80 94	6	5196 55	1	96 90	5
5475 ·08	0	74 ·71	1	5193 72	0		
		71 54	0			91 09	0
		65 ·20	0			88 55	0
		57 ·08	0	5180 93	0	81 14	2
		54 86	0	5174 ·98	0	75 16	2
		52 35	1			71 ·78	0
		46 75	0			68 99	0
5438 ·06	0	39 88	0			65 49	0
5435 ·06	1d	34 ·83	5			57 15	0
		30 ·96	0			54 ·76	2
		28 84	0	5149 10	0	47 ·28	3
5426 20	1	26 ·00	5	5143 ·68	0	43 64	3
5420 10	4	20 ·03	6			37 45	0
5418 ·05	0	18 36	1			34 46	0
		10 26	0			32 ·34	0
5409 ·02	0	09 18	0			28 ·09	0
		07 26	0	5122 ·64	0	23 ·36	1
		05 50	0	5120 86	0	21 41	0
5401 28	1	01 48	2	5113 36	1	14 ·09	4
5399 24	0	99 56	2			09 ·25	1
		96 59	0			07 ·26	1
5394 36	0	95 ·15	0			03 58	3
5392 55	0	92 ·67	0			99 66	0
		91 ·51	0			96 33	0
5388 35	3	88 53	6			95 ·00	0
		87 ·05	2			90 34	0
5372 ·07	0	73 59	2	5085 ·03*	1	55 ·36	5
5366 20	0	66 ·00	3			81 ·80	4
5356 ·13	0	56 ·78	0	5080 73	0	80 ·62	4
5344 ·01	0	44 ·07	0	5075 ·62	0	75 70	2
5343 ·43	0					73 ·62	1
5336 ·69	1	36 77	3			70 33	1
5334 46	0	33 ·04	0	5068 ·34	1	68 26	5
		32 ·26	0	5064 ·08	0	64 12	5

* Ames, 5084 ·9.

Table I—*continued.*

Wave length	Intensity	Hasselberg	Intensity	Wave-length	Intensity	Hasselberg	Intensity
5061 92	0	62 02	2	4883 97	0	84 ·04	1
5055 25*	1	55 02	7	4878 30	0	78 ·06	0
		49 53	2	4876 ·09*	2	76 13	4
		47 ·91	2	4874 41	0		
5044 03	0			4873 20	3	73 30	4
		41 71	4	4869 58	1	69 68	0
5040 00	0	39 74	4	4867 16	1	67 24	0
5030 56	0	30 40	4	*4861 50**	*10*	*61 ·60*	
		20 00	0	4860 28	0		
5016 53	1	16 67	4	4858 91	0		
5015 27	2	14 ·93	0	4856 ·78	4	56 77	2
5013 23*	4	13 ·01	7	4849 47	4	49 57	4
5011 48	1	11 56	1	4843 65	1*d*	43 67	1
5008 24	1	08 34	4			42 45	1
5003 57	2	03 50	5	4838 39*	2	38 31	3
		98 06	2	4832 ·93	1		
		96 ·90	2	4831 73	0		
		90 43	1	4830 91	0		
		89 54	0	4824 67	1		
		83 44	0	4823 10	2	23 20	2
4980 ·55	1	80 52	4	4817 71	1*d*		
		79 ·06	0	4813 76	2	13 93	2
		78 14	0	4807 49	1		
		76 50	0	4802 14	0		
4973 43*	3	73 41	6	4797 ·90*	3	97 81	4
4969 35	0	69 44	3	4797 14	2	97 ·08	2
4966 98	1	67 13	4	4794 11	1	93 ·97	2
		61 42	0	4791 ·02	0	90 ·93	1
4956 ·90	0	57 ·02	4	4789 58	1	89 ·41	1
4955 94	0	55 92	4	4786 23	1	86 ·00	1
		53 ·03	0	4785 ·03	1	84 ·74	1
4945 ·07	0	45 21	0			82 68	0
		42 ·67	0	4781 12	2	80 77	2
4939 ·90	0	39 82	2	4777 59	2	77 ·36	2
4939 23	0					78 ·92	0
4935 ·29	0	36 70	0	4770 87	0	70 56	0
4934 43	4	34 44	8	4766 35	0		
4933 ·68	1			4767 43	0		
4932 45	1	32 40	2	4768 ·96*	3	68 58	3
4928 ·88*	5	28 85	8	4758 72	0		
		25 ·74	1	4756 ·89	1		
4924 20	0	24 48	0	4755 42	0		
4919 30	0	19 32	2	4751 72	0		
4909 ·00	1	09 ·06	2	4743 56	1	43 53	1
4906 ·39	1	06 40	2	4742 ·90	1	42 ·86	1
4901 90	0	01 ·90	0	4742 ·19	0		
4901 ·08	0	01 14	0	4741 14	1	41 ·31	0
4896 ·69	0	96 ·55	2	4732 ·96	0		
4891 44	1	91 86	2	4730 ·87	1		
4886 ·77	0	88 58	0	4729 ·13	0		
4886 ·62	0	86 35	1	4725 ·00	2		

* Ames, 5055 2, 5013 15, 4973 ·8, 4928 8, 4876 1, 4861 40, 4838 3, 4797 ·9, 4764 ·0

Table I—*continued*

Wave-length	Intensity.	Hasselberg	Intensity	Frost	Wave length	Intensity	Hasselberg	Intensity	Frost
4723 17	6	23 23	4	23 16	4585 21	0			
4721 76	0	21 33	0		4584 71	1			
4719·18*	6	19 23	5	19 11	4584·02	0			
4714 10	2	14·04	2		4582 79	6	82 83	4	82·78
4711 17	1	11 23	0		4581 76	2	81 63	0	81·75
4709 68	3	09 62	3		4580 19*	7	80 24	4	80 21
4708 70	0				4579 60	1			79 70
4705 42	0				4578 22	2	77 92	2	78 22
4702·78	1	02 53	0		4576 07	3	75 60	3	76 08
4692 18	1	92 12	1		4572 90	5	72 54	6	72 94
4690 80	3	90 29	2	90·29	4568 80	7	68 01	6	68 35
4686 94	2	86 67	1		4565 80	0	65 18	0	
4686 26	1	86 37	1		4563·88	1	63·66	2	63·96
4684 79	1	84 57	0		4562 41	2	62 21	2	62·45
4683·96*	4	83 90	4	83 97	4561 28	0			
4682 48	2	82 56	2		4560 40	0			
4681 56	0				4558 69	2*d*	58 65	2*d*	58·75*d*
4680 57	1	80 50	2		4557 43	2*d*	57 27	2	57 46
4679·26	2	79 20	1	79 29	4554 34	3	54 13	4	54 28
4675 41	0	75 48	2		4552 58	0			
4674 63	1	74·92	2		4551·84	0			
4673·26	1	73 41	0		4551 14	2	51 08	2	51 15
4671 46	2	71·66	2	71 47	4550 10	1	49·77	0	50·31
4670·79	1				4548 13	1	47·89	0	
4670·14	0				4547 37	0			
4669·39	1				4543 87	2	43·67	1	43 89
4667·90	1	67·90	0		4541 20	0			
4667 23	1				4539 35	2	39·19	1	39 30
4665·73	2	65 80	2		4538 51	0			38 43
4662·94	4	63 15	3	62 95	4537·91	0	37 85	1	
4661 55	3	61·64	2	61·64	4536 00	0			
4660·54	2	60 48	1	60 57	4534 80*	2	34 52	4	34·61*d*
4658 97	0				4534 33	2			
4658·02	0				4533 28	2	32 86	1	33·30
4654 16	1				4532 13	0			32·18
4653 18	4	53 16	3	53 18	4531 35	0			
4650 60	0				4529 26	2	28 87	2	29 37
4649 51	0				4527 81	0			
4648·74	0				4527 27	0			
4645 48	1	45 20	0	45 51	4524 31	2	23·82	2	24·29
4634 77	3	34 40	0	34 77	4523 37	0	23·07	0	
4634 20*	9	33·90	7	34 21	4521·63	1	21 20	0	21·62
4632·06	6	31 48	6	32·02*d*	4520 11	0			20·17
4631 62	3				4519 27	0			
4628 15	6	27·66	5	28 17	4517·62	1			
4625 53	2	25 14	3	25 53	4515·69	0	15 63	0	15 71
4620·90	0	20·74	0		4515 32	0			
4618 45	3	18 24	4	18·47	4514 51	0			14 48
4617 69	3	17·59	4		4513·96	0			
4614 72	0				4512·99	0			
4613·19	0				4511 89	2			
4607 56	2	07·44	1	07 58	4511 11	2	10 65	0	10·98
4605 54	1				4509 29	0			09 13
4598·68	1			98 78					07·05
4597 25	1				4505·80	3	05 68	0	05·80
4592·01	1								05·16
4590·06	0				4504 70	0			
4588·85	1				4503·99	0			03·98

* Ames, 4719 2, 4683·95, 4634 15, 4580 1, 4534 8

Table I—*continued*

Wave length	Intensity	Hasselberg	Intensity	Frost	Wave length	Intensity	Hasselberg	Intensity	Frost
4503 84	0				4443 08	0	43 93	0	
4503 11	3	01 83	0	02 17	4441 58	0	41 42	0	
4500 21	0				4438 61	0			
4499 63	0								37 77
4498 75	1			98 69	4436 17	0			
4498 27*	4	98 15†	4	98 25	4435 11	0			
		96 81	0	95 57	4434 43	0			
		95 12	0		4432 52	0			
4493 86	1	93 43	1	93 64	4431 61	0			
4490 62	5	90 35	4	90 65	4426 11	1	25·91	0	26 13
		89 19	0		4425 33	1			
4488·00	3	87·71	3	88 01	4423 38	1*d*	23 35	0	23 43
4486 22	3	85 87	3	86 21			22 75	0	22 86
4482 25	1	81 85	0	82 19	4421 23	0			
4481 44	0				4420 48	1	20 27	0	
4479 77	0	80 04	0		4419 66	1	19 44	0	
4479 18	1	78 65	0		4417 48	2	17 40	3	17 49
4477 27	2	76 95	1	77·24	4416 53	0			16 43
		75 75	0		4415 25	1			15 16
4474 42	3	74 11	2	74 42	4414 39	1			14 43
4472 18	1	71·68	0		4413 60	1			13 70
4471·70	1				4412 42*	3	12 37	5	12 47
4468 36	0				4410 75	1	10 56	1	
4467 30	3	66 93	2	67 33	4404 80	0			
4464 34	1*d*	63 80	0	64 19	4401 77	0			01 79
4461 14*	6	60 78	4	61 15	4400 89	1	00·02	2	00 94
4459·02	1	58 85	0	58·97	4398 30	0			98·28
4456 95	2*d*	56 80	2	56 91					93·02
4455 93	1	55·57	1		4392 07	0*d*			91 90
		54 36	0		4391 11	1	91 04	2	91 13
4453 24	1*d*	52 94	0	53 22	4389 26	1	89 23	1	
4450·99	1	50 81	0				87 56	0	86·41
4450·07	1	49 83	1	50 11	4384 56	0			
4448 20	1				4383 62	0			
4447 73*	4	47 65	4	47 77	4381·68	0			
4445·42	3	45 31	2	45 44	4379 59	1	79 37	2	
4444·42	1	44 24	0		4367·96	0			

Wave-length	Intensity	Hasselberg	Intensity	Ames	Intensity	Wave-length	Intensity	Hasselberg	Intensity	Ames	Intensity
4342 07	4					4311·90	0				
4340 60	*6*	*40 66*	*10*	*40 66*	*15*	4308 78	0				
		38 90	4			4306 41	1				
4305 70	1					4306·00	0				
4333·74	1					4304·09	1				
4330 41	0					4303·60	1				
4329 84	0					4295 67	0				
4328·07	0					4279 53	0				
4318·35	0					4268 48	1				
4313 16	0					4246·85	0				

* Ames, 4498 75, 4461 1, 4447 85, 4412 35

† These and the succeeding values were the result of photographic observation

Table I—*continued*

Wave length	Intensity	Hasselberg	Intensity	Ames	Intensity	Wave-length	Intensity	Hasselberg	Intensity	Ames	Intensity
4243 55	2	43 33	2			4123 ·90	0				
4237 ·61	0	36 52	2			4118 ·74	1				
4234 ·03	1	33 86	2			4111 12	1*d*				
4233 59	2	33 48	2			4110 24	1	10 23	0		
4232 85	1*d*	32 72	0			4109 47	1	09 36	0		
4231 23	1					4108 ·08	1*d*	08 14	0		
4228 ·94	0							07 87	0		
4227 55	2	27 43	0			4106 39	2	06 35	0		
4224 ·68	2	24 49	0			*4101 89*	*3*	*01 98*	*15*	*01 ·85*	*10*
4224 10	3	23 96	2			4097 ·61	2	97 58	1		
4223 70	2	22 56	4					96 64	0		
4222 32	2	22 22	4			4096 24	1	96 13	0		
4219 ·69	1					4095 61	1	95 59	0		
4216 ·33	0					4089 01	1				
4212 ·67	5	12 43	6	12 ·65	7	4087 91	4	87 89	3		
4210 ·29	3	11 ·87	0			4085 39	2	85 88	1		
		10 11	3			4084 15	0	83 ·08 ?	0		
4209 34	3	09 13	2			4082 53	3	82 55	1		
4208 ·57	0					4081 ·65	1	81 65	1		
4207 71	0					4079 ·03	3	78 ·02*	8	79 ·0	5
4206 26	1	06 ·06	1			4074 25	2	74 28	0		
4205 26	8	04 ·99	10	05 2	8	4073 12	2	73 11	0		
4202 49	0					4071 39	2	71 42	1		
4201 15	1					4069 80	6	69 ·87	6	69 ·75	7
4199 ·95	3	99 79	5			4067 ·05	6	67 10	5	67 ·0	7
4198 34	1	98 28	2			4065 ·75	2	65 39	0		
4195 62	3	95 58	5	95 9	6	4063 78	3	63 87	2		
4194 33	0					4062 64	6	62 77	4	62 ·6	6
4189 59	1					4059 ·47	0				
4188 ·42	0					4057 71	0				
4182 33	4	82 12	4			4056 55	1				
4180 ·30	3	80 ·09	4			4055 46	1				
4179 70	2	79 58	2			4051 43	0				
4177 91	2	77 71	3			4048 58	2				
4177 26	6	77 ·07	10	77 25	8	4048 ·71	3				
4175 33	3	75 11	4			4038 67	0				
4171 45	4	71 26	6	71 85	7	4035 73	1				
4168 ·80	0					4034 22	0				
4167 72	0	67 57	0			4031 90	0*d*				
4165 29	1	65 29	1			4028 46	3				
4163 ·67	2	63 70	1			4027 50	0				
4162 ·09	2	62 ·05	3			4026 76	0*d*				
4159 49	1	59 38	2			4024 ·86	1				
4156 ·86	2*d*	56 ·62	4			4021 89	0*d*				
4155 ·98	0					4019 ·07	0				
4153 84	0					4009 ·70	0				
4151 79	0					4008 ·95	0				
4146 38	1	46 ·06	0			4006 10	2				
4145 ·77	1	45 47	0			4005 57	2				
4139 ·84	0					4002 ·60	1				
4134 ·14	1					4000 ·99	1				
4132 15	1					4000 ·29	1				
4131 59	1										

* Hasselberg's line due to Hg 4077 ·99

Table I—*continued*

Wave length	Intensity	Ames	Intensity	Wave-length	Intensity	Ames	Intensity.
3997 28	3	97 25	4	3865 ·62	0		
3993 ·95	1			3864 34	1		
3993 36	1			3863 35	2	63 3	5
3992 ·00	3	92 ·0	4	3861 ·69	2	61 7	4½
3991 23	1			3860 87	1		
3990 17	3	90 15	6	3860 ·05	1		
3987 06	1	87 ·0	3	3858 ·69	1*d*	58 85	4
3982 73	1	82 75	4	3857 89	1		
3978 94	0			3851 41	1		
3978 27	0			3849 45	0		
3977 ·03	0*d*			3836 59	1	36 ·6	3
3975 80	1					*35 6*	?
3974 90	0			3812 90	0		
3974 30	1			3809 18	0		
3971 58	0					04 ·9	6
3970 21	*1*	*70 25*	*8*	3803 ·17	3	03 2	4
3968 81	0			3800 28	0		
3963 21	1	63 3	3	3799 20	1		
3962 48	1	62 4	2	*3798 05*	*0*	*98 0*	?
3960 24	0			3797 ·63	1	97 7	4
3955 36	0			3796 73	2	96 ·8	5
3951 ·63	0			3794 67	1		
3950 ·72	0			3791 58	0		
3947 36	0			3784 30	0		
3944 45	1	44 5	3	3777 22	0		
3933 ·90	0			3774 40	0		
3928 50	0			3771 ·64	1	71 7	3
3927 32	0					*70 7*	*0*
3924 57	2	24 5	3			70 3	2
3918 11	0			3754 77	0		
3917 13	0			*3751 98*	*1*	*52 ·05*	*0*
3910 29	0			3741 40	0	41 3	2
3907 ·68	0					*34 15*	*0*
3906 51	2			3732 25	1	32 2	3
3902 76	0			3722 27	0*d*	22 2	2
3889 ·55	1	89 3	4			*21 8*	*0*
3889 16	2*d*	*89 15*	7			16 ·05	1
3888 ·00	1					*11 9*	?
3886 42	0			3702 ·21	2	02 2	3
3884 22	1			3700 20	0*d*		
3882 19	0			3690 29	1		
3879 ·73	1	79 ·7	3	3684 45	4	84 3	5
3878 79	0*d*			3682 20	1*d*	82 ·05	4
3878 ·08	0			3678 20	0		
3877 48	0			3676 ·97	0		
3874 ·95	0			3674 53	3	74 5	5
3874 ·27	1			3673 ·78	1		
3872 54	2	72 ·45	4	3666 ·02	1		
3871 ·75	3	71 ·8	5	3665 ·26	0		
3870 ·98	1*d*			3664 24	1		
3870 ·09	1			3652 58	0		
3869 33	1					44 ·8	1
3868 20	0					33 5	2
3867 29	1	67 2	4				

Table I—*continued*

Wave length	Intensity	Liveing and Dewar	Intensity	Wave-length	Intensity	Liveing and Dewar	Intensity
3520 60	1			3091 29	1	90 ·6	
3472 65	0	71 ·9		3090 36	1	89 8	
3395 ·04	1			3089 ·76	2	89 3	
3368 64	0			3087 33	0*d*	86 7	
3295 49	0	95 5		3085 25	0	84 6	
3154 47	0*d*	54 ·0		3083 28	1	82 ·6	
3130 44	0	29 9		3081 56	2	81 0	
3129 42	0			3079 90	1	79 3	
3127 67	0	27 8		3078 43	1	77 ·9	
3124 ·90	1	24 5		2977 28	0		
3124 ·00	0	23 5		2974 42	1*d*		
3122 49	1	22 2		2972 78	1*d*		
3121 71	0	21 3		2971 20	0*d*		
3119 73	1	19 2		2968 44	1		
3117 ·70	1	17 4		2830 15	1		
3117 26	0	16 ·6		2828 67	1		
3113 87	0	12 8		2827 23	1		
3112 ·09	0	11 5		2824 76	1		
3109 20	0	08 8		2804 56	1		
3103 30	0	01 ·6		2698 ·97	2		
3100 ·97	0	00 6		2661 77	0		
3099 40	0	99 ·0		2660 39	0		
3096 20	1	95 8		2639 ·90	0		
3095 88	0	94 8		2488 ·95	0		
3094 ·60	0	94 2		2482 87	0		
3092 ·81	0	92 ·0					

Table II—Wave-lengths of the Mercury Lines seen in the Spectrum.

Wave length	Intensity.	Wave-length.	Intensity	Wave length	Intensity	Wave-length	Intensity
5790 86	8	3663 40	5	3021 ·62	6	2652 16	3
5769 80	7	3654 ·96	6	2967 42	7	2576 39	0
5460 ·93	10	3650 28	9	2925 52	2	2536 ·62	1
4916 38	0	3341 66	4	2893 74	3	2534 89	3
4358 40	9	3131 ·94	9	2847 ·88	0	2482 18	2
4339 37	1	3131 65	2	2803 62	3	2399 54	0
4077 99	6	3125 ·77	7	2752 97	1		
4046 72	8	3027 ·64	2	2655 26	2		
3984 16	1	3023 61	4	2653 83	2		

The Expulsion of Radio-active Matter in the Radium Transformations

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(Communicated by Prof. E. Rutherford, F.R.S. Received March 18,—Read March 25, 1909.)

Introduction

When the radium emanation is transformed into radium A, the process is accompanied by the emission of α-particles with a velocity of 1.70×10^9 cm per second.* The portion of the atom from which the α-particle has been emitted, which constitutes the radium A, must therefore be subjected to considerable shock and recoil in a direction opposite to that in which the α-particle is projected. If we further consider that the mass of the α particle is 4 ($H = 1$), and that of the active deposit of the order 100, it follows that at the moment of its formation this product must be travelling with a velocity of the order 10^7 cm per second. In ordinary circumstances, when the emanation is mixed with air at atmospheric pressure, the radium A particle will possess only sufficient energy to permit it to travel a fraction of a millimetre before being stopped by collision with air molecules. On the other hand, at very low pressures, these particles should travel considerable distances without being stopped by the rarefied air, and come to rest on the enclosure containing the emanation.

Since the formation of radium B from radium A is also accompanied by the expulsion of an α-particle, it might also be expected that, at the moment of its formation, the recoil of the radium B atom would cause it to travel an appreciable distance through an evacuated space. There is some evidence that radium B can escape from a surface which has been exposed to the radium emanation and which is therefore coated with a film of active deposit.† Now since the volatilisation point of radium B is above 600° C., it seems unlikely that this phenomenon can be due to the volatility of radium B at ordinary temperatures as was at first suggested, and it may well be that the radium B leaves the surface as the result of the recoil when formed from radium A by the expulsion of an α-particle as has been suggested by Rutherford.‡

* Rutherford, 'Phil. Mag.,' October, 1906.
† Miss Brooks, 'Nature,' July 21, 1904.
‡ Rutherford, 'Radio-activity,' p. 392.

During the course of our work these experiments have been repeated by Hahn and Meitner,* who have shown that the phenomena are even more marked when a negatively charged plate is suspended over the active surface In the same paper evidence is given of the projection of radium C into the gas surrounding a surface coated with active deposit Both in that paper and in another,† evidence is adduced of similar phenomena with actinium and thorium

The experiments to be described in this paper afford direct evidence of the recoil of radium A, radium B, and radium C, when formed respectively from the emanation, radium A, and radium B Although hitherto no direct experiments have been made to determine the distance which the radium A

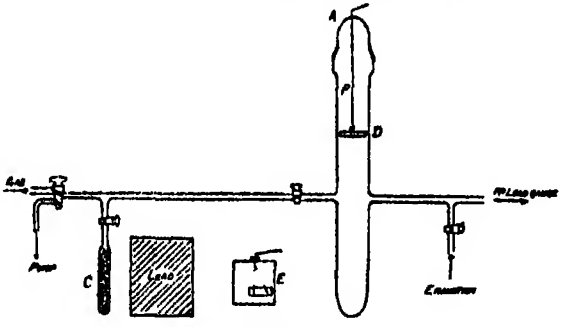

Fig 1

atom can travel when shot off from the emanation, a rough estimate of this quantity has been deduced by Jackson‡ from some previous experiments on the *method of transmission of the active deposit to the cathode.*§ It is well known that when radium emanation is exposed to an electric field the active deposit appears almost entirely on the cathode, indicating that, by some process which is not yet fully understood, the atom of radium A, when formed from the emanation, acquires a positive charge. Whatever view is adopted as to the manner in which the charge is acquired, it is certain that the velocity of the radium A atom when shot off from the emanation must be considerably diminished before a moderate electric field can exert a directive influence on it, and it has in fact been found that alike in the case of the active deposit from thorium,‖ radium,¶ and actinium,** the

* Otto Hahn and Lise Meitner, 'Verhandlungen der Deutschen Physikalischen Gesellschaft,' XI Jahrgang, No. 3.

† Otto Hahn, 'Physikalische Zeitschrift,' 10 Jahrgang, p. 81

‡ Jackson, 'Phil Mag,' November, 1905

§ Makower, 'Phil. Mag.,' November, 1905

‖ Rutherford, 'Phil Mag.,' (5), vol 49, 1900.

¶ Makower, *loc. cit*

** Russ, 'Phil. Mag,' May, 1906.

electric field exerts practically no influence if the pressure of the gas with which the emanation is mixed is sufficiently low Confining our attention to the case of radium, it has been calculated by Jackson* that, in the case considered by him, when the emanation is mixed with air at a pressure of 1 cm of mercury, the atom of radium A travels 7 mm before it has made sufficient collisions with air molecules to allow it to be directed by the electric field It will be of interest to compare this estimate with the results of the present experiments The case of the formation of radium C from radium B is a somewhat different one from those just considered, for radium B does not emit α-particles,† but only β-particles with low velocity ‡ The atom of radium C will therefore be set into far less violent motion when shot off from radium B than is the case either with radium A or B In spite of this circumstance, it will appear from experiments to be described later, that even in this case a radium C atom can travel a considerable distance through an evacuated space

The Radiation of Radium A and B through a Vacuum

We have seen that when radium A is produced from the emanation by the expulsion of an α-particle, there is reason to expect that the atom of radium A is projected with sufficient velocity to cause it to travel a considerable distance through an evacuated space The first object of these experiments, a preliminary account of which has already been given,§ was to obtain direct evidence of this radiation of radium A through a vacuum For this purpose the apparatus shown in fig 1 was employed. A cylindrical glass vessel of length 40 cm and diameter 1 7 cm. was closed at one end and provided with a ground glass stopper A, to which a brass disc D (diameter 1·6 cm) could be attached by a hook to a platinum wire P fused into the stopper A Before placing the disc D in position, a suitable quantity of radium emanation could be admitted into the vessel and condensed at the bottom by immersing the closed end of the tube in liquid air. The quantity of this emanation could be estimated by means of a γ-ray electroscope (E) situated near the apparatus. After a sufficient time had elapsed for the greater part of the emanation to be condensed, and for radio-active equilibrium to be set up, the apparatus was exhausted, first by a mechanical

* Jackson, *loc cit.*

† H. W Schmidt, 'Physikalische Zeitschrift,' 1905, p 897

‡ It has recently been suggested by Harvey that radium B emits α-particles, but these results require substantiation.—'Physikalische Zeitschrift,' January, 1909

§ Rose and Makower, 'Nature,' January 21, 1909.

pump and then by means of a bulb C, containing cocoanut charcoal immersed in liquid air so as to remove as far as possible any emanation which might have escaped condensation Pure dry air was then re-admitted to the vessel, the disc D replaced in position and the vessel re-exhausted to as high a vacuum as possible by means of the charcoal

Since we have condensed at the bottom of the tube the emanation together with its products of disintegration, it will be seen that if the view put forward above is correct, the disc D is exposed to the bombardment of the particles of radium A and B and possibly radium C, as they are successively formed, the one from the other On removing the disc D and testing its activity by measuring the α-radiation given off from it in the ordinary way with a quadrant electrometer, the side of the plate exposed to the radiation should be found to be radio-active This was in fact always found to be the case. Now this in itself would afford no evidence of the effect sought, for it is well known that when a large quantity of radium emanation is condensed in liquid air, the condensation is by no means complete, and there always exists in the vessel, above the condensed emanation, a considerable quantity of emanation in the gaseous state. A plate situated above the emanation as described above must therefore of necessity become radio active on this account. But it was always found that the activity of the surface of the plate facing the emanation was greater than that of the opposite side, and it seems quite certain that this excess of activity is due to the direct radiation of the active deposit on to the plate. The ratio of the activity of the surface turned towards the emanation to that turned away from it has been found, under suitable conditions, to be as great as 50 to 1. The exact ratio obtained depends, of course, on a variety of experimental conditions, but in all circumstances the activity of the surface of the plate turned towards the emanation exceeded that of the reverse side. Moreover, by interposing a screen between the emanation and the plate, the excess activity collected on the surface of the plate turned towards the emanation could be completely obliterated

A further experiment was tried of attaching to the bottom of the tube shown in fig 1 a narrow tube 3·8 cm long and 2 mm. diameter The emanation was then condensed in the narrow tube and the plate again exposed to the radiation from the emanation. Although the surface of the disc turned towards the emanation (henceforth called the front of the disc) was always more active than that turned away (henceforth called the back of the disc), the excess of activity was greatly diminished, as might have been anticipated, for in this case the aperture through which the radiation could reach the plate was greatly reduced.

These experiments were carried out with different quantities of emanation, and the results are given in Table I The quantity of emanation is expressed in terms of the amount of emanation which would be in equilibrium with a definite number of milligrammes of radium bromide The experiments were all made at pressures less than 0 01 mm of mercury, and the time of exposure of the discs was 40 minutes

Table I

I Milligrammes of $RaBr_2$ in equilibrium with emanation used	II Activity on front of disc in arbitrary units	III Activity on back of disc	IV Excess of front over back
16 35	2004	895	1109
7 12	1010	785	225
3 44	528	342	186
1 64	251	196	55
0 86	57	32	25

It will be noticed that, although the excess of the activity of the front of the disc over that on the back was not as marked as in previous experiments with a larger aperture through which radiation could take place, there was always an appreciably greater activity on the front than on the back, no matter what quantity of emanation was used

The Vapour Pressure of the Emanation

The early experiments were carried out with the emanation from 30 or 40 milligrammes of radium bromide in order to get as large an effect as possible, but although the radiation increased with the quantity of emanation used, it was soon found to be more advantageous to work with considerably smaller quantities, on account of the large quantity of uncondensed emanation existing above the liquefied emanation when working with these large quantities. In fact, the emanation when liquefied does not behave as an ordinary liquid, exerting a constant vapour pressure under constant conditions of temperature and pressure, but the apparent vapour pressure depends upon the quantity of emanation condensed. Although no complete investigation of this rather remarkable phenomenon was at the time possible, a few experiments were made to see what quantity of emanation would give as small a "vapour pressure" as possible, consistent with the condition of giving a measurable radiation of active deposit particles As there appear to be no available data on the variation of the apparent vapour pressure of radium emanation with the quantity used, it seems

desirable to give the results of some experiments which have been made on the subject. Reference to column III of Table I shows the variation of the vapour pressure with the quantity of emanation used, for the activity which reaches the back of the plate is due entirely to the uncondensed emanation present in the apparatus. The activity of the back of the plate after removal therefore gives an estimate in arbitrary units of the quantity of uncondensed emanation. It will be seen that by diminishing the quantity of emanation from 16·35 to 0·86 milligrammes, the apparent vapour pressure is reduced to about one-thirtieth.

A few experiments were next made to determine in absolute measure the quantity of emanation remaining uncondensed above different quantities of liquefied emanation. For this purpose the emanation from a known quantity of radium was condensed in a U-tube immersed in liquid air. The emanation was first condensed in the U-tube and sufficient time allowed to elapse for radio-active equilibrium to be set up. The quantity of emanation so condensed was measured by an adjacent γ-ray electroscope which had previously been calibrated. The U-tube was then connected to a mercury reservoir of 86·4 c.c. capacity and exhausted to a pressure less than 0·01 mm. of mercury. After some hours the mercury reservoir was disconnected from the U-tube and the emanation in the reservoir forced into a small glass bulb. After radio-active equilibrium had been established, the quantity of emanation in the bulb was measured by comparing the γ-radiation from the bulb with that from a standard quantity of radium. The results obtained are given in Table II.

Table II

Milligrammes of $RaBr_2$ in equilibrium with emanation condensed	Milligrammes of $RaBr_2$ in equilibrium with emanation vapour per c.c.
44·4	0·0026
32·5	0·00126
10·6	0·00066
6·18	0·0007*

* This observation is less reliable than the others, as the measurement of the quantity of emanation in the bulb was made two and a-half weeks after filling. A large correction had therefore to be applied for the decay of the emanation during the interval.

No doubt the apparent vapour pressure of the emanation may depend on a variety of conditions not here investigated, but although the experiments quoted are few and the conditions unvaried, they suffice to indicate that the quantity of uncondensed emanation above the liquid may be considerable when working with large quantities of emanation. On this account it was

decided to work with as small quantities of emanation as possible in subsequent experiments.

The Decrease in Radiation with Distance from the Source.

When some radium emanation is condensed at the bottom of the tube (fig 1), the activity acquired by the front of the disc D should depend upon its distance from the emanation It was found difficult to condense the emanation at the bottom of the tube, for when the tube was immersed in liquid air the emanation tended to condense in a fluorescent ring round the tube on a level with the surface of the liquid air In the experiments to be described, the end of the tube was immersed in liquid air to a depth of 1 or 2 cm and maintained nearly at a constant level, and the assumption made that all the emanation was condensed in a ring at this level The disc D was suspended at different distances above the emanation and measurement made of the activity acquired by it in half an hour, when at different distances from the condensed emanation. During these exposures the pressure was adjusted to 0 037 mm of mercury The results obtained are given in Table III As might have been expected, the activity acquired by the disc varied roughly inversely as the square of the distance from the emanation.

Table III

Distance of disc in centimetres from condensed emanation d	Activity of disc after ½ hour exposure A	Ad^2
4 7	638	14,100
8 8	187	14,500
13 5	77 8	14,200
17 25	62 9	18,700
20 3	43 1	17,800

In making these observations, a difficulty was encountered which made itself felt throughout the experiments with the emanation In order to remove the brass disc from the apparatus to test its activity, it was necessary to fill the apparatus with air to atmospheric pressure and then to re-exhaust after replacing the disc to be tested, by another similar one. If during this process the least trace of moisture or other impurity capable of being condensed in liquid air was accidentally admitted to the apparatus, this impurity condensed in a thin film over the surface of the liquid emanation and caused a diminution of the radiation emitted. An exceedingly minute quantity of such impurity is sufficient to produce a large diminution in the radiation

reaching the disc, for we shall see later that the radiation can penetrate only very small thicknesses of matter before it is completely absorbed

After this disturbing factor had been discovered, great care was taken to purify the air as far as possible before admitting it to the apparatus On this account the air was made to bubble through sulphuric acid, was then further dried by passing it over pumice stone soaked in sulphuric acid, and then over a considerable length of caustic soda to remove carbon dioxide The air was then finally caused to pass through a glass spiral immersed in liquid air. Even with these precautions it was not possible to completely eliminate the trouble mentioned, but with care it was found possible to get sufficiently consistent results for the present purpose

The Absorption of the Radiation by Air and Hydrogen.

When the residue of an emanation atom recoils as the result of the emission of an α particle, the kinetic energy of the residue will be far less than that of the α-particle, on account of its greater mass On this account, the power of the residue of penetrating matter will be small compared with that of the α-particle Considering the case of the formation of radium A from the emanation, and taking the masses of the α-particle and radium A atom as 4 and 218 respectively, we should expect that the penetrating power of the radium A atom would be only about one-fiftieth of that of the α-particle from which it is produced Since the range in air of the α-particle from radium emanation is 4 35 cm., the atom of radium A should be completely stopped by less than 1 millimetre of air at atmospheric pressure

The case is similar for the production of radium B from radium A, for the range of the α-particle from radium A is nearly the same as that from the emanation, being 4 82 cm Thus, when we have radium emanation condensed at the bottom of a tube, we have projected from it particles of radium A and radium B of about the same penetrating power

The following experiments have been made to determine the stopping power of air and hydrogen for this mixed radiation The consideration of the question of the relative quantities of radium A and radium B projected up the tube is left till later The disc D (fig. 1) was exposed to the radiation of a constant quantity of emanation and the pressure of the gas between the source of radiation and the disc varied The first experiments were made with air, and the disc was situated 6 5 cm. above the emanation. The pressures were measured on a calibrated McLeod gauge The discs were in each case exposed for 40 minutes to the radiation and tested between 15 and 20 minutes after removal from the vessel.

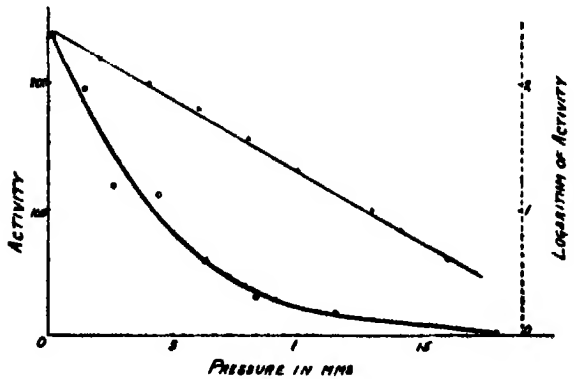

FIG 2

The results obtained are shown in fig 2, in which the excess of activity on the front of the disc over that on the back is plotted against the pressure of the air The quantity of active deposit projected on to the disc varies as an exponential function of the pressure, as may be seen from the logarithmic curve also given, the points on which have been calculated from the smooth curve drawn through the experimental points Although the general law of absorption is evident from this series of observations, it will be noticed that the points obtained do not lie very closely on a curve This inconsistency has been attributed to the cause mentioned above, for after each experiment the apparatus had to be filled with air to remove the disc, which was replaced by another During these operations the emanation must have become coated with a film of some impurity in the air that would be condensed in liquid air, and no amount of care in purifying the air seemed to suffice to give quite consistent results Although it was not thought worth while to repeat this particular series of observations, in the subsequent experiments to be described, the plan was adopted of carrying out the observations in a definite order, with a view to eliminating errors due to the condensation of impurities on the surface of the emanation At the same time great care was taken to purify the air admitted to the apparatus.

The next experiment was made with the disc at a distance of 13 5 cm. from the condensed emanation The observations were made in a manner similar to that just described, but a series of readings was taken first with continuously ascending values of the pressure and then with continuously descending values.

The order in which the observations were taken is indicated by arrows (fig. 3). As before, it was impossible to repeat any particular observation,

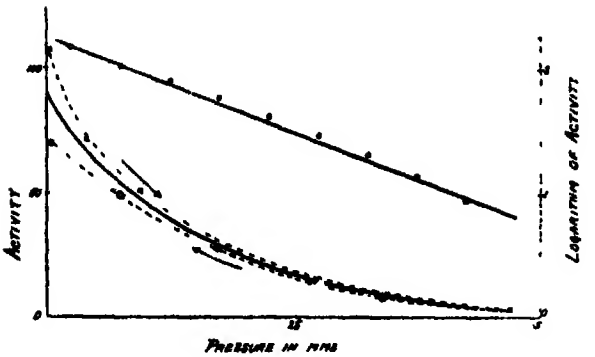

Fig 3

for as the experiment continued the radiation received by the disc at any definite pressure became less and less, so that the curve with diminishing values of the pressure fell below that with increasing values

The observations on each curve are, however, consistent with themselves, and the mean of the two curves was taken as the true curve representing the relation between pressure and activity

From the logarithmic curves and Tables IV and V it will be seen that when the disc is at 13 5 cm from the emanation the absorption of the air is about 2 6 times as great as when the disc is 6 5 cm from the emanation.

Table IV *—Air Disc, 6 5 cm. from emanation.

Pressure in millimetres	Activity of disc
0 ·009	238
0 140	195
0 254	119
0 440	112
0 625	58
0 ·830	39
1 150	16 5
1 800	2

Table V —Air Disc, 13·5 cm from emanation

Pressure in millimetres	Activity of disc
0 ·008	108
0 ·040	71 5
0 ·095	50 ·5
0 274	15
0 472	3
0 842	8
0 175	28
0 ·075	49
0 ·005	70

A further series of observations was made with hydrogen instead of air with the disc 13·5 cm from the emanation The hydrogen, which was prepared from pure sulphuric acid and pure zinc, was passed first through

* In this Table and in Tables V and VI the second column represents the excess of the activity on the front of the disc over that on the back, which, in a high vacuum, never amounted to more than a few per cent. of that on the front.

a solution of potassium permanganate, then through sulphuric acid, and then over caustic soda The purification was completed by passing the hydrogen through a spiral immersed in liquid air The observations with hydrogen were on the whole more satisfactory and consistent than with air, but although there was no gradual decrease in the activity due to a layer of impurity being deposited over the emanation, thus causing a steady variation in the quantity of radiation, it was found that rather capricious changes in the radiation took place After a series of consistent observations had been made, the radiation from the emanation would suddenly be reduced and subsequent readings, though consistent among themselves, were less than would have been anticipated from the earlier observations

No doubt these sudden changes were due to the accidental admission of some condensable impurity with the hydrogen With care it was found possible to obtain a sufficiently long series of consistent observations to

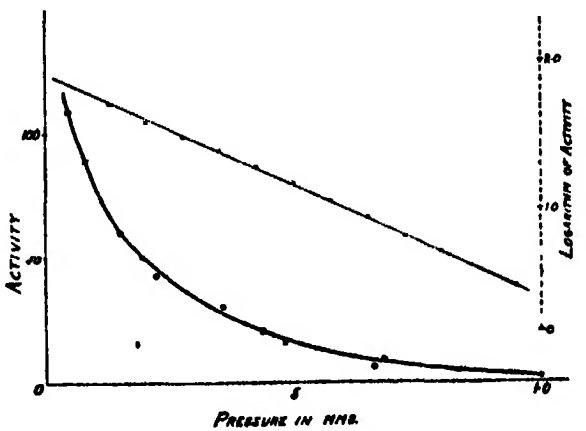

Fig 4

determine the absorption due to hydrogen The results of two sets of observations are shown in fig 4, the points on the logarithmic curve being calculated as in the case of air

It will be seen by an inspection of figs. 2, 3, and 4 that when the disc is situated at a fixed distance from the emanation, the radiation (r) reaching the disc is given by the equation

$$r = r_0 e^{-\lambda p},$$

where r_0 is the radiation reaching the disc in a perfect vacuum, p is the pressure of the air or hydrogen in millimetres, and λ is a constant coefficient which gives a measure of the degree of absorption of the radiation in passing

through the gas. Calculating the value of λ from figs. 2 and 3, we get $\lambda_{6\cdot5} = 2\,62$ when the disc is situated at a distance 6 5 cm from the emanation, and when situated 13 5 cm from the emanation $\lambda_{13\cdot5} = 6\,9$. We thus see that $\lambda_{6\cdot5}/6\,5 = 0\,40$ and $\lambda_{13\cdot5}/13\,5 = 0\,51$ Having regard to the difficulty of knowing exactly where the emanation is condensed and, therefore, of determining the distances of the discs from the emanation, these numbers are in fair agreement

In the case of hydrogen, experiments were carried out only at a distance of 13 5 cm from the emanation The value of the coefficient (λ_H) for this distance will be seen, by reference to fig 4 and Table VI, to be 4 2 Thus for a distance 13 5 cm from the emanation, the ratio

$$\lambda_{air}/\lambda_{hydrogen} = 6\,9/4\,2 = 1\,64$$

Table VI.—Hydrogen Disc, 13 5 cm from emanation

Pressure in millimetres	Activity of disc	Pressure in millimetres	Activity of disc
0·045	109	0 482	15 5
0 080	89	0 663	5 5
0 150	60	0·682	10
0 195	50	0 835	4
0·223	42 5	1·010	1 5
0 357	30		

Considering that the density of air is 14 44 if that of hydrogen is taken as unity, we see that hydrogen is much more efficient in absorbing this radiation than could be expected from its density ·

The Composition of the Radiation from the Emanation

So far no evidence has been recorded as to the composition of the radiation through a vacuum from the emanation It was of importance to decide whether both radium A and radium B were projected through the vacuum or whether the observed effects were due to one of these products alone To decide this question the decay curves of the activity collected on the disc suspended above the emanation were determined

Although several other observations were made, the results of only two experiments are given, which suffice to demonstrate beyond doubt that both radium A and radium B are expelled from the emanation into the vacuum. Both experiments were made with the disc subjected to the radiation, situated 6 5 cm. above the emanation, and the pressure of the air between the emanation and the disc was maintained below 0·01 millimetre of

mercury In the first experiment the disc was exposed to the radiation from the emanation for 40 minutes. The results of these experiments are shown in fig 5, curve I, in which the activity of the disc is plotted against the time, reckoned from the instant of removing it from the source of radiation On the same diagram curve 1 (a) is shown the well-known α-ray decay curve of the active deposit obtained by exposing a negatively charged conductor to

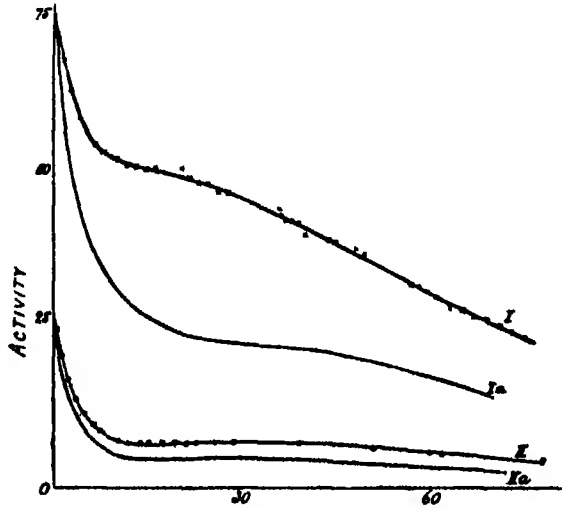

FIG 5

the emanation for the same time The difference between the two curves is striking and can at once be explained if we suppose that, whereas when a negatively-charged conductor is exposed to the emanation only radium A is deposited, which subsequently disintegrates, in the present case both radium A and radium B reach the plate by radiation.

A similar pair of curves II and II (a), for the case of an exposure of 10 minutes, is plotted in the same diagram. It will be seen that in this case, too, the ordinary decay curve falls below the curve obtained by the radiation method As might be expected, the difference between the two curves for this short time of exposure is less marked than with a long exposure

A complete analysis of the decay curves is at present withheld

The Number of Particles of Radium A and Radium B projected from the Emanation into a Vacuum

Having established that both radium A and radium B are projected on to a disc through a vacuum from the emanation condensed at the bottom

of a tube, it becomes possible to calculate the ratio of the number of particles reaching the disc in a known time to the total number of particles formed from the emanation This can be done by considering the activity of a disc exposed to the radiation from the emanation for a time sufficient for radio-active equilibrium to be set up between the emanation and the particles of radium A and radium B on the disc A disc was therefore exposed for more than three hours to the emanation from 1 18 milligrammes of radium which had been condensed at the bottom of the tube for sufficient time for equilibrium to be set up between the emanation and its products of disintegration Now it was found that *immediately* after removing the disc from the radiation from the emanation it produced a deflection of 828 scale divisions per minute on the scale of the electrometer used to measure its activity. A plate upon which polonium had been deposited, when similarly situated in the measuring apparatus, gave a deflection of 1800 scale divisions per minute. Now it was known that the quantity of polonium on the plate emitted $5\,07 \times 10^4$ α-particles per second * Since 1 gramme of radium emits $3\,4 \times 10^{10}$ α-particles per second,† the polonium standard gave off the same number of α-particles as would have been emitted from $1\,49 \times 10^{-3}$ milligramme of radium. The active disc tested therefore gave off the same number of α-particles as would have been emitted by $6\,9 \times 10^{-4}$ milligramme of radium But at the moment of its removal from the radiation from the emanation, the number of α-particles emitted by the radium A and radium C on the disc must have been nearly equal Consequently there must have been on the disc the quantity of radium A and radium C in radio-active equilibrium with $3\,45 \times 10^{-4}$ milligramme of radium Since 1 18 milligrammes of emanation were used in the experiment, the quantity of radium A and radium C on the disc was $2{\cdot}92 \times 10^{-4}$ of the total quantity of radium A and radium C in equilibrium with the emanation. Considering that the disc was at a distance of 7 cm. from the emanation, and that the area of the disc was 2·01 square centimetres, it is clear that even if every particle of radium A and radium B escaped from the emanation as it was formed by the expulsion of an α-particle, the fraction of the whole radiation reaching the disc would have been $3{\cdot}3 \times 10^{-3}$. We see, therefore, that about one-eleventh of the particles projected from the emanation, in directions included within the solid angle subtended by the disc, actually reached the disc. Having regard to the easy absorption of

* We are indebted to Dr Geiger for supplying us with this number, which had been determined by him by counting the number of scintillations produced per second on a zinc sulphide screen situated at a known distance from the polonium

† Rutherford and Geiger, 'Roy. Soc. Proc.,' A, vol. 81, p 174.

these particles by matter, there seems little doubt that with a sufficiently thin layer of emanation free from all impurities and in a sufficiently high vacuum every particle of radium A and radium B emitted from the emanation would be radiated from the emanation

The Radiation of Radium B and Radium C.

The experiments with radium emanation condensed at the bottom of a tube have shown that active deposit particles are projected through a vacuum on to a disc exposed above the emanation Since in these circumstances we have, besides the emanation, also radium A and radium B at the bottom of the tube, the conditions are somewhat complicated It was seen from the nature of the decay curves of the activity collected on the disc, that not only radium A but also radium B was projected on to the disc It is, however, not very easy to say whether there was any radiation of radium C It seemed, therefore, desirable to make experiments with simpler conditions to directly demonstrate the radiation of radium B and also of radium C, if this latter should exist To do this, observations were made with the active deposit on a plate which had been exposed as the negative electrode to the emanation in an electric field. When this plate was suspended in a vacuum, no radium A could be projected from it unless, as was generally the case, a trace of emanation remained adhering to the plate Neglecting, for the moment, this disturbing factor in the experiment, a disc exposed above the active plate should become active on account of the projection of radium B on to it as the radium A on the plate decays. If, however, sufficient time be allowed for the radium A to disappear before exposing the disc, no radium B can be projected up the tube, but in this case there still remains the possibility of radium C being projected up the tube as radium B decays Working in this way we thus have the possibility of investigating the phenomena in their simplest form. The apparatus used is shown in fig. 6. The plate D, which had been exposed to a considerable quantity of radium emanation for a definite time, was suspended by a small hook from the end of the wire w, the other end of which was sealed through the ground glass stopper S of the glass

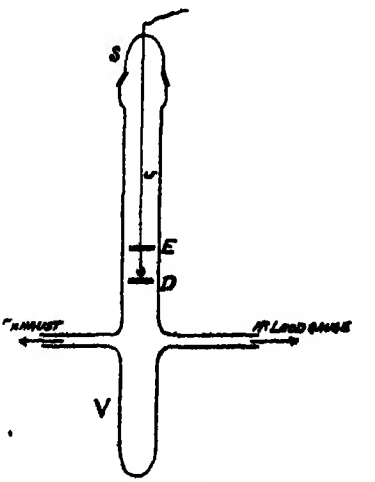

FIG. 6

vessel V of diameter 0·7 cm. Attached to the wire *w*, and at a distance of about 1 cm. above the plate D, was a disc E. By inserting the stopper S the plate and disc were brought into the position shown in the figure. By means of a bulb containing cocoanut charcoal immersed in liquid air and connected to the vessel V by a side tube, the vessel could be rapidly exhausted to as high a vacuum as desired. After a suitable exposure of the disc E to the radiation from D, air was admitted to the apparatus.

The disc E was then detached from the wire *w*, and its α-ray activity tested by a quadrant electrometer, care being taken to prevent the disc and plate touching during their removal from the vessel.

Before proceeding to an account of the results obtained in these experiments it is of importance to consider the phenomena which would be expected on certain simple assumptions. Consider the case in which the plate D has been exposed to the emanation so that it has deposited on its surface radium A, together with radium B and radium C in certain definite proportions depending on the time of exposure to the emanation. The disc E will receive a certain percentage of the radium B particles which are produced as the radium A on the plate D decays. If we assume that the radium C particles are not projected from the small quantity of radium B, initially on the plate D, then the activity of the disc E at any time after removal may be calculated by a simple extension of well-known equations.* This has been done for the case of an exposure of five minutes of the disc E to the radiation from the plate D, and the theoretical curve plotted in fig. 7, curve I.

The observations now to be described were made to see how nearly the experimental results obtained under different conditions agree with this theoretical curve.

In the first series of observations the plate D was exposed to the emanation for 10 minutes, then removed and placed in a vessel which could be quickly evacuated to remove any emanation adhering to the disc. Although it was not found possible to completely remove the emanation in this way, the amount left on the disc after this treatment was greatly reduced. Within a minute and a half of withdrawing the plate from the vessel in which it was exposed to the emanation, it was fixed in position below the disc E as shown in fig. 6. The vessel V was quickly evacuated as completely as possible, and the disc E then exposed to the radiation from D for five minutes. The disc E was then removed, and the side which had been facing D found to be strongly radio-active, its activity was tested during the next hour and a quarter.

* See Rutherford, 'Radio-activity,' p. 332.

The side turned away from D was only slightly active as a result of a trace of emanation adhering to the plate D which escaped during the exposure of the disc E In subsequent measurements allowance was made for this slight activity which must also have reached the front of the disc E

The results obtained are shown in curve II, fig 7. It will be noticed that the activity of the disc as measured by the α-radiation coming from it increased for the first half hour and then fell off with time This is what would be expected if radium B had been projected on to the disc E from the

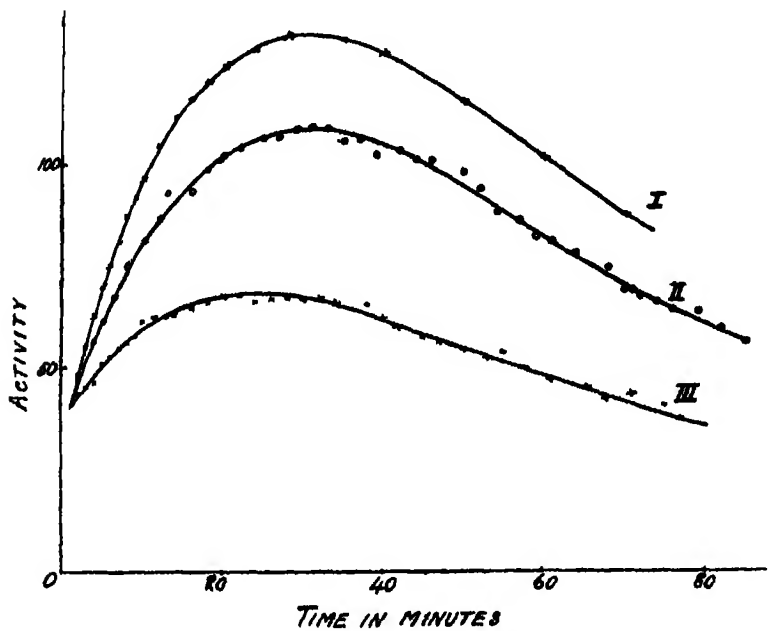

Fig 7

plate D during the five minutes' exposure, but it will be noticed that the curve II falls considerably below the theoretical curve I. Now the plate D during the previous exposure of 10 minutes to the emanation will have had deposited on its surface not only radium A but a certain amount of radium B If, during the decay of this radium B, some radium C is projected on to the disc E, the curve giving the activity at times subsequent to its removal from the radiation will fall below the theoretical curve I This has been seen to be the case and it thus appears likely that in the experiment just described radium C as well as radium B was radiated from the plate

This was a somewhat unexpected result, for unless α-particles are emitted in the transformation of radium B into radium C, this result could only be

explained by supposing that the radium C was projected from the plate by recoiling when a β-particle was emitted. It was therefore necessary to test the point more fully before this explanation could be accepted as correct

A similar experiment to that just described was therefore made in which the plate D was exposed to the emanation for nearly three hours instead of 10 minutes. In these circumstances the quantity of radium B on the plate relative to the quantity of radium A was greatly increased and the activity curve subsequently obtained with the disc E should fall even further below the theoretical curve than curve II This is seen to be the case from curve III, fig 7

Although these experiments leave little doubt that radium C is projected through a vacuum from a plate on which radium B is deposited, it seemed desirable to get more direct evidence on this point. The plate D was therefore again exposed to the emanation for about two hours and a half, and was then removed from the emanation and kept in a vacuum for 19 minutes to get rid of emanation adhering to it. During this time practically all the radium A on the plate had decayed. The plate D was then placed in position under the disc E and the vessel evacuated This disc E was removed 50 minutes later and tested during the next hour It was found to have deposited on it pure radium C, for in this case the activity at once diminished exponentially with time, falling to half value in 19 minutes (fig 8, and Table VII)

We thus see that a plate exposed in a vacuum to another one upon which there is only radium B and radium C becomes coated on the side facing the first plate with a deposit of pure radium C. This can only happen by the projection of the atoms of radium C on to the plate as radium B disintegrates No doubt radium D is also projected on to the second plate, but its rate of decay is in any case too slow to affect the observations

Since the recoil of radium C when produced from radium B is due to the emission of an electron instead of an α-particle, as is the case with radium A and radium B, we should expect that the velocity with which an atom of radium C is projected through a vacuum would be very much less than that of either radium A or radium B The power of the atom of radium C to penetrate matter should therefore be correspondingly less than was found for radium A and radium B A few experiments have been made to test this point. The disc D, with radium B and radium C deposited on it, was situated as in the previous experiments 1 cm. below the disc E, which was therefore subjected to the bombardment of the radium C proceeding from the plate D. Instead of working in as high a vacuum as could be obtained, some air was left between the plate D and E. In this way part of

the radiation from the plate D was cut off and the plate D became less active than in a perfect vacuum When there was air at a pressure of 0·14 mm. of mercury between the two plates at a distance of 1 cm. apart, the radiation was cut down to about one-tenth Other observations were made at different pressures, but the law of absorption has as yet not been determined with any precision

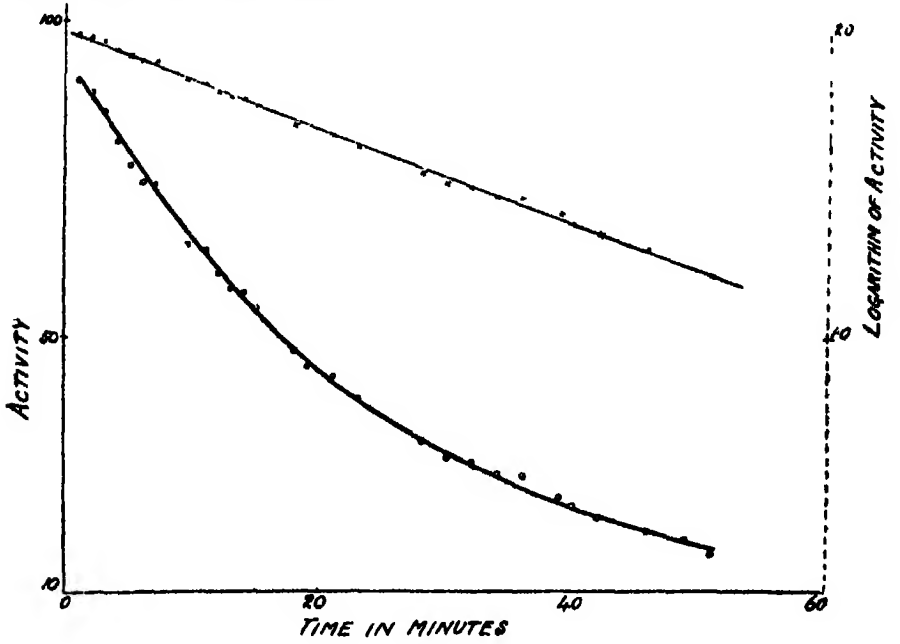

FIG 8

Table VII

Time in minutes after removal	Activity	Logarithm of activity	Time in minutes after removal	Activity	Logarithm of activity
1	90 5	1 957	19	45 3	1 656
2	88 ·6	1 947	21	43 8	1 ·642
3	85 ·6	1 932	23	40 4	1 ·606
4	81 ·0	1 ·908	28	33 5	1 525
5	77 3	1 888	30	30 ·9	1 490
6	74 5	1 872	32	30 1	1 ·479
7	74 2	1 870	34	28 2	1 450
9½	64 8	1 812	36	28 ·0	1 447
11	63 ·9	1 806	39	24 ·6	1 391
12	60 1	1 ·779	40	23 ·2	1 366
13	57 ·7	1 ·761	42	21 4	1 330
14	57 ·1	1 ·757	46	19 ·2	1 ·283
15	54 ·6	1 ·737	49	17 8	1 ·250
18	47 ·8	1 679	51	15 ·7	1 196

Summary

The principal results obtained in this paper may be summarised as follows:—(1) When radium emanation, in radio-active equilibrium with its products of disintegration, is condensed at the bottom of an evacuated tube immersed in liquid air, active deposit particles are radiated up the tube This phenomenon is ascribed to the recoil of the residual atom when an α-particle is emitted

(2) The law of absorption of this radiation both in air and hydrogen has been investigated The radiation reaching a surface at a fixed distance from the condensed emanation is an exponential function of the gas pressure

(3) From the rate of decay of the activity collected on a surface exposed to the radiation from the emanation, it appears that both radium A and radium B reach the surface

(4) Radium B and radium C are both radiated through a vacuum from a surface previously rendered active by exposure to the emanation Supposing that radium B emits only β-particles, the radiation of radium C must be due to the recoil of the atoms when β-particles are emitted

The work has been greatly facilitated by the kindness of Prof Rutherford in supplying us with the emanation necessary for these experiments. We take this opportunity of expressing our thanks to him

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An Attempt to detect some Electro-optical Effects

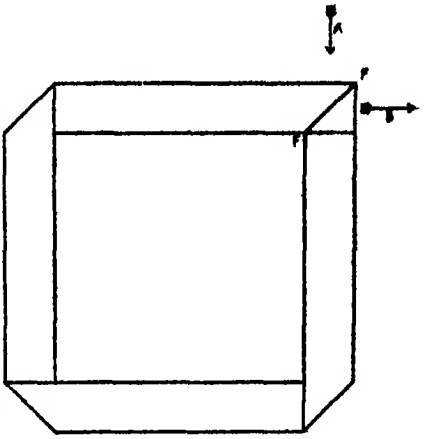
By Prof Harold A Wilson, F R S, King's College, London

(Received March 5,—Read March 18, 1909)

The following paper contains a description of some experiments made with the object of detecting possible effects due to electric and magnetic fields and moving matter on the velocity of propagation of light in glass The results obtained were negative, but it seems worth while to publish a short account of the experiments

The optical part of the apparatus is a simple form of interferometer which proved very easy and convenient to work with It consists of a square glass frame made up of glass bars of square cross-section cemented together with canada balsam Three of the corners are cut off at 45°, as shown in the figure, and the fourth corner contains a half silvered surface FF. Light entering in the direction of the arrow A is divided into two beams by the silver film, which pass round the frame in opposite directions, being totally reflected at the cut-off corners Half of each beam emerges in the direction of the arrow B, and the two beams at B are in a condition to interfere with each other

The frame is so constructed that an eye at B observing a source of white light at A by reflexion round the frame sees a system of interference bands If the frame is supported horizontally and is at an uniform temperature the bands are nearly circular with the centre of the system in the middle of the field The bands are more or less distorted by slightly warming the frame unequally, *e g* by taking hold of one of the bars, but the distortion is small and varies slowly The glass bars are 2 cm thick, and the outside breadth of the frame is 14 cm The path of a ray going once round the frame is 44 cm long inside the glass from the silver film and back to it The refractive index of the glass for sodium light is 1 67.



In all the experiments described below the interference bands were observed in a telescope provided with cross wires. The bands were widely separated, and I think a displacement or broadening of a band amounting to one-fiftieth part of the distance between two bands could have been detected. To produce this effect would require the velocity of one ray to be increased and that of the other to be diminished by one part in ten millions

The effect of a rapidly alternating induced electromotive force acting round the frame was first tried A coil of insulated wire of about 150 turns and 9 cm in diameter was wound on a glass cylinder and put through the frame so that the axis of the coil was perpendicular to the plane of the frame

A powerful jar discharge from two quart jars and a 10-inch coil was passed through the coil to produce an intermittent rapidly alternating induced electric force round the glass frame Since the alternating field is only on for a small fraction of the time that the coil is working, if a steady source of light were used to produce the bands a displacement would be difficult to observe This difficulty was avoided by using the sparks of the jar discharge as the source of light. The light from the sparks is a maximum when the rate of variation of the induced electric force is a maximum If a varying electric force acting round the frame increased the velocity of one ray and diminished that of the other, the alternating field, using the sparks as the source, would have produced an apparent doubling of the bands, or at least a broadening of them. A faint steady source of light was used in addition to the sparks, and the bands due to the sparks appeared to coincide

exactly with those due to the steady source. This experiment shows that there is no effect due to a rapidly varying electric force acting round the frame The rate of variation of the induced electric force in the glass in this experiment was about one million volts per cm. per sec.

Another experiment was then tried, using sparks from a secondary coil of two turns wound round the coil of 150 turns as the source of light. In this experiment the light was a maximum when the induced force was a maximum. Again no effect was observed.

The same two experiments were then repeated with a coil of two turns instead of the coil of 150 turns, with similar results. A small Hertz oscillator, with its axis along the axis of the frame, was then tried, using the sparks of the oscillator as the source of light, without effect.

A single layer of about 100 turns of wire was then wound on the bars of the frame so as to produce a magnetic field round the frame. A powerful jar discharge was passed through this coil without effect, using both primary and secondary sparks as the source of light.

The frame was now fixed up round the poles of a large Du Bois magnet which were put close together. The core of the magnet is 8 cm in diameter, and it is wound for 100 volts. The magnet was connected directly to an electric supply at 200 volts, and the current started, stopped, and reversed without effect* The connections to one of the coils of the magnet were then reversed so that both poles could be magnetised in the same way so as to produce a radial field at the glass frame No effect was obtained

The frame was next put between two parallel metal plates, 2 cm. apart, which were connected to a Wimshurst electrical machine, and charged up till sparks passed between them A very slight motion of the bands was observed when the sparks passed, which was no doubt due to the frame being strained by the electrostatic forces Holes were cut in the plates so that the frame and plates could be put round the poles of the Du Bois magnet In this way the effect of a radial magnetic field with a perpendicular electric field was tried Reversing the magnetic field while the electric field was steady produced no effect In this experiment, according to Poynting's theorem, there was a flow of electromagnetic energy round the frame inside the glass. Indiarubber tubing was then wound round the bars of the frame until the hole through the frame was nearly filled with the turns. A rapid stream of water was passed through the tubing without any effect on the interference bands. The glass frame was very excellently made and adjusted by Adam Hilger, Ltd., and it has remained in perfect adjustment and never given the slightest trouble.

* The total current flowing round the core was 70,000 amperes. The axis of the frame coincided with the axis of the core.

Syntonic Wireless Telegraphy; with Specimens of Large-scale Measurements.

By Sir Oliver Lodge, F R S., and Dr Alex. Muirhead, F R.S

(Received November 20, 1908,—Read January 21, 1909)

Introduction by Sir Oliver Lodge.

The absence of effective tuning is one of the marked features of wireless telegraphy as at present usually conducted in practice

In many cases messages are disentangled from a crowd of superposed disturbances, *i e* from other messages, largely by the skill of the receiving telegraphic operator, who, by the exercise of selective attention, manages to interpret and read what is intended for him, the process being identical with the ordinary human faculty whereby a conversation can be listened to amid general talking and a crowd of other noises at a dinner table

A certain amount of forbearance is also exercised in certain localities, and just as it is etiquette to wait till one conversation across the table is finished before beginning one on your account, so it is with neighbouring wireless operators—they wait for each other, and each finishes as quickly as he can In fact, it is not very different from the procedure of telephonic operators working through one and the same line, which for a time may be "engaged"

Tuning is not indeed absent, though usually far less perfect than it might be made The systems which work with a closed circuit vibrator, and use sufficient power, can effect a considerable amount of tuning, so that other wave-lengths can be weakened in comparison with those desired, and so also that the messages received have about them a more or less characteristic note or quality which disentangles them from others, and renders the picking of them out through the necessary selective attention an easier matter than it otherwise would be It is indeed extraordinary what the skill of an experienced operator will enable him to read, in spite of the imperfection of the tuning For, just as practice enables people to read bad and nearly illegible writing, or to understand very imperfect or ill-heard speech, so it is with the listener at the telephonic receiver to the Morse code signals in wireless telegraphy A purely physical instrument, such as a siphon recorder, would fail to give anything like a clear record under such circumstances, indeed, nothing short of an instrument with the selective power of the human ear could be successfully used in the process of receiving as it is frequently practised to-day

But it would appear that the physical elimination of other messages is never perfect, since, even with the most skilled operator as receiver, there are cases when conversation becomes impossible, owing to the superposition of disturbances from a newly established station; and then, by International Convention, the station first in the field has the power of calling upon the Government to withhold a licence from the newcomer or disturber of the peace, and to require him to remove or so to moderate his signals that they shall no longer be a source of trouble and confusion

And of all such stations it is natural that the stations of the Admiralty, and those which may be needed in warfare, shall demand and shall be accorded the precedence, so that they have the power to order away anything which they find deleterious to their free power of conversation with the ships—say in the German Ocean or elsewhere

But it is hardly necessary to point out how dangerous such a condition of things inevitably is If in peace time the free power of communication depends only on the ability to order away disturbing elements, what is likely to happen in time of war? Will it be possible to order away the disturbances caused by a foe? Surely the fact that the physical elimination of such disturbances is known not to have been as yet properly tackled scientifically in the best way that is possible, but has merely been got rid of by legislation, will encourage an enemy to cause purposed disturbance and to make all communication hopeless In fact, by dint of continual interruption, to render confusion worse confounded

And even apart from the exigencies of war, it is clearly desirable so to arrange each station that it can speak to the one it wants to speak to without the necessity for overhearing those which are simultaneously speaking in its moderate vicinity. And to do this not only by the practised skill of an operator but by selective means of a physical kind, so that an automatic receiving instrument such as a siphon recorder could also receive the message intended for it without superposed disturbance from others

In that case there would be no need to wait till others have finished speaking, but correspondence could be conducted simultaneously between several pairs of stations—not *too* close together and yet not really far apart.

Now all this can be managed quite easily The present trouble is caused by the utilisation of the earth as one terminal of the aerial system, both in sender and receiver I do not expect this to be immediately admitted; but so it is—at any rate at land stations. With the use of the earth as part of the main electric vibrator no perfect tuning is possible. The most economical and simplest plan is to use the same thing as vibrator and radiator: though at present in many cases they are separated If the earth is used as part of

the radiator, a closed circuit vibrator is necessary, since by its power it can force the radiator to emit a tuned disturbance, even though one end of it is earthed But with an open circuit vibrator alone, the earth must not be used, for if it is, the succession of waves is damped out, and the waves form so rapidly decadent a series that, though the initial impulse may be strong and able to carry a long way—provided it be emitted with great power—no reception on a truly syntonic plan based upon the accumulation of a large number of isochronous impulses—each alone inoperative—from a source of gentle power is possible

Yet, on the principles laid down by me in 1897, with a completely self-contained vibrator, consisting of two capacity areas both elevated above the earth as high as may be conveniently possible, accurate tuning is easy enough. A 5-per-cent change of frequency—indeed less—is sufficient to negative all response to other stations, whenever the receiver is purposely so adjusted as to be exactly in tune with one definite frequency and insensitive to all others Its sensitiveness under these conditions to the properly attuned wave-length is something surprising, and arouses the astonishment of the hardened operator The small induction coils which transmit the signals to each other in the signalling room, by mutual induction from collector to responder, may be separated in some cases more than two feet In that case it is manifest that none but the most accurately tuned impulses can be detected All others are completely eliminated, however violent they may be.

For the purpose of being called up, the coupling of the inductive connection can easily and quickly be made *close* again, and the receiver is then sensitive to all pitches But once called up it can, for the purpose of receiving the communication, be instantly, by a movement of the hand, switched over to the less sensitive state, in a state of attunement with that frequency which happens to characterise the speaking station, and to that alone.

The system thus perfected as regards tuning, through the labours of Dr Alex. Muirhead and myself and our assistants, is so satisfactory in this respect that I see no reason why I should any longer hesitate to publish a few of the records of observation and experiments made with it, on the ground of a possible but hitherto entirely non-existent pecuniary interest conceivably appertaining thereto, for they are measurements which cannot be made in a laboratory or without some similar apparatus, and they are worthy of record. With this introduction I submit the following paper:—

On the General Principles of Syntonic Wireless Telegraphy

To obtain telegraphic response between two disconnected stations in a selective manner, so as not to be disturbed by other sending stations in the neighbourhood, it is necessary—

(1) That the disturbance emitted consist of a regular or sinuous succession of impulses sufficiently numerous to accumulate their effect, consequently the emitter must be a fairly free vibrator, with a definite period of its own.

(2) That the receiver be made insufficiently sensitive to respond to a single one or two of such impulses, but sufficiently sensitive to respond to the accumulated effect of a fair number of them.

(3) For this to be possible the receiver also must be capable of fairly free and undamped vibration, so as to preserve and store the impulses as they arrive, till their combined effect attains the needful climax, and excites response

(4) Nevertheless, excessively long-continued vibration is useless In most cases the accumulation of from 20 to 40 swings should be both necessary and sufficient For if any large number of swings has to be depended on, say 1000, it is impossible to preserve the tuning sufficiently accurate to enable them all to help. For instance, if the tuning were "out" by so little as 1/10 per cent, the last 500 of such a series would be undoing the effect of the first 500

(5) In the case of radiated electric waves, the radiating power increases with the openness of circuit of the condenser or other discharged system, being greater from a linear vibrator, and least from one with a closed contour On the other hand, the persistence of vibration, or absence of damping, due to radiated energy, is greatest with a closed circuit, and very small with a linear one This is because in true waves the electric and magnetic energies must be equal In nearly closed circuits the magnetic energy greatly dominates the electrostatic, everywhere except between the plates of the condenser In open circuits the electrical lines spread much further, and so by their alternation interact with the magnetic lines to establish the system of advancing waves over a surface more distant from the radiator, for the radiating power with given wave-length increases rapidly with the extent of the radiating surface.* But of course it has less initial electrostatic energy than a condenser charged to the same potential with plates close together, and therefore of greater capacity

(6) Accordingly there must be a compromise, and in order that the syntonic radiator may also be the vibrator it must itself be a condenser

* Compare Sir G. G. Stokes, "On the Communication of Vibrations to a Gas," 'Phil. Trans,' 1868.

consisting of two moderately large capacity areas, separated from each other by a considerable space full of electrostatic lines, able, therefore, to give a moderate number of free swings—such a number as 30—after having been set in vibration. Such an arrangement was depicted in practical form, and the details indicated, by me in 1897. The mathematical principles of this and much more were discussed by Dr Larmor, in 1894,* as well as by Hertz and Fitzgerald †.

(7) The emitter and receiver should also be elevated some reasonable height above the earth, so that the perturbing and uncertain effect of its capricious and variable conductivity may be eliminated. Recognition of the evil effect of the earth has become clearer since 1897, for although at that time a sound arrangement, of two insulated and elevated capacity areas, was depicted in my first tuning arrangement, it was thought that the earth might in some cases be used as one of those areas, even at syntonic stations.

(8) Experience shows this not to be the case. We find the influence of the earth—such earth, for instance, as we get in Kent—wholly deleterious to accurate syntony, and we obtain our good results by disconnecting everything from it, and raising even the lower aerial or capacity area a sufficient distance above the earth's surface. The best distance is found to be that at which the resultant capacity of the lower plate, with reference to the upper aerial on the one hand, and to the earth below on the other, is a minimum.

(9) To avoid damping by resistance, all leads are very thick and composed of a multitude of silk-covered No 40 wires of high conductivity, made into a cable, and great care is taken with all joints, since the E.M.F. of the incipient swings at beginning of resonance is very low.

(10) *Arc* v. *Spark*.—It has sometimes been said recently that tuning can only be maintained by an arc, and that a spark is incapable of being tuned. A short snappy spark is incapable, but that is not the sort of spark that we employ. On the contrary, we use a comparatively long-continued flame-like spark, between points rather than knobs, and we take it in an enclosure, so that the ionised air escapes immediate dissipation, and is for the necessary time practically a conductor. We also prolong the discharge of the capacity areas by a sufficient amount of the inertia of self-induction.

Under these conditions, if the spark is analysed by a rotating mirror, its image is seen to be spread out into a long beaded band, with ample opportunity for being tuned. The old snappy spark or whip-crack plan of Hertz, with clean polished knobs, is unsuited to a syntonic station.

* 'Lond. Math. Soc. Proc.'

† See also Lodge and Howard, "On Electric Radiation," 'Phil. Mag.,' July, 1889, p. 56, 57.

The precision of tuning which can thus be obtained in conjunction with very effective radiating power is remarkable, the sensitiveness of such a receiver to accumulated impulses is noteworthy; and the resulting ease and discrimination of selective or syntonic signalling is so marked that the measurements establishing the fact are considered by us worthy of communication to the Royal Society

PRACTICAL DETAILS IN CONNECTION WITH TUNING *

Fig 1 shows diagrammatically the arrangement of upper and lower aerial, both well insulated from the earth and each in the form of a Maltese cross, with complete insulation and independence at the corners, as shown in the little enlargement on the right-hand side

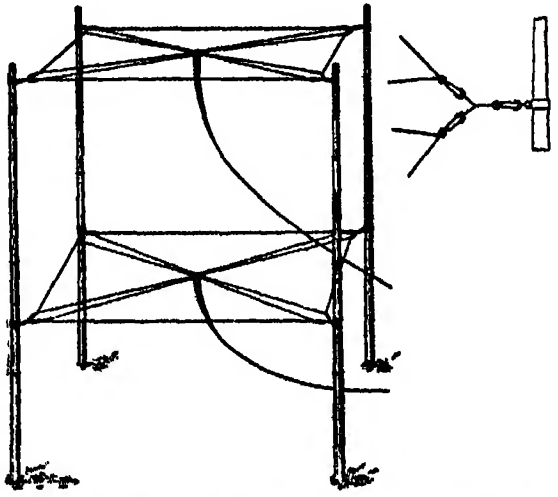


FIG 1.—Arrangement of Capacity Areas for Fixed Station

Dr Muirhead finds that if metallic connection is made between the loops of the cross at these corners the effect falls off greatly

There must be a surging of the pulse from the centre of the cross to the middle of each of its sides, and so this arrangement, if approximately adjusted to correspond with the wave-length, gives a remarkably powerful result, being very efficient both as sender and as receiver The figure only shows a model of the real arrangement, wherein, of course, the posts have to be supported by stays

Fig. 2 is a diagram of the connections Here we see the lower capacity area of the aerial connected to a small, adjustable, open, high-conductivity, self-induction spiral I. The upper capacity area can be switched either to the sender on the left or to the receiver on the right

* Owing to hesitation and delay in sending in this paper, the following details have now been published as an Appendix to a book, 'Signalling without Wires,' by Lodge, and accordingly they cannot be offered to the Society as new matter; but they are explanatory. The paper itself may be understood to begin again at p. 240.

Taking the receiving side first, we see the primary and secondary of the transformer; the secondary coil of the latter is in circuit with the adjustable condenser k which shunts the coherer WR, and is likewise in circuit with the recorder coil and a battery giving a fraction of a volt through a potentiometer PT. Both these accessories are prevented from interfering with the reception of messages by a large condenser K acting as shunt, so as to allow free passage to the impulses, though not to steady currents.

On the left-hand side we have first the spark-gaps—several, sometimes five, in series—then the sending transformer T, then the safety switch and signalling key, then the inductance coils for tuning this part of the arrangement. We see also the ammeter, the voltmeter, and the alternator, and, still further to the left, we see the arrangements for regulating the excitation of its field magnets and speed. It is to be understood, as

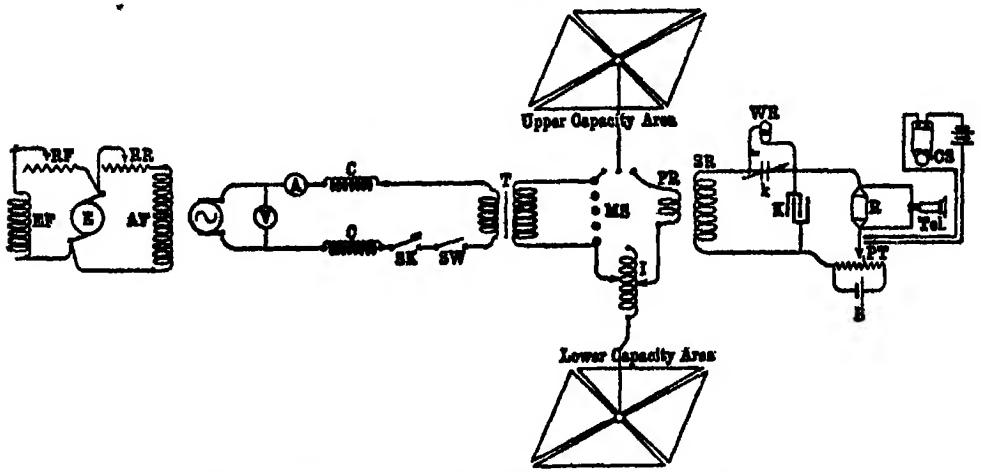


FIG 2.—Diagram of the Connections of a Complete Installation

I.	Sending and Primary Receiving Inductance	AF	Alternator (Field) Winding	k	Adjustable (overflow) Air Condenser
MS	Subdivided Spark Gaps.	RR	Regulating Resistance for Alternator Field Winding	WR.	Coherer (Wheel type)
T	Sending Transformer	E	Exciter Armature	K	Receiving Condenser
SW	Safety Switch	RF	Regulating Resistance for Exciter Field	R	Recorder
SK	Sending Key	EF	Exciter Field Winding	Tel	Telephone Receiver
CC	Inductance Coils (adjustable)	PR.	Primary of Receiving Transformer	PT	Potentiometer
A.	Ammeter	SR	Secondary	B	Battery (Receiving)
V	Voltmeter			CS	Call up Set.
⌀	Alternator (Armature)				

explained below, that the frequency of this alternator is in tune with its circuit, and likewise in tune with the secondary circuit of the sending transformer T connected to the aerial. But when the spark occurs it short-circuits the aerial through the spark gaps MS, and confers upon it a totally different frequency, enormously more rapid, which is responded to by the distant corresponding aerial switched on to its receiving circuit, every part of which is in tune with that frequency. The small power required by this arrangement, and the exactness of the tuning and the ease with which undesired messages and disturbances can be eliminated, is surprising and somewhat beautiful.

In order to enable a small-power coil to charge a sufficiently large capacity to bursting point, it is sometimes necessary to accumulate several of its impulses, which, by means of a *valve*, is readily done. The kind of valve which we interpolate between coil and aerial in such cases is shown in fig. 3. It is exhausted by a special and peculiar process, and it enables the capacity to be readily filled to sparking point several times a second, even by quite a small coil with rapid trembler.

The message to be sent can either be transmitted by hand or it can be punched on tape in the ordinary way and passed through an automatic transmitter

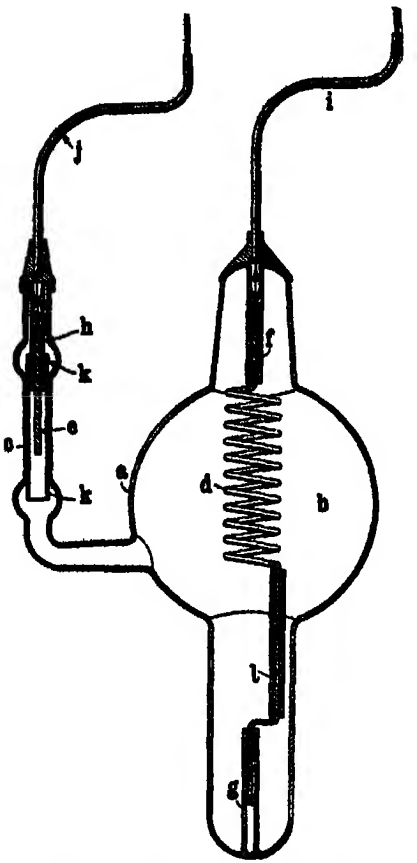


Fig. 3.—Electric Valves used in Portable Set

As receiver, one can use either the telephone or the recorder, but for all accurate purposes the recorder is much the best.

The coherer employed in connection with the recorder is the remarkably sensitive and trustworthy wheel coherer shown in fig 4, of which the important part is a steel wheel, kept slightly oiled and dipping into mercury by an adjustable amount The recorder clockwork keeps it slowly rotating

For telephone use, a simpler though less sensitive coherer suffices, and the one commonly used is a modification of my original single-point-and-plate, immersed in oil, and able to be tilted so that the pressure of the oil shall vary, thus constituting an easy adjustment It is depicted in fig 5 We also use an electrolytic coherer, which is highly sensitive but not so suitable for tuning

In any case, the coherer is arranged as a shunt to a Leyden jar or adjustable air-condenser, whose circuit is closed through self-induction, so that it has a definite frequency period The coherer is not affected till the jar overflows, which it does by reason of accumulation of impulses.*

An adjustable air-condenser is shown in the diagram of connections, fig 2, as a condenser with a slant arrow across it, k It can be quickly adjusted to give the proper tuning, and then the impulses accumulate, so that the condenser overflows through the coherer It is this overflow method which causes the coherer to be very sensitive to the proper tune, and to be insensitive to others. It represents my old syntonic Leyden jar experiment.†

In fig. 6 the handle of the adjustable condenser is shown on the left-hand side, and in the middle is seen the self-induction which completes its circuit. This same self-induction is employed as the secondary of a transformer, whose primary is in the circuit of the receiving aerial This primary has only a few turns, and is shown on the right-hand side

When the right tune is attained, there is a great margin of power, and the primary and secondary can then be separated very considerably, so that everything except impulses of the right frequency are excluded. They are often separated a large fraction of a foot.

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* 'Roy Soc Proc.,' vol 50, p. 15.

† 'Nature,' 1890, vol. 41, p. 368.

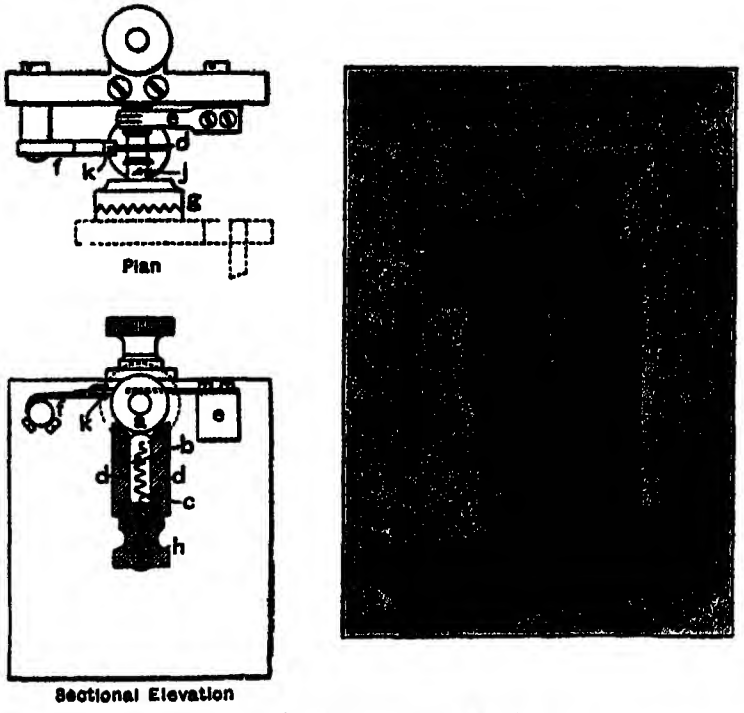


Fig 4.—Lodge-Muirhead Wheel Coherer

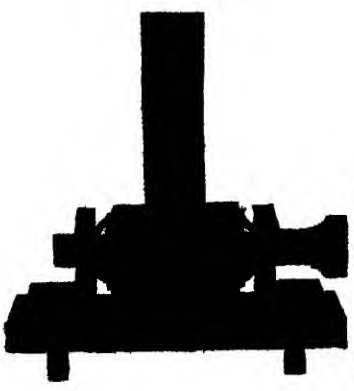


Fig 5.—Latest Form of Single Point Coherer in Oil.

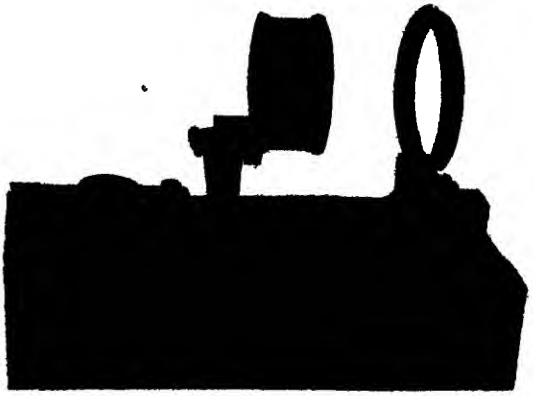


Fig 6.—"Selector" type of Receiving Transformer. This Transformer is arranged in series with an adjustable condenser, which is connected to the coherer so as to work on the "overflow" principle *

* 'Roy. Soc. Proc.,' vol. 50, p 15.

Statement of the Points to be attended to

The first point requiring attention is the tuning of the two capacity areas, in series with the secondary of the Ruhmkorff coil, and the exciting them by an alternating dynamo running so as to give the same frequency

This adjustment has been found a useful one, and in order to make it possible to treat the Ruhmkorff as an independently oscillating circuit, it is necessary to have it specially constructed with a great interspace between the primary and secondary bobbins, so that the secondary bobbin shall be almost free and isolated and have very little capacity when considered as a Leyden jar with the primary as the other coating No doubt some effectiveness is removed from the coil by this great separation, which is often as much as an inch between the inducing and the induced bobbin, but there is no loss of "efficiency," and the diminished effectiveness is more than compensated by the tuning which is thereby rendered possible in the secondary circuit Less power is called for in the primary circuit in such a case, since the secondary is then on open circuit, and not, so to speak, short-circuited by its capacity with the primary coil, a capacity which is merely useless and disturbing Practically all the capacity which is now inserted in the secondary circuit is the useful and effective capacity of the radiating aerials, so that no more power is consumed than is needed, and the secondary circuit being free, it can continue oscillating so long as the dynamo potential retains a sufficiently high value, thereby prolonging the spark to a surprising extent, and giving it a very bright and arc-like appearance, as if it consisted of a great number of oscillations in succession And the fact that it does so consist of a succession of oscillations is proved by photographing the spark in a revolving mirror or on a rotating plate, the number of alternations is then seen to be 30 or more semi-oscillations, each recorded on the plate, and after these oscillations there is a long stream, visible to the eye, of greenish coloured light, such as does not much affect a photographic plate, but which may be a continuance of the alternations beyond *

The spark must not, therefore, be treated as a simple snap it is nothing of the kind, it is a maintained spark, and it is maintained by connection with the tuned Ruhmkorff, which continues to supply the energy during the whole of the continuance of the sending alternations.

It is not to be supposed that these sending alternations are of the frequency of the Ruhmkorff or of the alternator they are of a totally different order, they are the alternations proper to the aerials short-circuited through the spark-gap by the connecting wires, and must be reckoned in fractions of a million per second. The tuning of the alternator and the Ruhmkorff only enables them to be maintained.

At the same time the maintenance could not be effected unless the lower area were elevated above the ground, for if it be lowered nearer the ground, the chain of oscillations is diminished, if it touch the ground, the series is cut very short, and if it is connected with the ground, nothing is left of it but just a few at the beginning—often not more than two or three—a sequence which is quite insufficient for anything like proper tuning Actual tuning experiments confirm this, for it is found that with the lower aerial on the ground, or even too near the ground, the tuning is imperfect, in fact, there is barely any tuning at all when there is earth connection. But as the lower aerial is raised the tuning improves very rapidly, and at the best position becomes extraordinarily sharp, the disturbances, when plotted, rising to a sharp peak, when everything is properly adjusted. It must be understood that these experiments were conducted on land, not at sea

* Such photographs, taken at Dr Muirhead's station at Elmer's End, were exhibited to the British Association at Leicester by Mr. Duddell in his Evening Lecture to the Association in 1907

It is on the top of this peak that a tuned station usually works, that is to say, the received effect is purposely diminished until the instruments only just respond, or respond with a small margin, so that a slight change of tune would render them entirely insensitive. Under these conditions they can receive from one similarly tuned station, but from no other, others are thereby tuned out, unless the tops of their peaks (if they have any, which a good many have not) happen to coincide with the tune of the receiving station

Another precaution that must be taken to keep the tuning sharp—that is to say, not to damp out the oscillations at these very high frequencies—is to have the conducting wires so composed that they shall offer plenty of surface to the ether To this end they must be finely subdivided into insulated strands, because, otherwise, if they were solid conductors, only their exterior surface would take any part in conducting the current, and therefore the resistance would be very high, and the oscillations would be killed But by using as conductors a bundle of a very large number of excessively fine insulated copper wires, of high conductivity, sufficient total surface is exposed to give admirable conducting power, even to disturbances of the highest frequency used All parts of apparatus used for this purpose, such as receiving transformers and the like, should be made with wire of this kind, and never with a merely solid conductor, of which the sectional interior is useless Moreover, since the received E M F. is very low, all questionable joints and imperfect contacts must be scrupulously avoided

Adjustment of Tuning

As for tuning at the receiving end it is managed in the first instance by putting between the two capacity areas a self-inductance coil, of a small number of turns, which are adjusted until the response, to the sending station desired, is a maximum A small part of this inductance is arranged to be easily variable, and capable of adjustment, but the main part of it (which, after all, is still not much) is used as the primary of the receiving transformer The number of turns depends on the wave-length desired, but for short waves as few as five turns is sufficient, and in order to compensate it a coil of a similar number of turns must be inserted in the sending circuit But such insertions are of no advantage they are rather a disadvantage, and are therefore kept as small as may serve to make the transformer efficient

The number of turns in the secondary of the receiving transformer again depends on the wave-length, but in a special case it consists of 35 turns connected in series with a large capacity or air-condenser, which is capable of continuous adjustment in order to tune this also to the required frequency, and the coherer is arranged as a shunt or overflow to this capacity, so that as soon as the excited oscillations have reached a certain value, the coherer has to respond This is an application of the overflow-method which I described in 'Proc. Roy Soc.,' vol 50

The capacity at which these receiving air-condensers are worked is generally found to be about a few milli microfarads Their external appearance is shown in fig 6

The primary and secondary of the receiving transformer are mounted in such a way that they can be moved to and from each other; and by this means the disturbance received by the coherer can be minimised till it will only just respond

That is the arrangement adopted for working on the top of the curve, and thereby getting the benefit of the accuracy of tuning, which accuracy—when all things are so adjusted—every part of both sending and receiving circuit being carefully tuned, and the lower area elevated above the ground—is exceedingly remarkable. A difference in wave-length of one-half per cent. on either side is sufficient to throw the tune completely off the top of the curve; and the response may be stopped even by so small a change as that. Of course, if we do not want to work with such accurate tuning there is no need

to limit oneself to the top of the curve, it is easy enough to utilise more power, but in that case one might pick up—and, as a matter of fact, we do pick up—signals from other stations By using the whole of the power available in the receiving circuit, we can hear the signals of all the stations round about, but by diminishing the power until we are working on the top of the curve of some one selected station, we tune out all the others, and listen to that alone Even when the others are not tuned to silence a skilful operator with a telephone receiver can pick out the message intended for him, just as a conversation can be carried on in a crowded room, but the tuning enables much more than that it is so perfect that a recording instrument can record the one upon tape and completely ignore all the others.

Measurement of Wave-length

To measure the wave-length, a method based on the sympathetic or resonant Leyden jar is employed, that is to say, an adjustable air-condenser is put in series with a self-inductance coil, and the coil brought into the neighbourhood of a single turn of wire in the sending circuit, so that it shall be operated upon inductively and thrown into oscillation when it is tuned to the right frequency The tuning is done by moving the plates of the air condenser, with an index pointing to a graduated scale, until the response of this sympathetic or subordinate closed circuit is a maximum The method is an application by Prof Braun of my syntonic Leyden jar experiment. In the older form of the experiment the existence of this maximum resonance was shown by actual overflow in the form of a spark, or in some cases by the use of a coherer, but the simplest metrical method of observing it is to intercalate in its circuit some simple measuring instrument, such as a hot-wire ammeter, of which there are many suitable kinds The indications of this instrument easily enable the maximum position to be found, and by a calibration scale the wave-length is at once read off on the adjustable condenser The maximum reading on the hot-wire instrument also serves to indicate in a comparative manner the amount of energy being sent into the aerials

Measurement of the Capacity of the Aerials

The capacity can be measured in straightforward manner by the Wheatstone bridge method, two arms of the bridge being resistances, the other two arms being the aerials on the one hand and an adjustable air-condenser on the other By means of a secohmmeter the system can be charged, insulated, and discharged some 30 or 40 times a second—a suitable galvanometer being placed in the bridge wire, and the air-condenser adjusted until its reading is zero

At Elmer's End and at Downe the aerials can be changed so as to enclose either an area represented by 3 or an area represented by 4 The ratio of the linear dimensions therefore, in the two cases would be as $\sqrt{3}$ to $\sqrt{4}$, that is, $\frac{1}{2}\sqrt{3} = 0{\cdot}866$ The observed capacities measured in the above way were approximately as 6 to 7, which is equal to the ratio 0 86 But one cannot say that it is exactly 6 to 7, because it varies with different elevations to some slight extent One pair of readings made it 6 4 to 7 3, which gives rather a greater ratio than the linear dimensions—as, indeed, is quite right.

Effect of the Earth on Capacity

Measured in this way, the capacity of the upper aerial, when the lower is lying on the ground, we will call 12.

If the lower is thoroughly connected with the ground, the effect is usually just the same, tested in this fashion, though at very high frequency it is certain that merely lying on the ground does not make fully efficient connection.

On insulating and raising the lower aerial 3 feet, which is about the least feasible when sag is allowed for, the capacity of the system falls to 8. On raising the lower aerial 9 feet, the capacity falls, say, to 7·4, which as an illustration we may take as the minimum, for if it be raised to 16 feet, the capacity begins to increase again, owing to its commencing proximity with the upper aerial. Thus, if raised too high, the capacity is increased for one reason, if depressed too low, it is great for another reason; an intervening position gives minimum capacity, and this is the position at which the sending is found to be best.

Measurement of Energy at the Receiving End.

For the purpose of measuring the received energy, a Duddell radio-micrometer is employed, consisting of a dead-beat instrument on a stone pillar, after the fashion of Boys' radio-micrometer. The instrument has a single loop of wire, in a strong magnetic field, carrying a thermal junction which is actuated by the hot air rising from a very fine short wire. This short fine wire is included in the receiving circuit—that is, between the upper and lower capacity areas—an adjustable self-inductance box being also inserted for the purpose of lengthening the wave as required, or an adjustable capacity being inserted if it is found necessary to shorten the wave. The received signals heat the fine wire and produce a steady deflection on the scale, which is then read, the self-inductance plug is then shifted and another reading taken, and so on until the maximum is found.

This is the tuned position. The sharpness of tuning is recorded by plotting the readings over a fair range. In the case of imperfect tuning, such as is experienced when the lower area is too near the earth, the value of the deflection, though it does rise to a low maximum, is not very different for a small distance on either side, although if the peg is moved several holes the deflection can fall off to something like half its value. When earth connection is actually made there is hardly any particular maximum or tuned position.

But when the lower aerial is insulated and sufficiently elevated, the curve runs up into a high peak at one particular position of the plug, and the smallest shift on either side brings the value down enormously.

That, of course, is exactly the characteristic of sharp tuning, and having found the top of the curve in this way, which is very quickly done, the receiving inductive connection can be so weakened as only to allow the coherer to respond to something very near the maximum disturbance. For under those conditions it will respond to no other station thereby eliminating unwished-for disturbances.

But if at any time it is desired to receive from other stations, or, indeed, from all stations round about—or, say, from stray ships—then the inductive connection can be closed up again, so as to make the receiving instrument sensitive and capable of responding, not only to the maximum, but to any disturbance, no matter how much out of tune it may be. This, for instance, is the condition in which a coast station would usually be kept, and it would only be screwed up to precise tuning when it was desired to speak to one particular station without interference from any other.

Some few of the numbers thus obtained are recorded and plotted further on. It must be understood that when these energy measurements were going on, telegraphic operations were suspended. These measurements are not necessary in practice they were made for scientific purposes.

It is quite easy to arrange a series of frequencies or wave-lengths corresponding to a series of different stations, any one of which can be spoken to at pleasure by pegging in a plug in the appropriate place.

Perfection of Tuning.

In this way it was possible to receive at Hythe from Elmer's End, while a much more powerful and nearer station at Dover was making a disturbance which was entirely eliminated It is easy to hear the ships in the Channel, but it is also easy to tune everything out and listen to the desired station alone A 5-per-cent. change can be, and in actual practice is, made to throw a given station out and throw a neighbouring one in. There is no difficulty in contracting to work as close as that.

With changes of that order of magnitude, several neighbouring sending stations can be made to send to several neighbouring receiving stations without interference That is to say, "diplex" telegraphy is possible, though at present not "duplex"

The accuracy of tuning on the Lodge-Muirhead system is such that messages are sent between Burma and the Andaman Islands—a distance of about 300 miles—with less than a horse-power

The power we use is small, but the radiation is so strong from an open circuit radiator that in places I fear we perturb the Admiralty We can tune them out quite easily, but we are informed that our radiation cannot be tuned out, their idea appearing to be that it is too strong I suspect, however, that the real fact is that their use of a sea or earth connection prevents their collector from being adequately tuned Our radiation is of one perfectly definite wave-length, and of that alone

Effect of the Earth.

If the earth were a perfect conductor, it would presumably act like a mirror, preventing the waves spreading in that direction, and thereby doubling the intensity of any radiator above it, except that in certain places there would be liable to be interference bands, where the difference between source and image was equal to half a wave-length Such interference, however, chiefly occurs in the case of those long trains of waves appropriate for tuning For single pulses—that is to say, the snaps needed for untuned signalling—the effect of a perfectly conducting earth would probably be good, and in so far as the sea is a moderately good conductor, connection with the sea may be advantageous for such signalling, but for tuned relation between stations it is becoming clear that even the sea is not a good enough conductor, and the land is certainly much too bad, so that, instead of prolonging the oscillations, its resistance wipes them out and kills them It is far better to ignore the earth and work independently of it, both at the sending and receiving end, taking care to keep everything insulated We thereby gain the advantage of being independent of fluctuations in the quality of the soil, in respect both of permanent geological quality and of variable heat and moisture, and we also get far better tuning

On the train of waves which is passing between distant stations, the earth probably has no particular influence, except by reason of its irregularities and obstructions; but over great distances it is possible that they may be reflected advantageously in the good conducting upper regions of the atmosphere. But with extremely great distances Mr Marconi has chiefly dealt My object has been to perfect the tuning for moderate distances

Experiments on the Effect of varying the Height of the Lower Aerial at both Sending and Receiving Stations. By Sir Oliver Lodge, F.R.S., and Dr Alex. Muirhead, F.R.S.

We will now quote some measurements exhibiting both the accuracy of tuning obtained and also the destructive influence of earth connection on

tuning. and we desire to record the energetic and most efficient help we have received from our chief assistants, Mr E. E. Robinson and Mr. W J Blenheim, who have spared no pains to make and to repeat accurate series of measurements and to obtain trustworthy results, being themselves greatly interested in the surprising accuracy of the tuning and in the measurements of the considerable fraction of emitted power received at a distant station when real syntony is established

Specimen of Measurements made on Feb 22, 1907, *as a Sample Set.*

Elmer's End sending, Downe receiving

Elmer's End sending in accordance with the following programme of work —

Keep upper capacity at masthead all the time

From 12 15 to 12 45 send with lower capacity 46 feet from the ground

„	2 30	„	3 0	„	„	35	„ „
„	3 30	„	4.0	„	„	24	„ „
„	4.30	„	5 0	„	„	13	„ „
„	5 30	„	6 0	„	„	46	„ „

Keep the condenser reading on wave-length-measurer at K = 4 2, and adjust wave-lengths to this

Keep the power at 450 actual watts, and the spark-length at five times $\frac{1}{16}$ths of an inch (A series of short spark-gaps being found better than one long one)

Measurements made at Elmer's End during the Sending

Upper aerial at masthead, 60 feet up at centre and 67 feet at corners

Lower aerial at various heights as given below.

Alternator volts ..	104	Speed of alternator	2530 revs per min
Alternator amperes	7 5	Frequency of alternator	210 per sec.
Apparent watts ..	780	Spark-length	5 × $\frac{1}{16}$ inch
Actual watts ..	475		

Distance between primary and secondary of wave measurer	6½ inches.
Height of lower aerial, changed between each set of observations and being	46, 35, 24, 13, 46 feet respectively
Hot-wire ampere meter reading in wave measurer	2 6, 2 475, 2 375, 2 25, 2 65
Ampere meter in aerial . .	9·5, 8 8, 8 3, 7 7, 9 5
K (condenser) reading	4 2 for all except the last, when it was 4 25
Wave-length	441 metres for all except the last, when it was 444 metres.
Ampere meter in aerial	7·7 amps, 7 15, 6 8, 6 25, 7·7.

Measurements made at Downe during the Receiving

With the upper aerial also at masthead all the time (corners 54 feet, centre 49 feet above the ground), the lower aerial at varying heights, not quite the same height as at Elmer's End, because the aerials are smaller, but at corresponding heights.

The self-induction was varied by plugging in different studs, and a set of readings taken for each stud.

In each case the reading recorded is the mean of five actual readings taken on the Duddell hot-wire instrument placed in the receiving circuit.

The stud expected to give best tuning was in the first series stud 24—25, in the second 25, in the third 27, in the fourth 28

First Set—Lower aerial 34 feet up

Stud	Received energy	Stud	Received energy.
24	143	27	24
23	106	25	100
25	102	24	140
26	51	23	112

Second Set—Lower aerial 27 feet up

Stud 25 reckoned best beforehand

Stud	Received energy Duddell instrument	Stud	Received energy Duddell instrument
25	326	29	48
26	207	24	192
27	170	23	101
28	83	22	58

Third Set—Lower aerial 20 feet up

Stud 27 reckoned to be best.

Stud	Received energy (hot wire)	Stud	Received energy (hot wire)
27	559	26	468
28	365	25	275
29	185	24	149
30	93·5	23	82

Fourth Set—Lower aerial 13 feet up.

Stud 28 reckoned to be best.

Stud.	Received energy.	Stud	Received energy
28	770	23	110
27	756	29	565
26	480	30	293
25	276	31	145
24	167	32	90

Fifth Set.—Lower aerial once more 34 feet up.

Stud	Received energy	Stud	Received energy
24	113	25	124 5
23	55	26	93
22	29	27	47
21	18	28	25

Plottings of the Readings

In figs. 7 and 8 all these five sets of measurements are plotted. Fig 7 shows the figures as here recorded, fig 8 shows them as amended by aid of the measurements plotted in fig. 9.

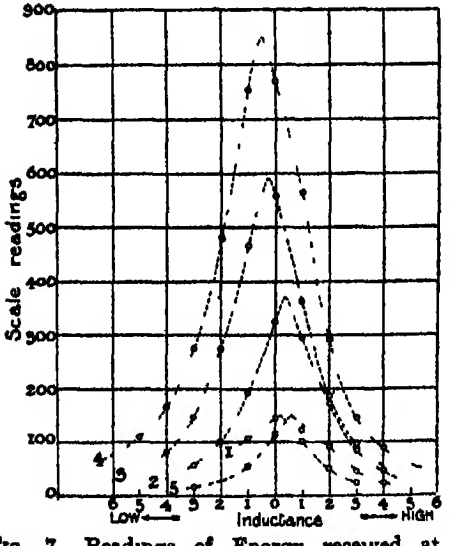


Fig 7.—Readings of Energy received at Downe, in terms of studs of Inductance Feb. 22, 1907

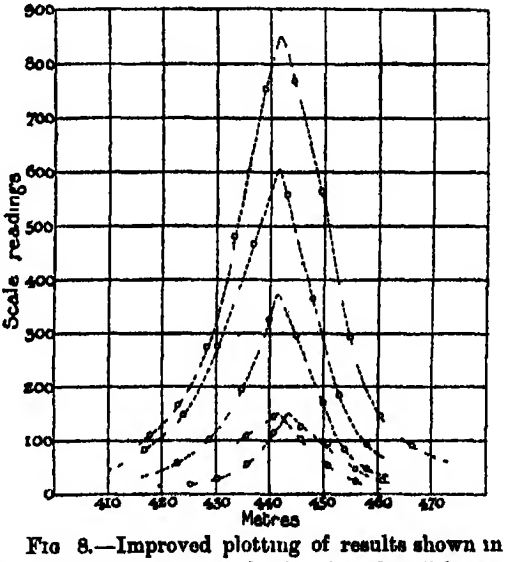


Fig 8.—Improved plotting of results shown in Fig 7, in terms of Wave-length. Feb 22, 1907

Deflexions of the micro-ampere meter, or hot-wire instrument, are plotted vertically. Variations of wave-length, in terms of studs of the inductance box, are plotted horizontally.

O corresponds to a wave-length of 440 metres. The curves are numbered in the order in which they were taken. The last curve (No 5), it will be noted, is shifted to the right, but that is entirely accounted for by the fact recorded in the set of *Elmer's End* (not *Downe*) readings (see first list of measurements recorded, p. 241). The sending condenser happened in that case to be adjusted to 4·25 instead of 4·2, so that the wave-length was

increased by nearly 1 per cent Hence the apparent defect is really a corroboration of the correctness of the measurement, for this change was not known by the observer at Downe, except from the fact that he received that same slightly increased wave-length

Improved Plottings of the Measurements.

But the plotting of the curves by stud readings is imperfect. they should be plotted accurately to wave-lengths as explained below, and in fig 8 they are so plotted The real precision of tuning, and the occurrence of the maximum at a definite wave-length, in all cases between 441 or 442 metres—except the last case which is 444 for a reason explained—come out here much more prominently. Otherwise fig 8 represents the same facts as fig. 7

Wave-length

For the purpose of getting the wave-length accurately, wave-lengths were measured corresponding to the different studs for different elevations of the lower aerial Only one set of plottings is shown as a specimen—that for the lower aerial at 13 feet above the ground, and for this also the corresponding readings are given

Sample Readings of Wave-length plotted in fig. 9

Stud	K or adjustable condenser readings.	Wave-lengths.	
		By measurement	By curve
23	3 8	418	418
24	3 9	423	423
25	4 0	427	428 5
26	4 1	435	433 5
27	4 2	440	439
28	4 3	445	444 5
29	4 4	450	449 5
30	4 5	455	455 0
31	4 6	461	460 5
32	4 7	467	466 5

The last column shows the wave-length as obtained from the curve, and therefore with mere irregularities of readings smoothed out.

The curve in fig 9 looks straight, though more extended examination in other cases shows that it is really a small portion of a curve, as it ought to be, but these other measurements are not reproduced.

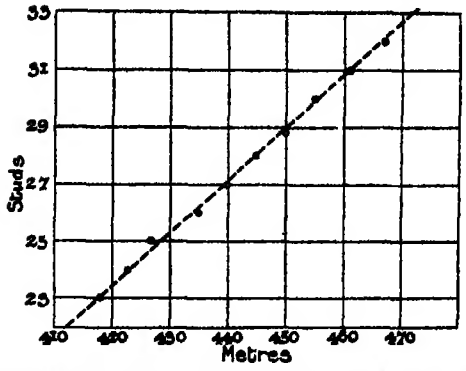


FIG 9.—Wave-length determinations corresponding to different self-induction studs. Feb 21, 1907

In the diagram, fig 10, another series of measurements is shown, selected at random from some taken on other days One group of this series was obtained with the lower aerial actually on the ground and connected to earth, this is plotted as the horizontal flat curve at the bottom, and shows how

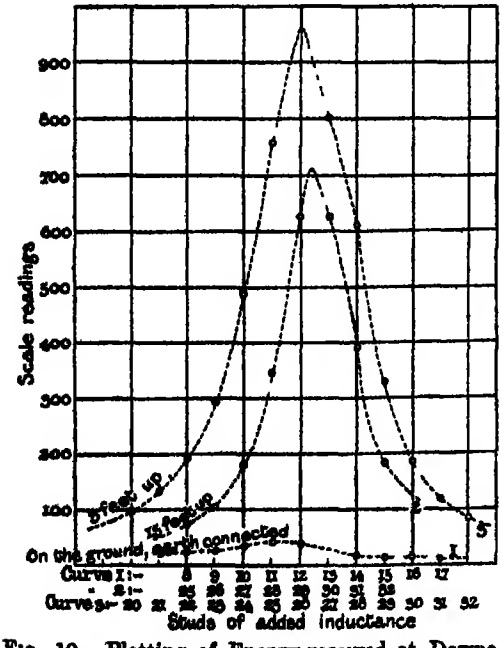


FIG 10.—Plotting of Energy received at Downe with Lower Aerial at different heights. March 4, 1907 Wave-length 440 metres.

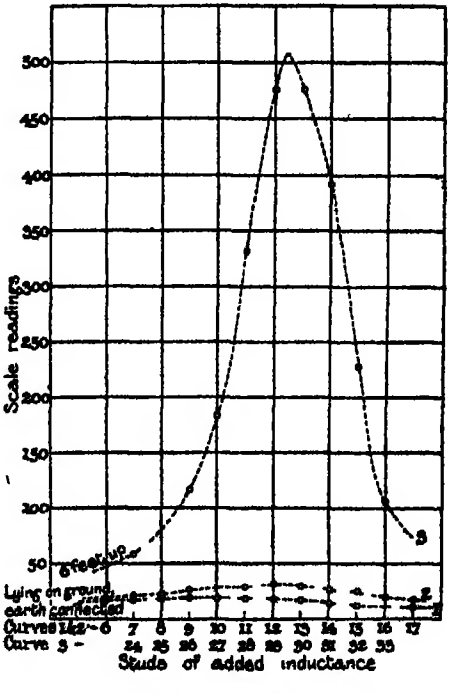


FIG 11.—Another set of plottings with Lower Aerial on the earth and elevated. March 7, 1907 Wave-length 440 metres.

almost entirely all tuning has disappeared, having been destroyed by earth connection Another set was taken with the lower aerial raised only 3 feet, and instantly the sensitiveness is vastly increased, the same power produces an enormous result at the proper tune, and the measurements show that the curve of received energy for different wave-lengths rises to a high peak at one definite place. On raising the lower area rather too high, to 15 feet, the peak is a little lower, because the radiating power is diminished by the closing up of the capacity areas, but the tuning is even sharper

During this series, Elmer's End was sending with 475 watts, $5 \times 3/16$ inch sparks, wave-length 440 metres, in each case.

Diagram 11 shows the energy received at Downe on March 7, with the lower aerial in different positions —(1) Thoroughly earth connected, (2) lying on the ground, but not otherwise connected, (3) elevated to a height of 6 feet.

It is to be understood that the upper aerial continued at the same height throughout. Elmer's End was sending with 400 watts, $5 \times \frac{1}{8}$ inch gaps, wave-length 440 metres

On this day the recording ammeter was purposely reduced in sensitiveness

The immense improvement of a capacity area even slightly raised from the ground (a few feet only) is in this as well as in the preceding diagram conspicuous And the difference between lying on the ground and good earth connection is also noticeable, and though both conditions are very bad, the latter is the worse

To give an idea of the absolute quantities involved, a calibration of the energy receiving instrument, carried out later, shows that a deflexion of 400 divisions meant 3 milliamperes the relation between the readings on the hot-wire instrument and the corresponding received current being plotted in the diagram Fig 12 represents this calibration of the instrument used to measure the received energy, and shows how well the Duddell hot-wire micro-ampere-meter answered the purpose

Received Fraction of Emitted Energy.

In a certain case the current received at Downe was 4 6 milliamperes, and the resistance of the hot wire which indicated it was 20 ohms.

This means, for the time, a power of 21 microwatts. The nominal power at the Elmer's End station was 400 watts, but of course this was emitted intermittently, and the actual power at the instant of sending must be quite otherwise estimated A very rough guess makes the factor of efficiency 10^{-9}, for this cross-country distance of 7 miles.

Measurements of Capacities at Different Heights

Another set of measurements of some interest records the effect on the capacity of varying the height of the lower aerial from 0 to 18 feet These are plotted in fig 13, the upper aerial being at masthead all the time. It will be seen that there is a shallow minimum at the height of 9 feet, and that is the height at which it is usually adjusted for most efficient sending

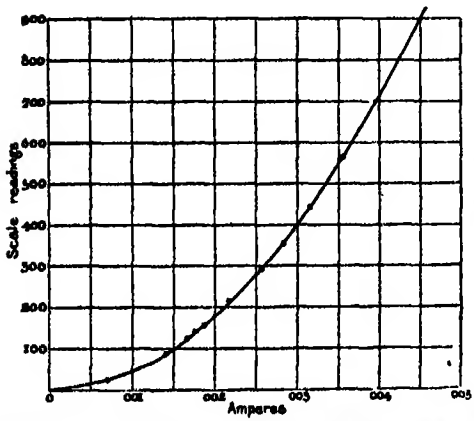


Fig 12.—Calibration of Ammeter used at Downe to measure Energy received March 21, 1907.

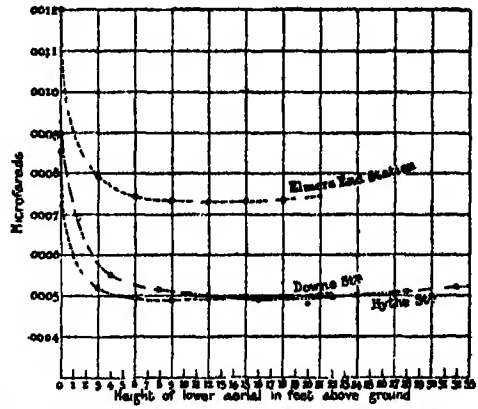


Fig 13.—Plottings of measured Capacity with the Lower Aerial at different heights above the ground

This minimum occurs at different heights at different stations, in correspondence with the sizes of the aerials and the heights of the upper aerials were also different At Elmer's End the minimum occurs at about 12 feet up, at Hythe, 20 feet, at Downe, 6 feet And in each of the three cases it was found by actual experiment that the station radiated and received most powerfully when the lower capacity was raised to these heights.

Effect of Varying Height of Upper Aerial

The placing of the lower aerial in or about the best position is an important matter in practice The elevation of the upper aerial is merely a matter of convenience and expense But it seemed desirable to ascertain how the radiating power was affected by varying the height of the upper capacity area, keeping the lower one constant, and accordingly the measurements plotted in fig. 14 were made

For this purpose the height of the upper aerial was the quantity modified in this set of observations, the charge being made at both stations so as to make the two stations correspond with the best tuning position It is to be understood that with each height of the sending aerial a complete set of

readings was taken at the receiving station, so as to get the sharpness of tuning depicted, as well as the best power received. It was, of course, some trouble to change the height of the upper aerial, and it would never

Fig 14.

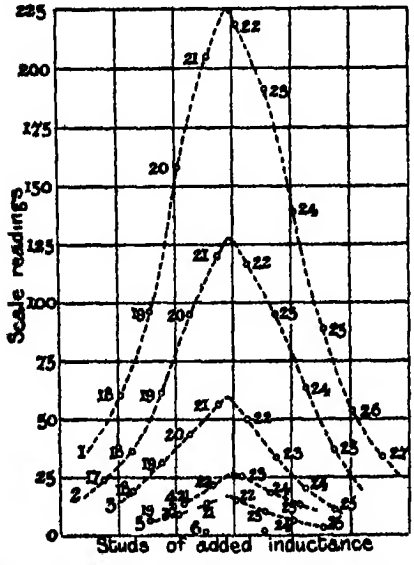


Fig 15.

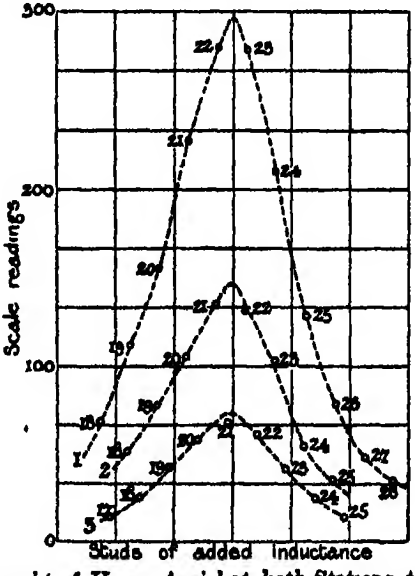


Fig 14.—Experiments on effect of varying height of Upper Aerial at both Stations, the Lower Aerial being at a fixed height at both stations.

1 Elmer's End, U A. full height, Downe, U.A. full height. W -l 410 m.
2. " " ¾ full height, Downe, U A full height. W -l 410 m
3 " " " " ¾ full height W -l 410 m
4 " " ½ " " " " 420 m
5. " " " " ½ " " "
6 " " ¼ " " " " 440 m.

Fig 15.—Another set of Experiments on effect of varying height of Upper Aerial at both Stations, the Lower Aerial being at a constant height.

1 Elmer's End, U A. full height, Downe, U A full height. W -l. 410 m
2 " " ½ full height; Downe, U.A ½ full height. W -l. 410 m
3. " " ¼ " " ¼ " "

be done in practice, but it was done for this series of experiments. It will be observed that the tuning becomes less sharp as the upper capacity area is lowered; and of course the received energy decreases fast as the lowering continues.

As these experiments are not likely to be repeated, the plotting of another set of observations is likewise given in fig. 15. And in fig. 16 the maximum energy received at Downe, when tuning was best, is plotted against the height of upper capacity area as abscissæ

Another set of troublesome observations was made in order to ascertain the effect of varying the *size* of the capacity areas, keeping their heights

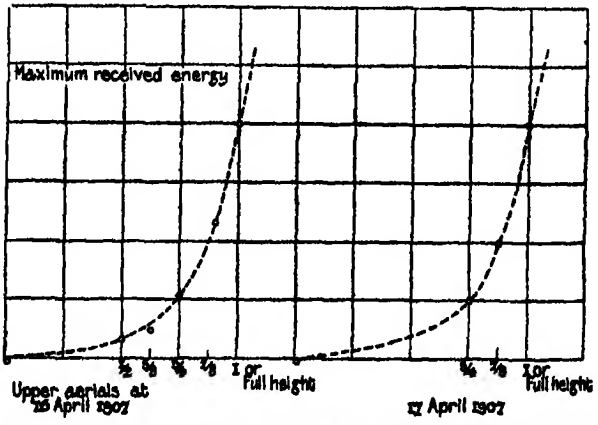


Fig 16.—Maximum received Energy at Downe, in terms of Heights of Upper Aerials.

constant, thereby, of course, changing the wave-length very considerably, but it was not necessary to change the aerials at the receiving end also. The size at the sending end alone was modified, the tuning at the receiving end being managed entirely by changing the amount of inductance plugged into the receiving collector The effectiveness of this method is depicted in fig 17.

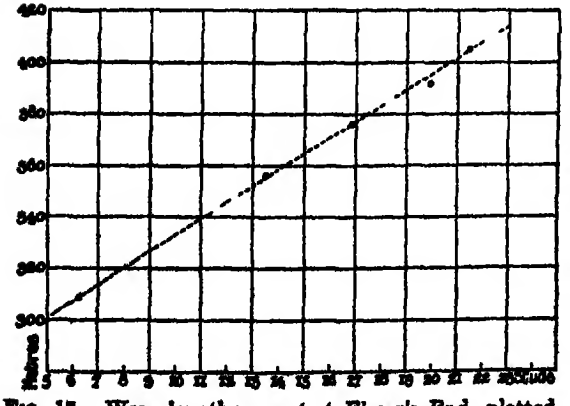


Fig. 17.—Wave-lengths as set at Elmer's End, plotted vertically Stud readings at Downe when stations in tune, plotted horizontally April 19, 1907

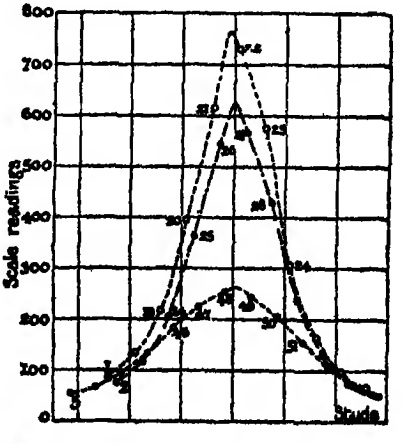


Fig 19.—Elmer's End sending on full-size Aerials. 405 metre wave, 500 watts Downe receiving—Expt. 1, on full-size or 60 ft. side, Expt 2, aerials 52 ft. side; Expt. 3, aerials 30 ft side April 25, 1907.

In this case, therefore, the receiver is virtually responding to different stations, each with its own wave-length, and by referring to fig 18 it will be seen how readily it could pick out one station and listen only to that, especially by working at the top of the curve—*i e*, at the position of least sensitiveness, with the induction coupling very loose For instance, between

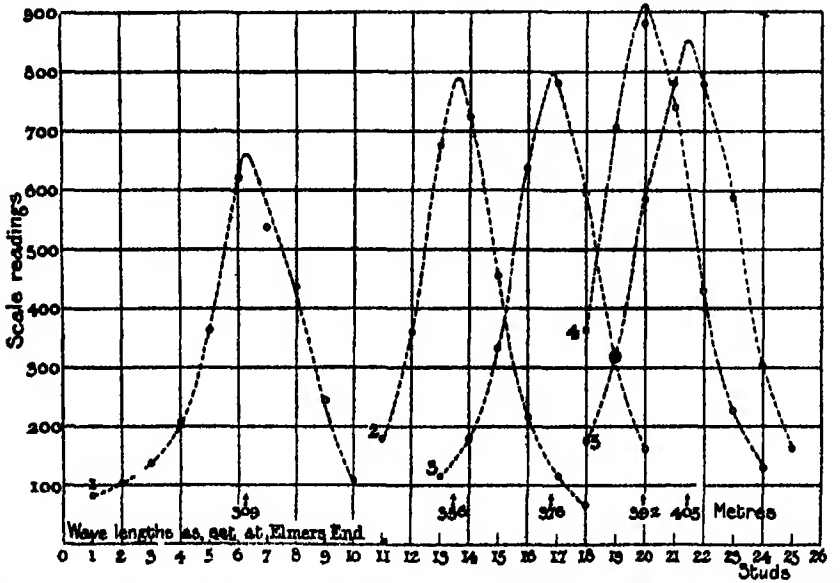


Fig. 18.—Downe receiving on full-size Aerials Elmer's End sending on different sizes, viz Expt. 1, 51 ft sides, 350 watts, Expt 2, 72 ft sides, 400 watts, Expt 3, 83 ft. sides, 425 watts, Expt 4, 91 ft sides, 500 watts; Expt. 5, 95 ft sides, 500 watts

the two right-hand curves—which are the closest together—the wave-length has changed from 392 to 405, that is to say a little over 3 per cent, but it would be practicable to tune out either of these stations and listen to the other alone.

In fig 19 the converse is shown. That is to say, the sender has a fixed aerial, and the size of that at the receiving station is modified; thereby making it correspond to different stations, each tuned up merely by plugging in or out induction in the shed—a very quick operation

To find whether the received energy depends more on the linear or on the superficial dimensions of the receiving aerials, the observations were plotted as in fig. 20—curve 1 is in terms of area and curve 2 in terms of length—and it is seen that the received energy depends on a mean between the length and the area—as would be expected

The measurements plotted in fig 21 represent a large amount of work.

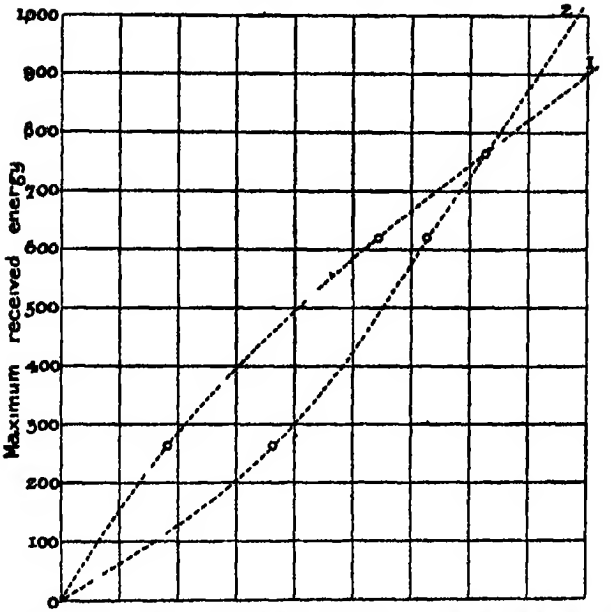


FIG 20.—Maximum received Energy plotted vertically Areas of Downe Aerials plotted horizontally in curve 1 Length of wire in Downe Aerials plotted horizontally in curve 2 Data obtained from fig 19

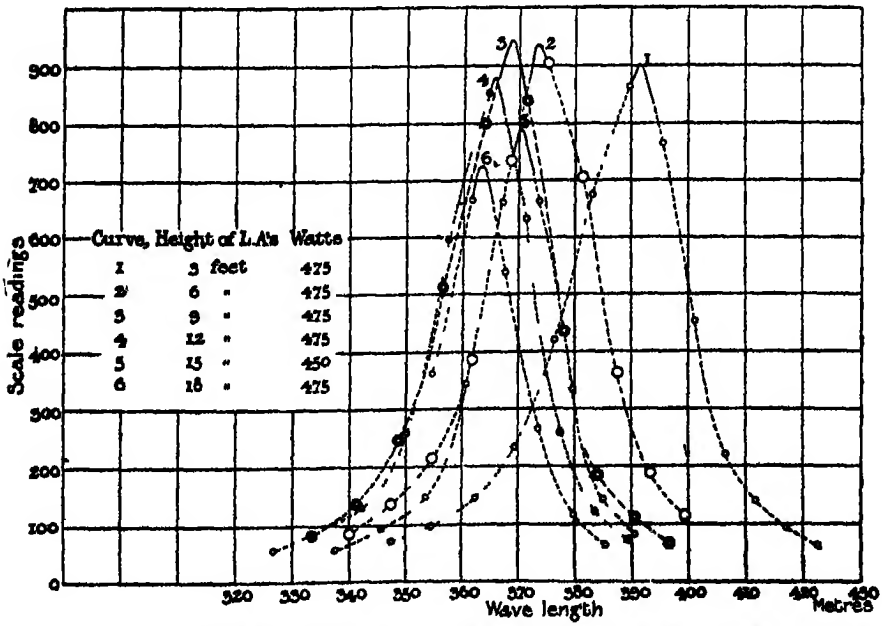


FIG. 21.—Plottings of Energy received at Downe with Lower Aerial at different heights. Elmer's End sending with Lower Aerial at corresponding heights. Elmer's End Aerials, 81·5 ft. sides, Downe Aerials, 57 ft. sides.

Each curve represents a series of measurements taken between a couple of stations with the lower capacity area of each adjusted to different but corresponding heights, sagging being prevented by a support in the middle as well as at the corners In these experiments the aerial was reduced in size to three-quarters what it had been, to see if that reduced the sensitiveness importantly, but it made very little difference. The stations were not readjusted each time to a constant wave-length, as in fig 7 or 8, and no self-induction was inserted at Elmer's End, but it was allowed to emit the natural wave-length pertaining to the aerial. After each adjustment of the lower aerial, therefore, it emitted radiation of a different wave-length, so as to constitute virtually a slightly different station The wave-lengths are plotted as abscissæ

A series of measurements was taken in each case at the sending end, so as to include the best wave-length and to observe the tuning The tuning in every case is quite good, and the lower area was not put into contact with the ground in this series, because it had become abundantly and constantly clear that tuning was thereby practically destroyed, no special wave-length being emitted from an earthed radiator or received by an earthed collector much more readily than any other Besides, the sensitiveness in such cases was reduced to a very insignificant value.

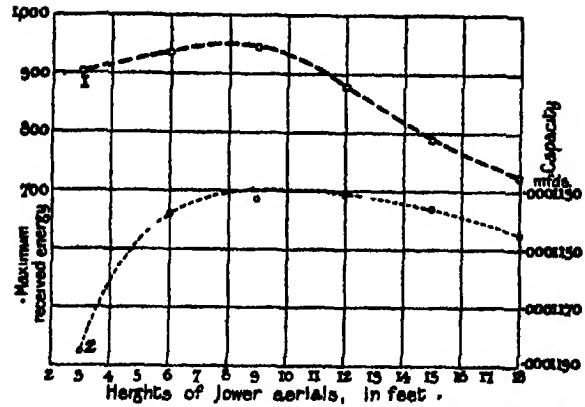
One object of the series was to verify which height of the lower aerial gave the best effect at a land station, and to ascertain whether the best height was really at the position of minimum capacity as had appeared likely

To this end the sum of the capacities of the aerials at the two stations was plotted, and is shown in curve No 2 of fig 22. Only it is plotted in an inverted position, corresponding to the inverted capacity values on the right-hand side of the diagram, so that the minimum capacity comes at the top of the curve instead of at the bottom. And above it is plotted curve No 1, the maximum values of the received energy in the different cases of fig. 21, in accordance with numbers on the left-hand side of fig. 22, the abscissæ in 22 being the heights of the lower capacity area—the upper one being kept constant.

It will be observed that the maximum received energy does really correspond fairly with the minimum capacity: that is to say, with the position in which the earth interferes as little as possible, while yet the radiator and collector are not so close together as to begin to be inefficient.

A great number of other measurements have been made, but they are all confirmatory of what is shown in the above selection, and it is thought unnecessary to publish more than the above sample of the work.

FIG 22



Curve 1 —Maximum Energy received, in terms of heights of Lower Aerials Data from fig 21

Curve 2 —Capacity of Elmer's End and Downe Stations, added together, in terms of heights of Lower Aerials. (Curve inverted)

Postscript—It is hoped to submit a communication on some theoretical considerations at a later stage

Added February, 1909—From the above paper an account of the most striking experiment to illustrate the facility and perfection of tuning on this system, when insulated capacity areas are employed without any earth connection, was omitted The experiment was made on May 14, 1907, and may be thus briefly described —

At the two stations, Downe and Elmer's End respectively, the upper capacity area of each aerial (fig 1) was bisected diagonally, the two triangular halves being insulated from each other, and each connected to its own independent sending or receiving arrangement. The lower aerial was not bisected, but was doubled; an additional insulated area being placed a few feet below the ordinary one. By this means each station was practically doubled, and the two halves at each station corresponded to a different wave-length.

Two automatic senders at Elmer's End were then set to work simultaneously, one to transmit the word "Liverpool" many times in succession, the other the word "Steamships" continuously in the same way Two independent receivers at the Downe station—one of them a siphon recorder and one a telephone, though both might easily have been automatic recorders—each of them inductively connected with one half of the aerial there, now received simultaneously, one of them a succession of

"Liverpools," the other a succession of "Steamships," without the slightest confusion or interference or overlapping of any kind

In other words, diplex telegraphy (as distinct from duplex) was found quite easy on this system of tuning which was specified by one of us in 1897.

Added March 8, 1909.—Another experiment more recently tried is the following.—Two stations were arranged at Downe, 1200 feet apart, either of which could speak with great ease to Elmer's End, and was strong enough to speak to a station 30 miles away. One of the Downe stations was then switched on to "receiving," and both Elmer's End and the other station at Downe were set speaking to it.

The wave-length of one was 300 metres, of the other, 660 metres, so as to compare Civil with Admiralty conditions.

By the mere motion of a handle the frequency of the receiving station could be altered at will so as to correspond either with the neighbouring sending station 1200 feet off, or with the distant sending station seven miles off,—which distance might, however, have been increased immensely without any difficulty. A few trees intervened between the neighbouring stations.

Under these circumstances, when properly adjusted, each station could be heard separately, that is to say, messages could be received first from one tuned-in station, and then from the other, without any disturbance from the station tuned-out, although both stations were sending all the time strongly and simultaneously. The ease and large margin with which selection could be achieved shows that the two neighbouring stations could have been put still nearer, while still retaining the power of complete tuning-out.

Received March 25, 1909.—Further experiments in the same direction were conducted as follows.—

The two stations at Downe, 400 yards apart, were re-arranged so that there were no trees between, only a few low hedges, thus making the test manifestly more severe. A given power was then employed for sending at one of these neighbouring stations, and the same power at the distant Elmer's End Station, while the other neighbouring station was arranged for receiving from either of these two at pleasure. Experiment was now directed to determining the conditions under which the neighbouring station could be completely cut out, while still the distant one could be clearly heard. In other words, to determine the separation between the primary and secondary of the inductive connection (fig. 6) which eliminated all disturbance from the neighbouring station adjusted to ordinary commercial wave-length, while it permitted perfect signals to be received on the siphon recorder from the tuned station of longer, or more nearly Naval, wave-length.

Case 1.—Elmer's End sending with a wave length of 580 metres. Neighbouring Downe sending with a wave-length of 300 metres. The receiving Downe station was attuned so as to cover a range of wave-length about 580 metres on the average, but extending more than 20 metres above and below. Under these conditions it was possible completely to cut out the local station on a coupling of 3½ inches, that is with 3½ inches

separating primary and secondary coil of the inductive connection Whereas from Elmer's End perfect signals could be obtained without disturbance on any coupling between 3½ and 7 inches Indeed as the exact pitch was reached at the receiving adjustment, the signals received boomed out, as it were, very strongly

Case 2.—The Elmer's End wave-length was shortened to 510 metres the local Downe station remaining at 300 metres and again a series of readings was taken at the receiving Downe station adjusted to an average of 510 metres wave-length. The coupling separation which now just managed to cut out the local station was 4 inches. Anything above 4 inches gave perfect signals from Elmer's End, and no disturbance.

Case 3.—*On shortening the distant wave-length still more, so as to make it 450 metres,* the neighbouring station could not be completely cut out without at the same time introducing a trace of superposed disturbance into the messages received from the distant station.

Case 4.—The difference of wave-length between the two stations was now therefore again slightly increased the Elmer's End wave-length being adjusted to 480 metres, with the local station still remaining at 300

In this case perfect and strong *signals* could be received *from* Elmer's End again, but the separation of the inductive connection had to be as much as 6 inches in order completely to cut out the local signals from the neighbouring station

It follows, therefore, that when two powerful stations are so excessively near each other as they were in this case (in adjoining fields), a distant signal can be heard with perfect clearness, *i e*, without any trace of disturbance, only when its wave-length is more than half as great again as the neighbouring station, but that undisturbed signalling is much more easy when it approaches *double that magnitude*, or, *of course*, when the neighbouring stations are not quite so close together

In no case was any trace of harmonic detected, *e g*, when a station was sending 300 metres, and the neighbouring receiving station was attuned to 600 metres, it did not necessarily feel any disturbance The waves emitted and received by these radiators appear to be practically pure

Summary

1 Perfection of tuning can be attained in spark telegraphy when the radiator and receiver are both persistent vibrators which can be accurately tuned together

2. These vibrators consist each of a pair of insulated capacity areas, well raised above the earth The effect of the earth, in so far as it is allowed to act, is to damp out vibrations and so to spoil tuning

3 To prove this, the received energy was measured at a distant station by a Duddell hot-wire meter, and a series of measurements was taken with the lower capacity at different heights above the earth, and also when connected with the earth

4. Directly land-earth connection is made, tuning of the radiator and collector is nearly gone, for they no longer have any persistent free vibration period. Samples of a large number of measurements are recorded in the paper.

5. The sensitiveness of a thoroughly tuned Lodge-Muirhead system is

extreme, small power is sufficient, and the inductive connection of the collector to the receiving instrument may be separated by a surprising interval without stopping communication

6 Two sending instruments can work at once on the same aerial bisected electrically, and at the distant station two receiving instruments can receive independently and simultaneously from the corresponding halves of an aerial there.

7 Under these non-earthed conditions every other station, even near and powerful ones, can be tuned out and their disturbance eliminated

8 The importance of the possession of power thus to hold conversation in the midst of disturbance, without being affected by it, and the naval danger of being without such power, are emphasised.

On the Influence of their State in Solution on the Absorption Spectra of Dissolved Dyes

By S E. Sheppard, D Sc (Lond), late 1851 Exhibition Scholar.

(Communicated by Sir William Ramsay, K C B, F R.S Received March 9,— Read March 18, 1909)

In a previous investigation of the absorption spectra and sensitising properties of some *iso*cyanine dyes,* the influence of the solvent was examined and it was found that the absorption maximum was shifted toward the red as the refractive index of the solvent increased This is in accordance with Kundt's law The absorption in water, however, differs markedly from that in organic solvents In the latter the spectrum consists of a prominent band in the orange and a half-shade nearer the blue † In water this half-shade has become a separate band comparable in intensity with the orange Absorption curves in alcohol and water are shown.

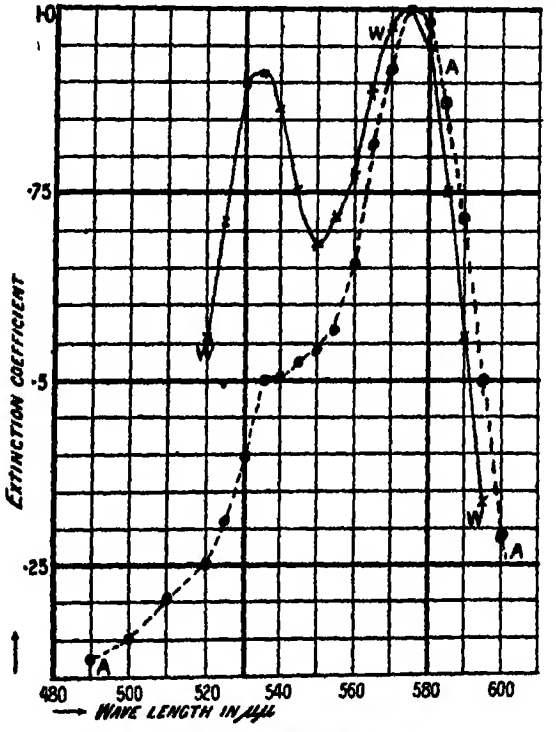
It is convenient to term the band near the red the β-(organic) band, the one nearer the blue the α-(water) band It appeared desirable to investigate this difference further.

* 'Chem. Soc Trans,' vol 95, 17 (1909), and 'Roy Phot. Soc. Journ.,' vol 48, p 300, 1908.

† The α-band is at *ca* 535 $\mu\mu$, the β-band at 575 $\mu\mu$, the positions depend on the isocyanine (*vide* papers quoted)

The Action of Alkali

Aqueous solutions of the isocyanines are very sensitive to traces of acid, even the CO_2 from air being sufficient to discharge the colour The change from a colourless to a coloured state is probably due to the salt formed having a different structure * But whilst the first additions increase the colour, beyond a certain point further increase of alkali lessens the intensity of the α-band, whilst leaving that of the β-band relatively unaltered. This



CURVE 1.—Isocyanine

W = absorption in water A = absorption in alcohol

change in the ratio of the absorptions was followed spectro-photometrically, the instrument being an improved form of the Hüfner spectro-photometer † In Tables I and IA measurements for two different isocyanines are given

The action of alkali is complex and not easy to interpret. At first it increases the total absorption, which we may attribute to the formation of the true dye, tautomeric with the colourless form The concentration of this in presence of excess of alkali should remain constant: actually the intensity

* This question is discussed in the papers cited.

† See 'Theory of Photographic Process' (Longmans, 1907), p 28.

Table I.

Conc. of alkali, KOH	Σ at 540 $\mu\mu$.	Σ at 587 $\mu\mu$	Ratio
Per cent			
0·4	0·946	0·871	1·085
0 8	0·960	0 874	1·092
1 6	0·962	0 871	1 102
3 2	0·917	0·765	1 198
6 4	0·819	0 620	1 500
13 4	0·749	0 432	1 780
13 4 after 30 minutes	0·692	0·379	1 720

Table Ia.

Conc of alkali.	Σ at 535 $\mu\mu$	Σ at 575 $\mu\mu$.	Ratio
Per cent.			
—	0 244	0 361	0 675
0 2	0·440	0 537	0·82
0 5	0 510	0 598	0 855
1·0	0 541	0 533	0·962
2·0	0 502	0 473	1·065
4·0	0 505	0 420	1 190
8·0	0·511	0 374	1 370
16·0	0 490	0·262	1·850

Σ is the Bunsen extinction coefficient, calculated from the relation $I = I_0 10^{-\Sigma d}$, where I and I_0 are the transmitted and incident intensities

d, the thickness, in this case 1·00 cm The concentration of the dyes was 1 in 100,000

of the α-band remains constant, that of the β-band diminishes, over a certain range, ultimately above a certain concentration of alkali the solution becomes turbid. In alcohol no action of this nature could be observed—

Conc of alkali .	0	0 5	1 0	4 0	8 0 per cent
Ratio $\Sigma\beta/\Sigma\alpha$ . .	2 27	2 16	2 20	2 23	2 23

This would seem to preclude the idea of a chemical modification The increasing turbidity, amounting finally to precipitation, suggested that the change involved was one of aggregation, the dye being present in various stages of aggregation, from true solution to suspension, and that the alkali coagulated the finer particles On this view the α-band is due to a suspension, the β-band to a colloidal solution, but one very near true solution To test this hypothesis, the "solutions" were examined with the ultramicroscope. A Zeiss microscope was used with a paraboloid substage condenser,* being the second arrangement of Siedentopf and Zsigmondy,†

* For the principles and technique, see R. Zsigmondy, 'Zur Erkenntniss d. Kolloide,' Fischer, Jena, and A. Cotton and H Mouton, 'Les Ultramicroscopes,' Masson et Cie., Paris.

† Cotton and Mouton, *loc. cit*, p. 52.

and this parallel investigation of the state of aggregation was extended to other dyes

For brevity, the results are expressed in tabular form, some are then discussed in greater detail —

Table II

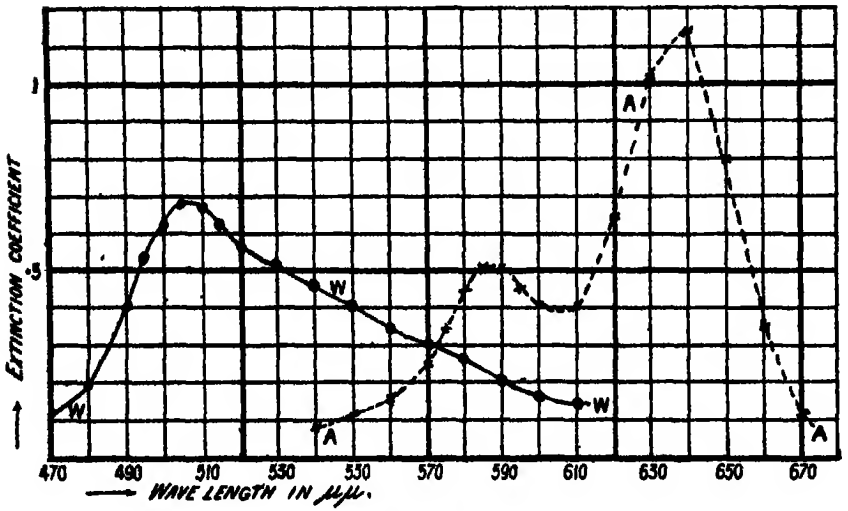
Solvent	Colour	Absorption maximum in $\mu\mu$	Appearance in ultra	Electrolytes
		Isocyanine *		
Water	Reddish purple	535 and 575	Large number of small granules	Increase in number and size of grains, especially KOH.
Alcohol	Purple	577, and half shade at 535	Few grains	No action
Chloroform	Purple	583, and half shade at 540	Nearly optically clear	
		Cyanine (lepidine-cyanine) †		
Water	Reddish violet	Max at 560, ill defined	Many grains	Increase in number and size of grains
Alcohol	Blue	605, well defined	Very few grains	No action

* *p* toluquinaldine-*p*-ethoxy-quinoline ethyl cyanine bromide
† Lepidine-iso amyl-cyanine

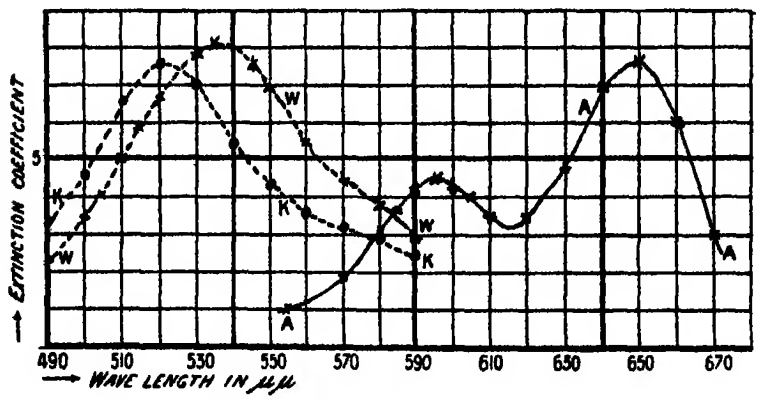
Table IIA

Solvent	Colour	Absorption maximum in $\mu\mu$	Appearance in ultra	Electrolytes
		Pinacyanol * Curve 2		
Water	Rose-pink	505--510, flat, ill defined	Many grains, some greenish blue, others brilliant yellow. Small suspended crystals also	Conc KCl gradually coagulates. KOH makes turbid Absorption shifted
Alcohol	Deep blue	640, strong, 587, weaker	Very few grains, much smaller†	No action
Xylol	Deep blue	—	Very few grains	
Chloroform	Deep blue, but brighter	643, strong; 592, weaker	Optically clear	
Quinoline	Greenish blue	660, strong; 605, weaker	Very few small grains	
		Pinachrome blue.‡ Curve 3		
Water	Rose	535, flattish	Similar to pinacyanol	Coagulated At first, shift of absorption.
Alcohol	Blue-green	650, strong; 595, weak	Very few grains	No action

* The pinacyanols are dyes of unknown constitution, but allied to the isocyanines. The one used was from *p*-ethoxy-quinaldine
† The size is judged from the brilliancy ‡ Allied to the pinacyanols.



CURVE 2.—Pinacyanol
A, 1 in 200,000 alcohol. W, 1 in 100,000 water.



CURVE 3.—Pinachrome.
K = absorption in *n*/50 KCl

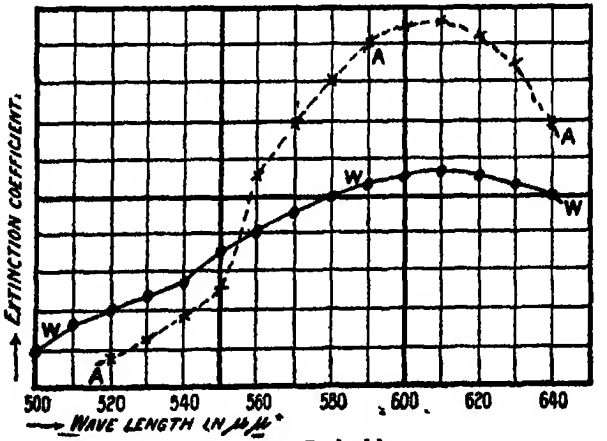
(To economise space, only the curves are given, and not the tables of extinction-coefficients.)

It is evident that the dyes are contained in quite different form in water and in alcohol, or organic solvents. The evidence that they are partially in suspension, partially in colloid solution, was as follows:—

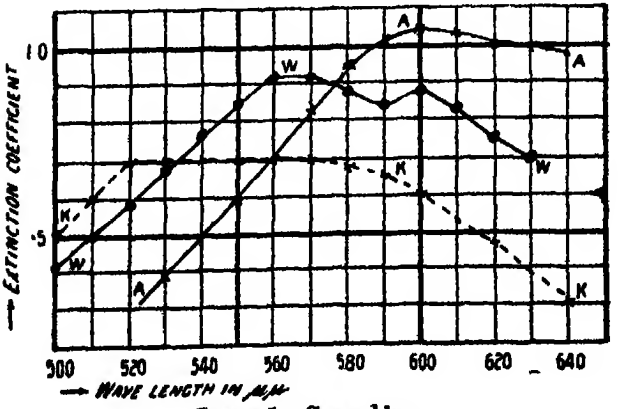
(*a*) They are coagulated by concentrated solutions of KCl, more rapidly by di- and trivalent cations, as Zn, Mg, Al. The process could be followed microscopically in the manner used by Linder and Picton with colloidal As_2S_3.

Table III

Solvent	Colour	Absorption maximum	Appearance in ultra.	Electrolytes
		Sky-blue extra (Licht-blau) See Curve 4		
Water	Blue	*Ca* 610, band very broad	Many grains	Gradual coagulation
Alcohol .	Deep blue	610, band sharp and clear	Very few grains	No action
		Congo blue See Curve 5		
Water	Turbid blue	560, dip at 585, max at 600	At $\frac{1}{10}$ per cent many grains small, at $\frac{1}{50}$ per cent few, but luminous field	Acid precipitates KOH, clears di- and trivalent cations precipitated
Alcohol	Blue	Max at 600, sharper shading to blue	Very few grains	



Curve 4 —Licht-blau



Curve 5 —Congo blue
K = absorption in 1 per cent. KOH

(*b*) Aqueous solutions are coagulated by repeated heating and cooling, though not so completely as by electrolytes

(*c*) The aqueous solutions show the Tyndall cone with an intense beam of light The further ultramicroscopic appearance is given in the tables.

The remarkable difference in the absorption-curves of aqueous and organic solutions of pinacyanol and pinachrome blue is shown in the curves. Further, it is interesting to note that the addition of KCl in quantity insufficient to precipitate the dye immediately shifted the absorption still further toward the blue This points to the particle-size as being one factor in modifying the absorption

Colloid Solutions of Dyes.

Recent investigations have shown* that a very considerable number of dyes exist in pseudo- or colloid solution in water Michaelis, from their ultramicroscopic appearance, divides them as follows —

(*a*) Optically, completely resolvable, showing granules at the greatest dilutions Such are many sulpho-dyes, as induline, violett-schwarz, aniline-blue

(*b*) Partially resolvable, showing granules to an increasing degree above a certain concentration Such are fuchsin, methyl violet

(*c*) Not resolvable Generally fluorescent As fluorescein, toluidine blue, eosine.

From the experiments detailed above it would seem that *pinacyanol* and *pinachrome* blue belong to the first class, *cyanine* and *isocyanine* and *sky-blue extra* to the second But all pass into the third in organic solvents, and the important point is, that *this change in the state of aggregation is accompanied by a marked change in the absorption spectrum* It appears that care must be exercised in correlating all colour changes with structural modifications

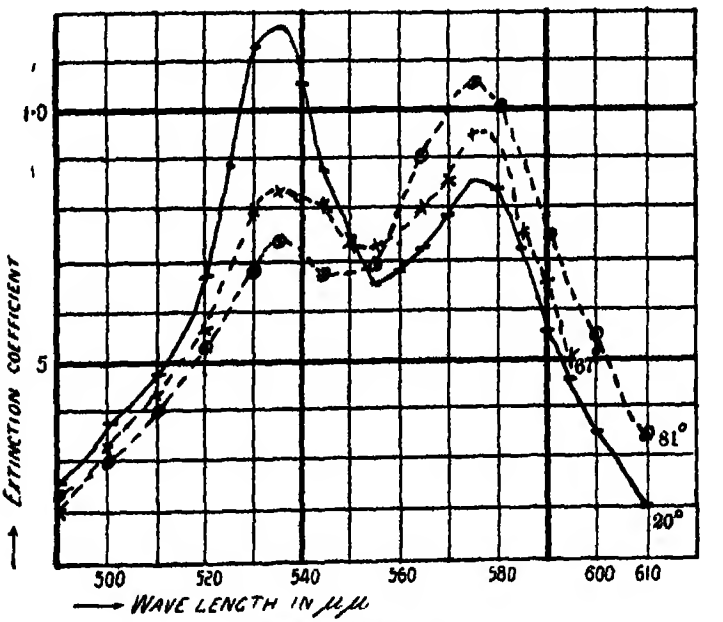
Action of Temperature

Whereas so far the disaggregation producing true solutions was only obtained by changing the solvent, the important result was obtained that on heating the aqueous solutions of pinachrome, pinacyanol, *iso*cyanine, and cyanine, the pseudo-solutions passed into the state of true solution, the colour gradually changing, and the optical heterogeneity disappearing With pinacyanol the change appeared to be complete above 100° The change was followed quantitatively for *iso*cyanine and pinacyanol.

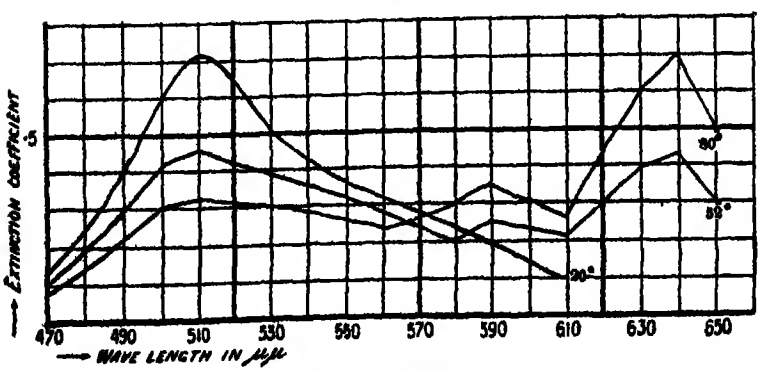
The Hüfner spectrophotometer not being quite suitable, a Konig instrument

* *Cf* B. H Buxton and O Teague, 'Zeitschr. phys. Chem.,' vol. 60, p. 4, 1907; L. Michaelis, 'Virchow's Archiv,' vol. 179, p. 195, 1905.

was employed The method was to compare two exactly similar solutions, one kept at 20°, the other heated to a known constant temperature Two exactly similar tubes of 20 cm length were constructed, the ends being closed with plate glass and having apertures for thermometers One was heated by a coil of wire through which passed the lighting current, by means of a variable resistance any desired temperature up to 80° could be obtained, constant to 0° 5 One slit of the spectrophotometer was illuminated directly, the other by reflection from a right-angled prism, and this was effected by two Auer burners fed from the same gas tap The readings gave



CURVE 6.—Isocyanine.



CURVE 7.—Pinachrome.

the differences between the hot and cold solutions. From the known extinctions for the cold solution at 20°, the curves for the different temperatures were constructed (curves 6 and 7)

It will be seen that there is a gradual transition from the spectrum of the pseudo-solution to that of the true solution as shown in alcohol (compare with curves 2 and 3). The change of temperature has precisely the same effect as the addition of an organic solvent miscible with water such as alcohol or acetone *In the hot solutions very few grains were visible and electrolytes had no effect. On cooling, the solutions returned to their former colour and state* The phenomena are similar to the behaviour of starch, which at or above the boiling point of water gives true crystalloid solutions of high molecular weight,* showing in the ultra-microscope only the diffuse beam due to "amicrons," or particles too small to be rendered visible.† On cooling, the starch solution becomes opalescent The behaviour is characteristic of reversible colloids forming at high temperatures unsaturated crystalloid solutions, at low ones very supersaturated and colloidal ones.‡ Apparently the pinacyanols and probably other dyes belong to this class, *i.e.*, they spontaneously dissolve in water to a colloid solution, or reach a very fine state of division but one still remote from molecular dimensions It is possible that this change of aggregation may in part account for some of the interesting changes in the absorption-curves of other dye solutions with change of temperature noticed by Nichols §

Influence of Acid on Alkali.

Michaelis noticed that nile blue sulphate and alizarin blue with soda lye give pseudo-solutions. A considerable number of dyes were found to change their degree of aggregation when acid or alkali was added, but there are of course in many cases chemical changes masking this The dyes were also examined for electric transport, and the results were in agreement with the previous datum of V Henri and A Mayer,‖ that OH′ and H′ ions flock or clear positive or negative colloids respectively.

Linder and Picton¶ noticed that congo red in alkaline solution filters readily through a porous cell but was retained in neutral or acid solution In the majority of cases there are undoubtedly chemical changes, but these are accompanied or preceded by changes in the state of aggregation.

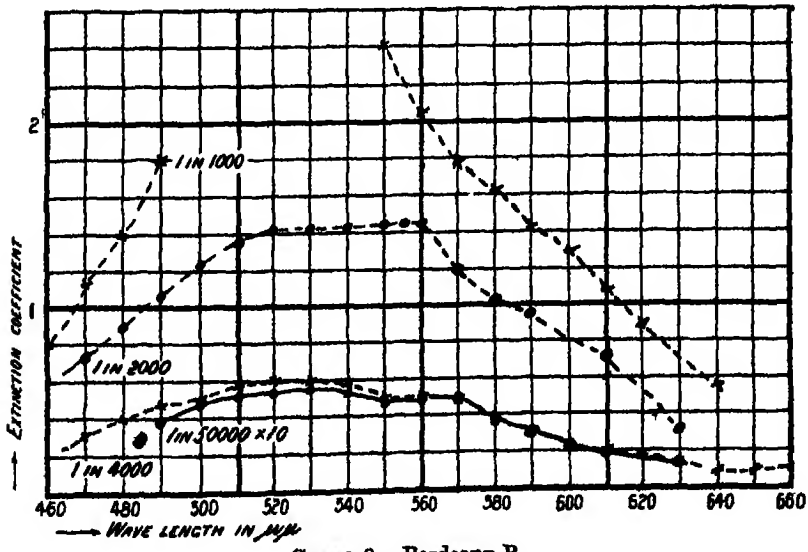
* *Cf* Lobry de Bruyn, 'Rec Chim. Trav,' vol 19, p. 253, 1900.
† R. Zsigmondy, *loc cit*, p. 87.
‡ *Ibid.*, p. 165
§ 'Zeitschr. phys. Chem,' vol 48, p. 271, 1900
‖ 'C. R. Soc. Biol.,' vol 55, p. 1671, 1903.
¶ 'Chem Soc Trans.,' 1892, p. 148.

Table IV.

Dye	Acid	Alkali	Transport.
Pinachrome	Clears, finally decolorises	Turbid	+ive colloid
Pinacyanol	Ditto	„	+ive „
Isocyanine	Decolorises	„	+ive „
Sky blue ex.	Turbid	Clears	−ive „
Diamil blue	„	„	−ive „
Congo blue	„	„ turns red	−ive „
Bordeaux B	„ but decolorised	„	−ive „
Benzopurpurin	„	Clearer	−ive „
Daphen blue	Clears	Turbid	+ive „

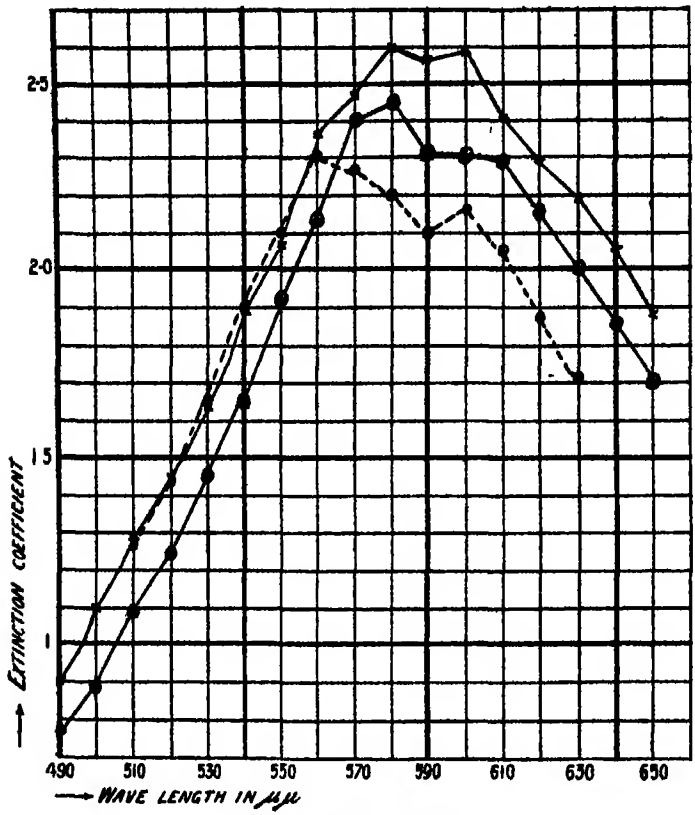
Influence of Concentration.

With the partially resolvable dyes (*v* p 262), the more concentrated the solution the greater the number of granules, dilution favouring the process of disgregation and solution Optically it results that certain deviations of the absorption spectrum occur The absorption curves for Bordeaux B and congo blue are given for different concentrations.



CURVE 8.—Bordeaux B.

The absorption in concentrated solutions, especially in the red, is *greater* than if Beer's law held.



CURVE 9.—Congo Blue

The curves should coincide, if Beer's law held

From these and other measurements it is concluded:—

(*a*) Completely optically resolvable dyes, *i. e.*, such as are in true solution, show the least change in the form of the curves, the maxima are not displaced. If chemically stable, Beer's law is followed *

(*b*) Dyes partially in colloid, partially in true solution or complete solution, show deviations, sometimes displacement of maxima, at very high concentrations. These results are in agreement with the observations of Kalandek †

The State of Dyes in Membranes.

Since the absorption-spectrum shows itself to some extent as a criterion of the state of aggregation in liquid "solutions," it is an obvious step to apply it to the investigation of the state of dyes in solid media In the course of

* See paper in 'Chem Soc. Trans.,' *loc. cit*, p. 16

† 'Physik Zeitschr,' 1906, May

the present investigation, the "colloidality" of the dyes was tested by their dialysis through collodion sacs, prepared according to Malfitano.* It is interesting to note that a definite osmotic pressure was obtained with certain colloid dyes, persistent after sufficient dialysis to have removed crystalloid impurities The dyes were congo blue, aniline blue, primuline, all resolvable optically That such colloids can show a definite osmotic pressure has already been shown by Linder and Picton, and by R S Lillie† Other colloid dyes were completely absorbed by the collodion, as pinacyanol And here the interesting fact was observed, that the dye, in pseudo-solution in water, *gave the colour and absorption spectrum of the true solution in collodion.*

The following table gives a brief survey of some of the absorption results The degree of absorption is indicated thus + = strongly, *m* = moderately, —not absorbed In addition, the nature of the spectrum is noted, corresponding to pseudo- or true solution

Table V‡

Dye	Collodion	Viscose	Gelatine	Caoutchouc
Pinacynol	+, bands as in alcohol	+, bands as in alcohol	—	*m*, spectrum as in water
Pinachrome	Same as pinacyanol			
Congo blue	+, band as in alcohol		+, band as in water	—
Rosaniline	+	+	+	+
Isocyanine (alkaline)	+, bands as in alcohol	+	—	*m*, uncertain
Crystal violet	+, band as in water	+	+	
Sky blue ex	+, as in alcohol	+	+	—

It is impossible to draw general conclusions as to the behaviour of membranes from these few experiments, a behaviour which appears to depend on a great variety of specific factors,§ but the important point is that in certain cases the dye is truly dissolved

This lends a certain amount of support to the view that in osmosis the membrane acts as a *solvent* for the permeating substance Returning to the absorption of dyes, colloid membranes themselves possess a charge,‖ and

* 'C R.,' vol. 139, p. 1221, 1904

† 'Amer. Journ. of Physiol.,' vol. 20, p. 167, 1907.

‡ *Collodion* is nitro cellulose, *viscose* = xantho-cellulose, the *caoutchouc* was pure rubber from coagulated latex.

§ *Cf* H Zangger, 'Vierteljahr. Naturforsch. Ges Zürich,' vol 51, p 432, 1906

‖ *Cf.* V Henri and A. Icovesco, 'C R. de la Soc. de Biol,' vol 51, p. 197, 1906.

this appears to be an important factor in absorption Collodion is negative to water, and this agrees with its strong absorption of positive colloids such as pinacyanol, but it does not explain the solution thereof Gelatine is +ive or −ive according to the alkalinity or acidity of the solution in contact. I have tested the absorption of a large number of dyes in neutral solution with a carefully dialysed gelatine, therefore as neutral as possible Here the colloidality of the dye appears to play a great *rôle*, *i.e.*, the more colloidal the less absorbed (see Table VI), but an absolute generalisation this is not

Table VI

Dye	Colloidality by dialysis and ultra	Absorption by gelatine	Absorption by caoutchouc	Spectrum in water
Naphthol green	Little or *nil*	+	m	Well defined
Methylene blue	,,	+	m	Steep
Indigo carmine	,,	+	+	,,
Chrysaniline	,,	+	m	,,
Crystal violet	Slight	+	+	,,
Rosaniline	,,	+	+	Sharp
Janus yellow B	,,	+	m	,,
Isocyanine	Moderate	m	−	,,
Cyanine	,,	m	−	Diffused
Bordeaux B	,,	+		Ill defined
Cærulein	,,	−		,,
Sky blue ex	,,	m	−	,,
Congo orange	,,	+	−	,,
Congo blue	Strongly	+	−	,,
Nigrosine	,,	−	−	,,
Wool blue 5B	,,	Slightly	−	,,
Pinacyanol	,,	−	m	Flattish.
Pinachrome	,,	−	m	,,
Benzopurpurin B	,,	m	−	,,
Aniline blue	,,	m	−	,,

The Action of Dyes on each other's Absorption Spectrum.

Colloid dyes of opposite sign may form so-called "complex colloids" stable in excess of one component* An interesting phenomenon was noticed with Baumwoll-gelb S and pinacyanol—

Solution	Absorption spectrum
Pinacyanol in cold water	Band at 510 $\mu\mu$.†
,, hot water	Bands at 590 $\mu\mu$ and 640 mm.†
Baumwoll-gelb S	Absorption in blue, violet, and ultra-violet (max).
Pinacyanol in excess of Baumwoll-gelb S	Bands at 510 $\mu\mu$, 590—600 $\mu\mu$, and at 660 $\mu\mu$.

* *Cf.* B. H Buxton and O Teague, 'Zeit. physik. Chem.,' vol. 50, p. 479, 1907.

† See Curve 2.

Now we have found the bands at *ca.* 590 and 640 $\mu\mu$ as characteristic of true solution, so that it appears, that just as in membranes, one component in so-called "complex-colloids" may be partially in true solution in the other Spectrophotometric examination of other complexes should yield valuable information as to the nature of this union

Dyeing and the Nature of Solution

The experiments detailed here seem to support the view that "solution" is a continuous phenomenon, from mechanical suspension down to true solution (Linder and Picton, Zsigmondy, and others). The author is of the opinion that the process of dyeing in solid media is strictly analogous to the "solution" of dyes in liquid solvents, in that all phases or grades may be found It seems permissible to parallel the solution states in liquid and solid media as follows —

Liquid solutions.	Solid solutions
Suspensions	Adhesion
Colloid solutions	Adsorption
Reversible hydrosols	Colloid complexes
Eg { Dyestuffs, Glycogen, Soluble starch	(Dyeing.)
Crystalloid solutions	Solid solutions proper (including isomorphous mixtures and alloys).

In solutions of dyes, either in liquid or solid media, all grades may coexist, the conditions for forwarding or diminishing the degree of solution may be grouped as follows —

Toward true solution Diminishing particles	De-solution Increasing particles
Dilution	Concentration.
Admixture of other solvent	Electrolytes.
Acid or alkali.	Alkali or acid.
Rise of temperature	Lowering of temperature

These conclusions are not novel,* but it seems worth while to point out that concomitant with "solution," in this sense, the absorption spectrum

* In particular, *cf* R. Zsigmondy, 'Zur Erkenntniss d. Kolloide' (Fischer, Jena), and C. Dreaper, 'The Chemistry and Physics of Dyeing,' 1906.

tends to change from broad and ill-defined bands (*cf* Licht-blau, p 261) to sharp ones, whilst in certain cases a quite new spectrum is formed. (I hope to point out elsewhere the resemblance between the absorption spectra of colloid dyes and those of colloid metals, such as gold) At the same time it appears probable that there is a progressively increasing combination between the "solute" and the solvent

SUMMARY

(*a*) In the aqueous solutions of certain dye-stuffs—isocyanines, pinacyanols, cyanine—the dye is present partially or wholly in colloid solution, and the absorption spectrum is quite different from that of the true solution The influence of various agencies, as heat, acid and alkali, electrolytes on the absorption was examined quantitatively

(*b*) In other dye solutions, the change from true solution to the colloid state is accompanied by broadening and diffusion of the absorption curve, consequent on the increase in number and size of the colloid particles. Deviations from Beer's law result

(*c*) The state of dyes in solid media is comparable with that in liquid, and the absorption spectrum is similarly affected The absorption of a number of dyes by membranes was studied

(*d*) The solution of dyes appears to be a combined process of disaggregation of the solute, accompanied by a progressive combination with the solvent. If the same stage of solution is attained in *different* solvents, the absorption maxima are displaced according to Kundt's law

This investigation was carried out in the spring and summer of 1908 in the laboratory of Physiological Chemistry of the Sorbonne, Paris, the author desires to express his great thanks for the hospitality and facilities afforded him: in particular to the director, Prof. Dastre, and to M. Victor Henri for his continuous advice and interest

"Dynamic" Osmotic Pressures

By the Earl of Berkeley, F.R.S., and E. G. J. Hartley, B.A. (Oxon)

(Received March 31,—Read April 22, 1909.)

1. The following paper is a preliminary account of what is apparently a new method of measuring osmotic pressures. The account is published now because during the course of the experiments we have unfortunately damaged the only two good semi-permeable membranes that we possess, and it will be some months before the damage can be repaired.

2. A word of explanation as to the use of "dynamic" to distinguish the osmotic phenomena we are about to describe seems necessary. In all discussions of osmotic pressures (except those involving diffusion) the locution "osmotic pressure" connotes some form of equilibrium between a solution and its solvent; in the experiments about to be recorded we have measured the rate at which the solvent flows into the solution, and the essence of the observations lies in the fact that there should be no approach to equilibrium. Thus the osmotic pressure here involved is substantially that assumed to act in Nernst's theory of diffusion.

3. The experiments described below will be seen to prove that the rate of flow is proportional to, and may be used to measure, the equilibrium osmotic pressures, at all events in dilute solutions. Moreover, the rate at which water passes through a semi-permeable membrane under a given hydrostatic pressure will be shown to be the same as the rate at which it would pass that membrane, when entering a dilute solution whose osmotic pressure has the same numerical value as the hydrostatic pressure formerly used. Thus in these experiments the osmotic pressures are directly correlated with hydrostatic pressure.

4. Briefly, the method is as follows. A porcelain tube, carrying a copper ferrocyanide membrane on the outside, is set up in the osmotic pressure apparatus,* with water on both sides of the membrane. The interior of the tube is in communication with a graduated capillary, while the outside water can be subjected to pressure; the first part of the experiment consists in noting the rate at which known hydrostatic pressures force water through the membrane. In the second part of the experiment the tube, with its capillary attached, is surrounded with the solution whose "dynamic osmotic pressure" is required, and a measurement is made of the initial rate at which the water is sucked through the membrane into the solution.

* See 'Phil. Trans.,' Series A, vol. 206, p. 483.

5 It is of the utmost importance that the initial rates be measured; for very early in the research it was found that the rate at which the water is sucked into a solution varies enormously with the time the tube remains in that solution, in some cases the velocity of the water is reduced to less than half the initial rate, and in all cases the rates slow down to a minimum, as found by Vegard,* the minimum depending on the previous history of the tube A great many experiments were made to elucidate the cause of this phenomenon, but they will not be detailed here as they do not seem to bear immediately on the main object of the research.

6 *The Experiments*—The first method tried was devised so as to find whether the rates of passage of the water into various strengths of cane-sugar solutions was proportional to the osmotic pressure A tube was set up in the osmotic pressure apparatus exactly as for a determination of the equilibrium osmotic pressure,† when equilibrium had been established between the water in the tube and the solution outside (by means of the mechanical pressure put upon the solution), the pressure was suddenly and completely released and the initial rate at which the water flowed into the solution was noted The following are examples of the observations The first column gives the concentration of the solutions in grammes per litre, and the second and third columns give the observed rates, in millimetres, of the capillary per second,‡ for the two tubes N and X The last column gives the known osmotic pressures.—

Concentration	Tube N	Tube X	Equilibrium osmotic pressure
grammes.	mm /sec	mm /sec	atmos
750	0 500	Not determined	134
660	0 365	0 369	101
540	0 276	0 275	67
420	0 185	0 190	44
300	0 126	0 124	27

It will be seen that there is good concordance between the rates for the two tubes, but they are not proportional to the osmotic pressures

7. Thinking that this want of proportionality was due to the solution not being stirred, the experiments were repeated in the following manner:—Tube X, fitted with ring stirrers which could be moved up and down, was fixed in a vertical position, and the various solutions were brought up from underneath to submerge the membrane. The observed rates were practically the

* 'Proc. Camb. Phil Soc.,' vol. 15, Part I, p. 17 † *Loc cit.*

‡ One mm of the capillary has a capacity of 0·0011 c.c.

same as in the previous case. It was noticed, however, that an increased speed of movement of the ring stirrers seemed to influence the rate slightly, so a more efficient stirring arrangement was devised

8 In the new apparatus the tube is fixed vertically between two horizontal brass plates by means of dermatine rings and screw couplings, in such a manner that the area of exposed membrane is the same as in the osmotic pressure apparatus proper Three four-bladed brass paddles are placed symmetrically round the tube with their axes vertical, and the edges of the blades, which are also vertical, pass within 1 mm of the membrane when the paddles are rotated The edges of the blades extend along the whole length of the membrane In the experiments, the number of revolutions of the paddles was between six and three per second, and it was now found that the difference between practically no stirring and these rapid speeds was, in the case of the strongest solution, about 10 per cent in the water rate

9 Before giving the results of the experiments with the new apparatus, we will record the measurements of the flow of water through the membrane under different hydrostatic pressures These were obtained with tube X in the osmotic pressure apparatus, using a Schaeffer and Budenberg standard dead weight pressure gauge to give the pressures In the table, the first column gives the pressure on the water, and the corresponding rate of flow is noted in the second column

Pressure	Rate	Ratio of rates.	Ratio of pressures
atmos	mm /sec		
20 41	0 1075	1	1
40 82	0 2203	2 05	2
61 24	0 3240	3 01	3
81 65	0 4308	4 00	4
102 06	0 5319	4 95	5
122 47	0 6378	5 93	6

The average rate per atmosphere pressure is 0·00528 mm per second. It is evident from these figures that the rates may be taken as proportional to the pressure, a conclusion which has some interest of its own; but we will reserve the discussion of this for another opportunity, when a more detailed account of the whole work can be given.

10. On setting up tube X in the new stirring apparatus and proceeding in the manner already outlined, we obtained the results tabulated below. The first column gives the concentration (grammes per litre), the second the observed rate, the third gives this rate divided by 0·00528 (the average rate per atmosphere hydrostatic pressure found in the last experiment) The

numbers in this column are taken as the "dynamic" osmotic pressures of the solutions. The fourth column gives the equilibrium pressures for the solutions—determined experimentally* for the stronger solutions, and calculated from Boyle's Law for those that are weaker

Tube X			
Concentration	Rates	Rates/0·00528 = dynamic osmotic pressure	Equilibrium osmotic pressure
grammes	mm/sec	atmos	atmos
750	0 571	108 2	134 7
660	0 472	89 5	100·8
540	0 315	59 7	67 5
300	0 134	25·4	26 8
96 2	0·0341	6 46	6 36
45	0·0155	2 94	2 97

It will be seen that the agreement between the last two columns is very good for the lower numbers, but that the larger values diverge considerably. We are not ready as yet to offer a satisfactory explanation of this discrepancy

11 A similar set of experiments was made with tube N, but unfortunately the membrane was damaged before the rate of flow under hydrostatic pressure could be determined. The results are tabulated as in the last table,

Tube N			
Concentration.	Rates.	Rates/0 00528 = dynamic osmotic pressure	Equilibrium osmotic pressure
grammes	mm/sec	atmos	atmos
750	0 552	104 6	134·7
558 5	0 324	61 3	71 8
300	0 1272	24 2	26·8
150 8	0·0552	10·5	11·8
93 75	0·03285	6 23	6 18
45	0 01537	2·91	2·97
20	0·006680	1 27	1 32
10	0·003443	0·65	0·66
2·02	0·000747	0 141	0 134

and we have felt justified in dividing the observed water rates by the same factor as in the case of tube X, because the two tubes have behaved throughout all our researches in an exactly similar manner,† and, moreover,

* *Loc. cit.*

† This is apparent in the table in paragraph (6).

a measurement of the hydrostatic pressure rate, when the tube was known to be slightly damaged, gave a value of 0 00540 mm /sec per atmosphere pressure, a value which is not greatly different from that used in the table.

Here, again, the agreement between the last two columns is good for the lower values, and attention may be drawn to the fact that the rates for the two tubes are very similar

12 All the experiments here recorded were carried out at 0° C, or as near as possible to that temperature, but it may be mentioned that some somewhat imperfect experiments show that the temperature coefficient is quite large—in fact, a difference of 1° (near 0° C) may cause a difference in the rates of 10 per cent.

13 There are two further points that seem worth mentioning One is that we have reason for believing that fairly accurate experiments can be made on more dilute solutions than 2 02 grammes per litre This may be of importance as giving an easier way of measuring very small osmotic pressures than that employed in the direct equilibrium method

The other point is that the determination of the rate of flow under a hydrostatic pressure is a more delicate test of the semi-permeability of the membrane than is the actual measurement of the amount of sugar which comes through during a direct equilibrium pressure experiment.*

14. In conclusion, it may be emphasised that these preliminary results show that osmotic phenomena can be measured kinetically (we do not mean that the kinetic theory of osmotics is thereby inferred), and that the results are, for dilute solutions, the same as when measured statically. The method puts directly in evidence the driving forces or partial pressures which have to be considered in the dynamical theory of diffusion of solutions

* *Cf loc cit*, Appendix A

The Wave-making Resistance of Ships a Theoretical and Practical Analysis.

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(Communicated by Prof. J Larmor, Sec R S. Received April 1,—Read April 29, 1909)

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§ 1 *Introduction and Summary*

The theoretical investigation of the total resistance to the forward motion of a ship is usually simplified by regarding it as the sum of certain independent terms such as the frictional, wave-making, and eddy-making resistances The experimental study of frictional resistance leads to a formula of the type

$$R_s = fSV^m, \tag{1}$$

where S is the wetted surface, V the speed, f a frictional coefficient, and m an index whose value is about 1 83

After deducting from the total resistance the frictional part calculated from a suitable formula of this kind, the remainder is called the residuary resistance Of this the wave-making resistance is the most important part, the present paper is limited to the study of wave-making resistance, and chiefly its variation with the speed of the ship. The hydrodynamical theory as it stands at present may be stated briefly

Simplify the problem first by having no diverging waves, that is, suppose the motion to be "in two dimensions in space," the crests and troughs being in infinite parallel lines at right angles to the direction of motion. Further, suppose that the motion was started at some remote period and has been maintained uniform. We know that, except very near to the travelling disturbance, the surface motion in the rear consists practically of simple periodic waves of length suitable to the velocity v of the disturbance. Let

a be the amplitude of the waves, and w the weight of unit volume of water; then the mean energy of the wave motion per unit area of the water surface is $\frac{1}{2}wa^2$ Imagine a fixed vertical plane in the rear of the disturbance, the space in front of this plane is gaining energy at the rate $\frac{1}{2}wa^2v$ per unit time But on account of the fluid motion, energy is supplied through the imaginary fixed plane to the space in front, and it can be shown that the rate of supply is $\frac{1}{2}wa^2u$, where u is the group-velocity corresponding to the wave-velocity v The nett rate of gain of energy is $\frac{1}{2}wa^2(v-u)$, and this represents the part of the power of the ship which is needed, at uniform velocity, to feed the procession of regular waves in its rear. An equivalent method of stating this argument is to regard the whole procession of regular waves from the beginning of the motion as a simple group, then the rear moves forward with velocity u while the head advances with velocity v, and the whole procession lengthens at the rate $v-u$ If we write Rv for the rate at which energy must be supplied by the ship, we call R the wave-making resistance, and we have

$$R = \tfrac{1}{2}wa^2(v-u)/v \qquad (2)$$

We notice that R is the wave-making resistance in *uniform* motion; it is only different from zero because u differs from v, that is, because the velocity of propagation depends upon the wave-length.

In deep water, u is $\frac{1}{2}v$, so that R is $\frac{1}{4}wa^2$ In the application of this to a ship at sea, it is assumed that the transverse waves have a certain average uniform breadth and height, and, further, that the diverging waves may be considered separately and as having crests of uniform height inclined at a certain angle to the line of motion, if the amplitude is taken to vary as the square of the velocity, it follows that R varies as v^4 Several formulæ of the type $R = Av^4$, or $R = Av^4 + Bv^6$, have been proposed, although these may be of use practically by embodying the results of sets of experiments, they are not successful from a theoretical point of view. Recently many such cases have been analysed graphically by Prof. Hovgaard;* the general result is that a fair agreement may be made for lower velocities with an average experimental curve neglecting the humps and hollows due to the interference of bow and stern wave systems, but at higher velocities the experimental curve falls away very considerably from the empirical curve

The method used here consists in considering the ship, in regard to its wave-making properties, as equivalent to a transverse linear pressure distribution travelling uniformly over the surface of the water Taking a simple form of diffused pressure system and making some necessary

* W Hovgaard, 'Inst. Nav Arch. Trans.,' vol. 50, p 205, 1908.

assumptions, we obtain an expression for the amplitude of the transverse waves thus originated, and for the resistance R, in which the velocity enters in the form e^{-a/v^2}, this function is seen to have the general character of the experimental curves. Adding on a similar term for the waves diverging from bow and stern, and, finally, in the manner of W Froude, an oscillating factor for the interference of these bow and stern waves, we find a formula for the wave-making resistance of the type

$$R = \alpha e^{-l/v^2} + \beta \{1 - \gamma \cos (m/v^2)\} e^{-n/v^2}$$

In this expression there are six adjustable constants, we proceed to reduce the number of these after transforming into units which utilise Froude's law of comparison We use the quantity c, defined as

$$(\text{speed in knots})/\sqrt{(\text{length of ship in feet})},$$

and we express the resistance in lbs per ton displacement of the ship. An inspection of experimental curves, and other considerations suggest that the quantities l, m, n may be treated as universal constants, with this assumption, a three-constant formula is obtained, viz,

$$R = \alpha e^{-2 \cdot 53/0c^2} + \beta \{1 - \gamma \cos (10 \cdot 2/c^2)\} e^{-2 \cdot 53/c^2}, \qquad (3)$$

where the constants α, β, γ depend upon the form of the ship

We then treat (3) as a semi-empirical formula of which the form has been suggested by the preceding theoretical considerations, several experimental model curves are examined, and numerical calculations are given which show that these can be expressed very well by a formula of the above type

Since the constant α is found to be small compared with β, it is not allowable to press too closely the theoretical interpretation of the first term, especially as the experimental curves include certain small elements in addition to wave-making resistance. If we limit the comparison to values of c from about 0 9 upwards, it is possible to fit the curves with an alternative formula of the type

$$R = \beta \{1 - \gamma \cos (10 \cdot 2/c^2)\} e^{-n/c^2},$$

and some examples of this are given.

The effect of finite depth of water is considered, and a modification of the formula is obtained to express this effect as far as possible. Starting from an experimental curve for deep water, curves are drawn, from the formula, for the transverse wave resistance of the same model with different depths; although certain simplifications have to be made, the curves show the character of the effect, and allow an estimate of the stage at which it becomes appreciable

In the last section the question of other types of pressure distribution is

discussed, and one is given in illustration of the wave-making resistance of an entirely submerged vessel.

§2 *Pressure System travelling over Deep Water.*

It is known that a line pressure-disturbance travelling over the surface of water with uniform velocity v at right angles to its length gives rise to a regular wave-train in its rear of equal wave-velocity * Take the axis of x in the direction of motion and let the pressure system be symmetrical with respect to the origin and given by $p = f(x)$; suppose that $f(x)$ vanishes for all but small values of x, for which it becomes infinite so that $\int_{-\infty}^{\infty} f(x)\,dx = \mathrm{P}$ The regular part of the surface depression η due to this integral pressure P practically concentrated on a line is given by

$$\eta = \frac{2g\mathrm{P}}{wv^2} \sin\frac{gx}{v^2}. \tag{4}$$

The part of the surface effect which is neglected in this expression consists of a local disturbance symmetrical with respect to the origin and practically confined to its neighbourhood.

If we suppose P constant, the amplitude in the regular wave-train and the consequent drain of energy due to its maintenance diminish with the velocity.

To obtain results in any way comparable with practical conditions it is necessary to suppose the pressure system diffused over a strip which is not infinitely narrow

An illustration is afforded by taking

$$p = f(x) = \frac{\mathrm{P}}{\pi}\frac{\alpha}{\alpha^2 + x^2}, \tag{5}$$

where α is small compared with the distances at which the regular surface effects are estimated This type of pressure distribution is shown in fig 1.

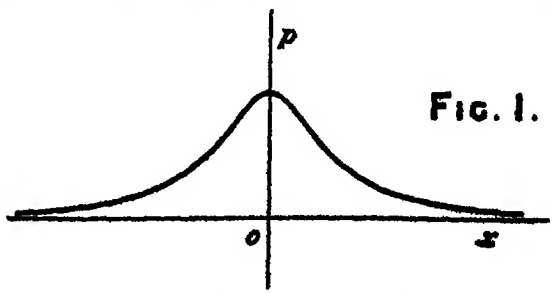


FIG. 1.

* For a discussion of the wave pattern, see Lamb, 'Hydrodynamics,' § 241 *et seq.*; or Havelock, 'Roy. Soc. Proc.,' A, vol. 81, p. 398, 1908.

The effect of thus diffusing the pressure system is expressed by the introduction of a factor $\phi(\kappa)$ into the amplitude of the regular waves, where $2\pi/\kappa$ is the wave-length and

$$\phi(\kappa) = \int_{-\infty}^{\infty} f(\omega) \cos \kappa\omega \, d\omega. \tag{6}$$

Using (5) in (6), we find

$$\phi(\kappa) = \mathrm{P}e^{-a\kappa} = \mathrm{P}e^{-ag/v^2}$$

Hence the amplitude of the waves is given by

$$a = \frac{2q\mathrm{P}}{wv^2} e^{-ag/v^2} \tag{7}$$

Further, since $\kappa = v^2/g$, the group velocity $u = d(\kappa v)/d\kappa = \frac{1}{2}v$. Hence the wave-making resistance R is given by

$$\mathrm{R} = \frac{g^2\mathrm{P}^2}{wv^4} e^{-2ag/v^2} \tag{8}$$

We have to examine the variation of these quantities with the velocity v under the supposition that the pressure system is due to the motion of a body either floating on the surface or wholly immersed in the water The pressures concerned being the vertical components of the excess or defect due to the motion, it seems possible to assume as a first approximation that P varies as v^2; this is the case in the ordinary hydrodynamical theory of a solid in an infinite perfect fluid, and a similar assumption is also made in the theory of Froude's law of comparison. This being assumed, we find

$$a = \mathrm{A}e^{-ag/v^2}, \qquad \mathrm{R} = \mathrm{B}e^{-2ag/v^2}. \tag{9}$$

We see that both the amplitude and the resistance increase steadily from zero up to limiting values

If we draw the curve representing this relation between R and v, there is a point of inflection when

$$\frac{d^2\mathrm{R}}{dv^2} = 0, \quad \text{or} \quad v^2 = \tfrac{1}{2}ga. \tag{10}$$

Writing v' for this velocity, we see that $d\mathrm{R}/dv$ increases as the velocity rises to v' and then falls off in value as the velocity is further increased.

We can write the relation now in the form

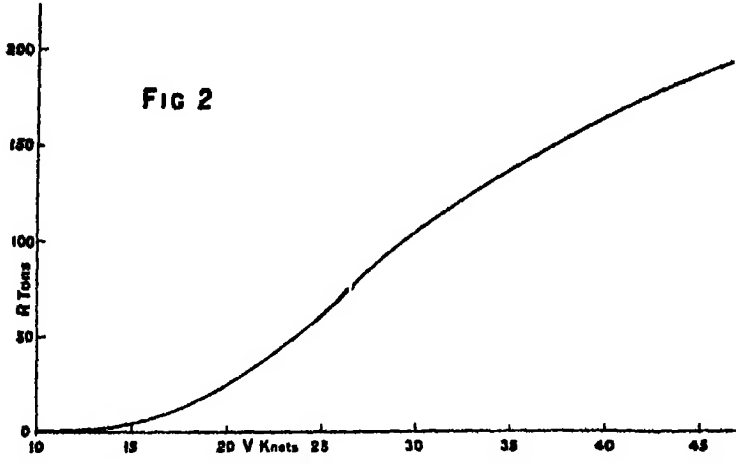
$$\mathrm{R} = \mathrm{B}e^{-\frac{1}{2}(v'/v)^2} \tag{11}$$

The character of this relation is shown by the curve in fig 2, which represents the case

$$\mathrm{R} = 315e^{-\frac{1}{2}(36/\mathrm{V})^2}, \tag{12}$$

R being in tons, and V in knots.

The values of the constants in (12) have been chosen for comparison with an experimental curve of residuary resistance given by R. E. Froude;* it was obtained from model experiments and by means of the law of



corresponding speeds and dimensions the results were given for a ship (model A) of 4090 tons displacement and 400 feet length The actual curve is given in fig 4 and is discussed more fully later, we neglect for the present the undulations which are known to be due to the interference of the bow and stern wave systems, and we consider a fairly drawn mean experimental curve denoted by R′ Table I shows a comparison of the values of R′ with those of R calculated from the formula (12)

Table I

V.	R	R′
10	0 02	1 8
14	2	4
18	14	16
22	38	39 5
26	70	70
30	106	107
34	132	130
38	157	156
42	176	175
46	195	192

From this comparison we see that the point of inflection given by V′ corresponds to the point at which the slope of the mean experimental curve

* R. E. Froude, 'Inst. Nav. Arch Trans.,' vol. 22, p. 220, 1881.

begins to fall off. This effect is general in residuary resistance curves; we see that it is really an interference effect, the character of the curve being due to the mutual interference of the wave-making elements of the pressure system. Superposed on the mean curve we have a further interference effect due to the combination of two systems, the bow and stern systems.

From Table I we infer that the mean curve agrees well with the calculated values R from about 18 knots upwards, but at the lower speeds the values of R are much too small, this suggests the addition of a term to represent the effect of the diverging waves.

§ 3 *Diverging Wave System.*

In the example considered above, the calculated values of R are much too small at the lower velocities. This might have been expected, for we obtained (12) by the consideration of line-waves on the surface, that is waves with crests of uniform height along parallel infinite lines. But the model experiments correspond more to a point disturbance travelling over the surface, with the formation of diverging waves as well as transverse waves. In fact, W Froude* infers from his experimental curves that the residuary resistance at the lower velocities is chiefly due to the diverging wave system, on account of the absence of undulations, for the latter signify interference of the transverse systems initiated by the bow and stern, and these become very important at the higher velocities.

We have to add to (12) a term representing the diverging waves, the comparison in Table I suggests for this a term of the same type, $e^{-k(V''/V)^2}$, with V'' much smaller than the corresponding velocity V' for the transverse waves. With the data at our disposal we might then determine the various constants so as to obtain the closest fit possible, however, we can make the process appear less artificial by the following considerations. We know that the wave pattern produced by a travelling point source consists of a system of transverse waves and a system of diverging waves, the whole pattern being contained with two radial lines making angles of about 19° 28′ with the direction of motion; a fuller investigation of the effects produced by a diffused source must be left over at present. In applying energy considerations as in the previous sections, the usual method is to suppose that the transverse waves form on the average a regular wave-train of uniform amplitude and uniform breadth; using the same approximation for the diverging waves we suppose that these form on the average a regular wave-train on each side, with the crests inclined at some angle θ to the direction

* W. Froude, 'Inst. Nav. Arch Trans.,' vol. 18, p. 86, 1877.

of motion of the disturbance Then the velocity of the diverging wave-trains normally to their crests is $V \sin\theta$ Now the same features of the ship are responsible for the character of both transverse and diverging waves; then if V' is the velocity at which there is a point of inflection in the resistance curve for the transverse waves, the suggestion is that $V' \sin\theta$ is the corresponding velocity for the diverging waves Taking as a first approximation the angle given above, viz, 19° 28′ or $\sin^{-1}\frac{1}{3}$, we test now a formula of the type

$$R = Ae^{-\frac{1}{2}(V'/3V)^2} + Be^{-\frac{1}{2}(V'/V)^2}. \qquad (13)$$

For the particular example already used (Froude, Ship A) we take V' equal to 26 knots, and determine A, B from two values of V. We obtain thus

$$R = 4\,5e^{-\frac{1}{2}(26/3V)^2} + 297e^{-\frac{1}{2}(26/V)^2} \qquad (14)$$

With this formula we find as good an agreement as before at the higher velocities, and we have now at lower velocities the comparison in Table II —

Table II

V	R	R′
10	1·6	1 8
14	4 1	4
18	16·5	16
22	40	39 5

In calculating from (14) we find that the two terms both increase continually, at low velocities the second term is practically negligible, then at about 15 knots the two terms are of equal value, and after that the transverse wave term becomes all important.

It must be remembered that the experimental curve was obtained from tank experiments, and it is possible that the width of the tank may have an effect on the relative values of the transverse and diverging waves. It would be of interest if experiments were possible with the same model in tanks of different widths, if the methods used in obtaining (14) form a legitimate approximation, the effect might be shown in the relative proportions of the two terms—provided always that one can make a suitable deduction first for the frictional resistance, and can then separate out the relatively small effects of the diverging waves, the eddy-making and other similar elements.

§ 4. *Interference of Bow and Stern Wave-trains.*

The cause of the undulations in the resistance curves was shown by W. Froude to be interference of the wave system produced by the bow (or entrance) with that arising at the stern (or run). His experiments on the effect of introducing a parallel middle body between entrance and run confirmed his theory, which may be stated briefly. Let the wave-making features of the bow produce transverse waves which would have at a breadth b an amplitude a, owing to the spreading out of the transverse waves they will be equivalent to simple waves at the stern of smaller amplitude ka, at the same breadth b Let a' be the amplitude there of the waves produced by the stern Then in the rear of the ship we suppose there are simple waves of amplitude ka superposed upon others of equal wave-length of amplitude a' At certain velocities the crests of the two systems coincide in position, giving rise to a hump on the resistance curve, and at intermediate velocities there are hollows on the curve owing to the crests of one system coinciding with the troughs of the other

In developing a form for the resistance, subsequent writers have generally taken R proportional to an expression of the form $a^2+a'^2+2kaa'\cos(mgL/v^2)$, where L is the length of the ship This means that the bow is supposed to initiate a system of waves with a first crest at a short distance behind the bow, and that similarly the stern waves have their first crest shortly after the stern, the length mL is the distance between these two crests, and is called the wave-making length of the ship. The determination of a value for m appears to be doubtful, but from interference effects it is said to vary for different ships between the values 1 and 1 2.

It has seemed desirable here to follow more closely the point of view in W. Froude's original paper already quoted * We regard the entrance of the ship as forming transverse waves with their first crest shortly aft of the bow, and the run of the ship as forming waves with their first trough in the vicinity of the middle of the run. It is suggested that this distance between first crest and first trough, in practice found to be about 0 9L, should be taken as the "wave-making distance", the cosine term in the formula is then prefixed by a minus sign instead of a positive sign. We return to this point later, we first work out a definite simple illustration in "two-dimensional waves," and then build up a more complete formula for comparison with experiment. With the same notation as in § 1, let the pressure system be given by

$$p = f(x) = \frac{1}{\pi}\left\{\frac{P_1 a^2}{a^2+(x-\frac{1}{2}l)^2} - \frac{P_2 a^2}{a^2+(x+\frac{1}{2}l)^2}\right\}. \qquad (15)$$

* W. Froude, *loc. cit ante*, p. 83.

This indicates two pressure systems, one of excess and the other of defect of pressure, each distribution is of the type already used, and their centres are separated by a distance l. Fig 3 shows the character of the disturbance

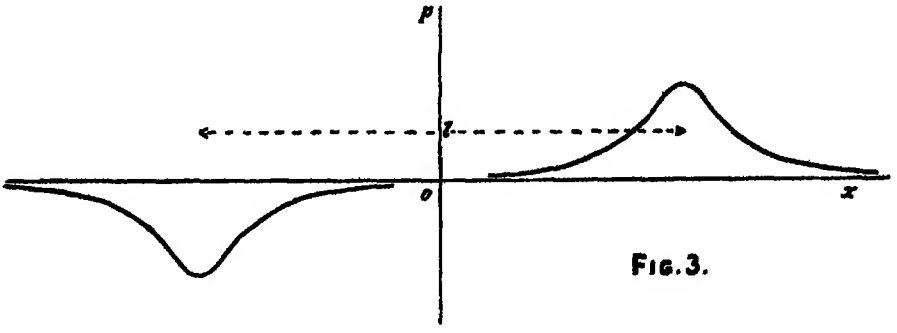


FIG. 3.

In the rear of the whole disturbance there is interference between the regular wave-trains due to the two parts. With the same methods as before we find that the resulting waves are given by

$$\eta = \frac{2gP_1}{wv^2} e^{-\alpha g/v^2} \sin \frac{g(x-\frac{1}{2}l)}{v^2} - \frac{2gP_2}{wv^2} e^{-\alpha g/v^2} \sin \frac{g(x+\frac{1}{2}l)}{v^2}$$

$$= \frac{2g}{wv^2} e^{-\alpha g/v^2} \left\{ (P_1 - P_2) \cos \frac{gl}{2v^2} \sin \frac{gx}{v^2} - (P_1 + P_2) \sin \frac{gl}{2v^2} \cos \frac{gx}{v^2} \right\}. \quad (16)$$

Hence the average energy per unit area is proportional to

$$v^{-4} e^{-2\alpha g/v^2} \{P_1^2 + P_2^2 - 2P_1P_2 \cos(gl/v^2)\}$$

Now, assuming as before that P_1 and P_2 vary as v^2, we find that as regards variation with the velocity the effective resistance R, which is the expression of the energy required to feed the wave-trains, is given in the form

$$R = \{A^2 + B^2 - 2AB \cos(gl/v^2)\} e^{-2\alpha g/v^2} \quad (17)$$

A more general expression might have been obtained by taking two quantities α_1 and α_2 in (15), corresponding to some difference in wave-making properties of entrance and run, this would have led to different exponential factors being attached to the bow and stern waves. However, we find (17), with a common exponential factor, sufficiently adjustable for present purposes.

In Froude's experiments in 1877 the effect of inserting different lengths of parallel middle body between the same entrance and run was examined, it was found that a hump in the residuary resistance curve corresponded to a trough of the bow waves being in the vicinity of the middle of the run, and a hollow to a crest being in that position.

For the model, Ship A, we have: Length = L = 400 feet, entrance = run = 80 feet.

Hence, in this case we may take, in formula (17), l as approximately 360 feet. We notice that this gives $l = 0\,9\mathrm{L}$, and in subsequent comparisons, instead of leaving l to be adjusted to fit the experimental curve, we find there is sufficient agreement if we fix it beforehand as 0 9 of the length of the ship on the water-line

Compare, now, the length l with the ordinary "wave-making length" of the ship, the latter is written as $m\mathrm{L}$ and is defined as the distance between the first regular bow crest and the first regular stern crest. From the present point of view (17) gives

$$m\mathrm{L} = l + \tfrac{1}{2}\lambda \quad \text{or} \quad m = 0\,9 + \tfrac{1}{2}\lambda/\mathrm{L}, \tag{18}$$

where λ is the wave-length in feet of deep-sea waves of velocity v ft /sec.

Calculating from this formula for Ship A, and writing V for velocity in knots (6080 feet per hour), we obtain Table III

We see that the statement that m lies between 1 and about 1 2 would hold for this ship if it were measured for ordinary speeds between about 14 and 22 knots.

Table III

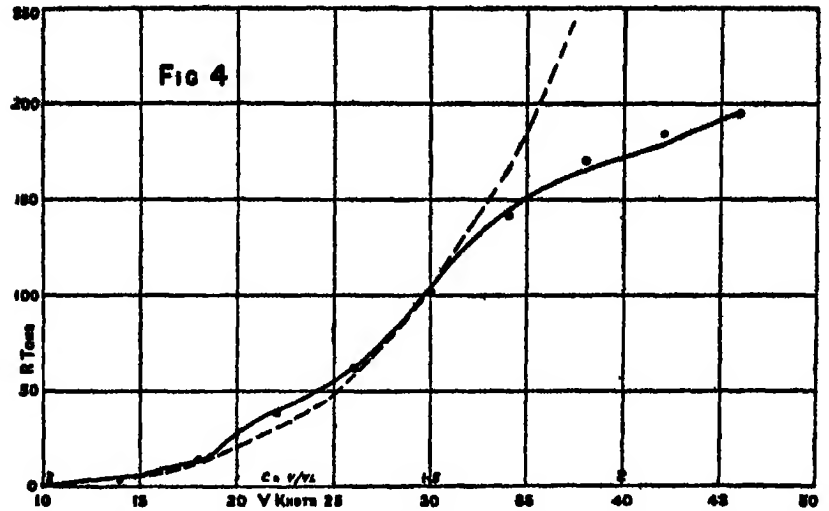
V	λ.	m
10	55 5	0 97
14	110	1 ·03
18	180	1 13
22	270	1 24
26	362	1 85
30	500	1 5

We proceed now to modify (14) by introducing into the second term a factor $1 - \gamma \cos(gl/v^2)$ With $l = 360$, we find gl/v^2 is approximately $4080/\mathrm{V}^2$, with V in knots; further, from one value from the experimental curve we obtain $\gamma = 0{\cdot}12$ Thus for Ship A we have R in tons given by

$$\mathrm{R} = 4{\cdot}5e^{-\frac{1}{2}(26/\mathrm{V})^2} + 297\,\{1 - 0{\cdot}12\cos(4080/\mathrm{V}^2)\}\,e^{-\frac{1}{2}(26/\mathrm{V})^2}. \tag{19}$$

Table IV shows some calculated values for R, and these are represented in fig. 4 by dots, the continuous curve is the experimental residuary resistance curve given by Froude, that is, the total resistance less the calculated frictional part.

It is the custom to give the results of model experiments in the form of a fair curve, so that the positions of actual readings and the possible



error are not known. The interrupted curve is a curve $R = AV^4$ sketched in for comparison

Table IV

V	R	V	R
10	1·5	30	102
14	4·2	34	142
18	15	38	171
22	44	42	185
26	62	46	195

§ 5 *Comparison with Experimental Results*

Before examining further model curves we must express the previous formula in a form more suitable for calculation, we use the system of units in which model results are now generally expressed. R is given in lbs per ton displacement of the ship, while instead of the speed V we use the ratio $V/\sqrt{L}$, V being in knots and L in feet; this is called the speed-length ratio, and we shall denote it by c. The advantage of these units is that they utilise Froude's law of comparison, from the experimental curve between R and c we can write down at once the residuary resistance for a ship of any length and displacement at the corresponding velocity, provided the ship has the same lines and form as the model. Thus the constants which are left in the relation between R and c depend only upon the lines of the model, not upon its absolute size. At present we make no attempt to connect these constants with the form of the model, as expressed by the usual coefficients

of fineness or the curve of sectional areas, or in other ways, we are concerned with the form of R as a function of c, and the constants are chosen in each case to make the best fit possible

First, as regards the exponential factor, we had $e^{-\frac{1}{2}(V'/V)^2}$, with V′ giving a point of inflection on the resistance curve, in the case of Ship A we had $V' = 26$, $L = 400$, so that $c' = 1\,3$ Now, it is just about this value of c that there is a falling off in most experimental curves, so that we try first $c' = 1\,3$ for the point of inflection on the R, c curve Then the exponential factor becomes $e^{-\frac{1}{2}\,c'/c)^2}$, or $c^{-2\,53/c^2}$

Secondly, as regards the cosine term which gives the undulations, we had $\cos(gl/v^2)$, we have decided to put $l = 0\,9L$, so that we have

$$\frac{gl}{v^2} = 0{\cdot}9gL\Big/\left(\frac{6080}{3600}V\right)^2 = \frac{10\,2}{c^2}, \text{ approximately}$$

Hence the previous relation for R reduces to the following general form:

$$R = \alpha e^{-2\,53/c^2} + \beta(1 - \gamma\cos 10\,2/c^2)\,e^{-2\,53/c^2}, \qquad (20)$$

where R is in lbs per ton displacement, and α, β, γ depend upon the form of the model

There are humps on the curve when $10\,2c^{-2}$ is an odd multiple of π, hollows when it is an even multiple, and mean values when it is an odd multiple of $\frac{1}{2}\pi$ For facilitating calculation, some of these positions are given in Table V, and, for the same reason, values of the exponentials and the cosine factor are given in Table VI

Table V

Humps	—	—	1 8	—	—	—	1·04	—	—	—	0 8		
Means	—	2 54	—	1·47	—	1 13	—	0·96	—	0 85	—	0 76	
Hollows	∞	—	—	—	1 27	—	—	—	0 9	—	—	—	0·73

Values of c

Table VI

c	$e^{-2\,53/c^2}$	$e^{-2\,53/c^2}$	$\cos(10\,2/c^2)$.
0 6	0·460	0·0009	+0 75
0 8	0·644	0·019	−0·97
1·0	0·756	0·080	−0 71
1 2	0 821	0 172	+0 70
1 4	0 866	0 275	+0 47
1·6	0 896	0 372	−0·65
1 8	0 916	0 458	−1·0
2 0	0 932	0 532	−0·88
2 2	0 943	0 592	−0·51
2 4	0·951	0·644	−0 20
3	0 970	0 756	+0 43

We examine, now, some examples of experimental curves, comparing them with the formula (20), several of the curves and other data, in particular for II, III, and V, have been taken from the collection in Prof Hovgaard's paper already referred to, in which he essays to fit formulæ involving V^4 or V^6 with the experimental curves.

I *R E Froude*, 1881, *Ship A*

Displacement = 4090 tons, length = 400 feet, cylindrical coefficient = 0 694

This is the case we have examined in the previous sections, so that we have only to change the numerical factors in (19) to cause R to be given in lbs. per ton displacement. We find the result is formula (20) with

$$\alpha = 2\,46, \quad \beta = 162\,6, \quad \gamma = 0\,12$$

II *W. Froude*, 1877

Displacement = 3804 tons, length = 340 feet, cylindrical coefficient = 0 787

The last two data include the cylindrical middle body The curve is given in fig 5, it was constructed by Hovgaard from the data of Froude's

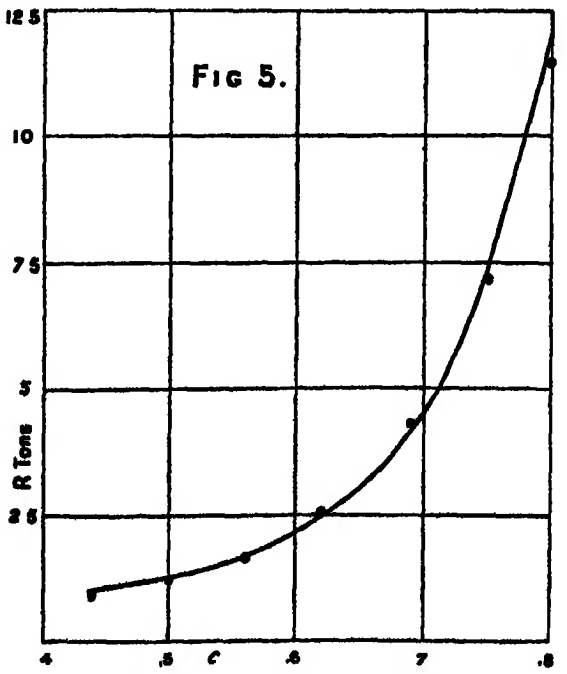


FIG 5.

experiments, and these were such that it was possible to make a mean residuary resistance curve, the effects of bow and stern interference being eliminated. The curve is given as total residuary resistance in tons on a base of V in knots If we work in lbs. per ton, we find there is a very fair agreement with formula (20) if we take

$$\alpha = 2\,24, \quad \beta = 279\,7; \quad \gamma = 0$$

Probably a closer agreement could be obtained by further slight adjustment of α and β Fig 5 shows a comparison of values of the total residuary resistance for the ship (in tons), the calculated values are indicated by small circles

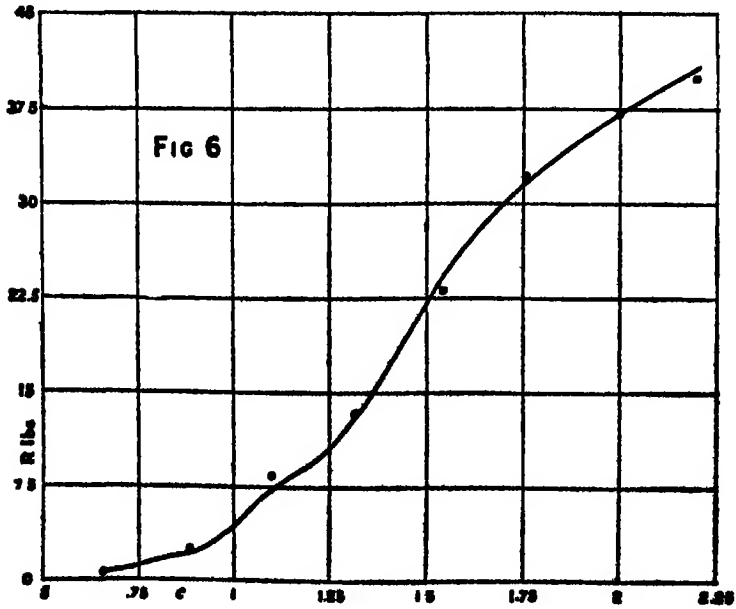
III *D W Taylor*, 1000 *lbs Model*

Length on water line = 20 51 feet, cyl. coeff = 0 680

The experimental curve in this case is given as residuary resistance for the model in lbs on a base of V in knots With the same notation as before we find

$$\alpha = 2, \quad \beta = 136\,6, \quad \gamma = 0{\cdot}14.$$

Putting these values in (20), we can calculate R in lbs per ton, and hence R_1 in lbs. for the model, fig 6 shows the comparison between R_1 and the corresponding values on the curve, the calculated values R_1 are indicated by dots



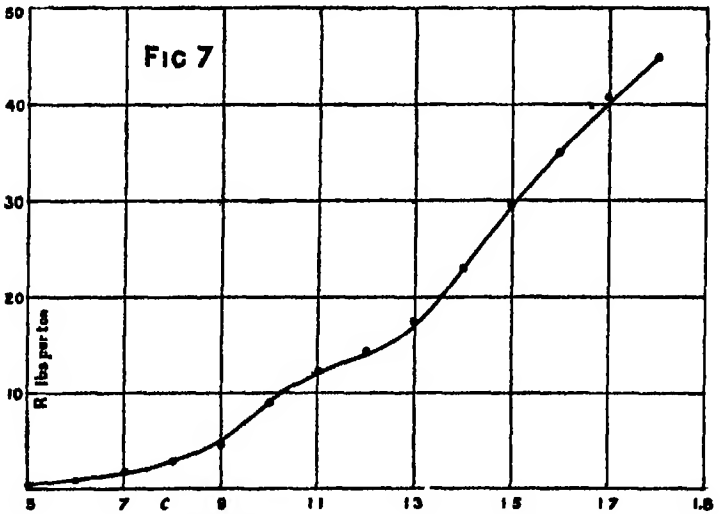
IV *D W Taylor, Model No* 892.*

Displacement = 500 lbs., length on water line = 20·512 feet, longitudinal coeff. = 0 68, midship section coeff = 0 70.

In this case the experimental curve is given as lbs. per ton displacement (R′) on a base of speed-length ratio (c) In the same manner as before, fig 7 shows the comparison with the formula (20) when we take

$$\alpha = 2, \quad \beta = 82\,5, \quad \gamma = 0\,14$$

Since the constant α is small compared with β, one is not able to lay much stress on the meaning of the first term For as the velocity functions



are of a suitable type, the constants possess considerable elasticity as regards fitting an experimental curve For instance, if we omit values of c below about 0 9, it is possible to represent the previous curves fairly well by a formula

$$R = \beta\,\{1-\gamma\cos(10\,2/c^2)\}\,e^{-\frac{1}{2}(c'/c)^2}$$

In the previous examples we took the value 1 3 for c' In Case IV above we find now the values

$$\beta = 87, \quad \gamma = 0\,14, \quad c' = 1\,3$$

For a similar curve taken from the same paper, viz, Model No 891, displacement 1000 lbs, we find a good correspondence, except for slightly higher values near $c = 1\,1$, with the values

$$\beta = 174, \quad \gamma = 0\,14, \quad c' = 1\,4$$

* D W Taylor, Society of Naval Architects, New York, November 19, 1908

V. *I. I. Yates, Destroyer Model C* *

Displacement = 575 lbs , length = 20 feet; cyl. coeff = 0·529.

The experimental curve is given in lbs. for the model on a base of V in knots, and is a total resistance curve, that is, it includes the frictional resistance. The curve is reproduced in fig 8.

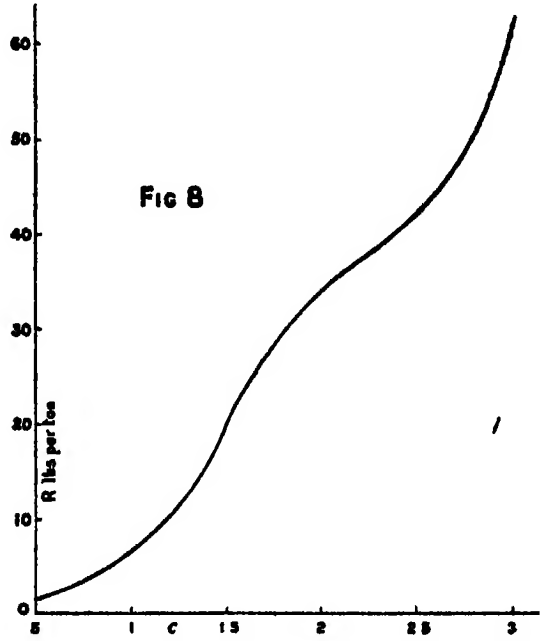


FIG 8

This curve is not analysed here so as to compare the residuary resistance with the formula (20), but it is included in order to draw attention to certain possible complications It may be noticed that the curve is carried to a high value of the speed-length ratio c, and that it continues to rise more rapidly after about $c = 2\cdot3$ than might be expected on the present theory Now in the first place it is possible that the frictional resistance may account partly for this rise The ordinary estimation of the frictional resistance assumes that it can be calculated separately from some expression like $fSV^{1\cdot85}$, now the legitimacy of this is beyond doubt in all ordinary cases, but at high speeds it is possible that the form of the expression may change, or even that it may not be a fair simplification to divide the total resistance into simple additive components

In the second place a more important consideration must be taken into account, and that is the depth of the tank. For the experiments now under

* I I. Yates, Thesis, 1907, Mass. Inst. Tech U.S A. See Hovgaard, *loc. cit. ante*

consideration the depth of water in the tank is not known. The deepest experimental tank appears to be the U S Government tank at Washington, which has a maximum depth of about 14 7 feet Now in that tank, with a 20-foot model, there would be a "critical" condition near the value $c = 2\,9$, before and up to that point the residuary resistance curve would rise sharply and abnormally. This effect is discussed more fully in the next section, and curves are given in fig. 11, with which fig 8 may be compared It appears, then, as far as one is able to judge, that it is possible the resistance curve in fig 8 is complicated by the effect of finite depth of the tank

§ 6 *The Effect of Shallow Water.*

We saw in the first section that the wave-making resistance R can be written in the form

$$R = \tfrac{1}{2}wa^2(v-u)/v,$$

where u is the group-velocity corresponding to wave-velocity v For deep water $u = \frac{1}{2}v$, and the formulæ are comparatively simple But for water of finite depth h the relation between u and v depends upon the wave-length $(2\pi/\kappa)$. We have

$$v = \sqrt{\left(\frac{g}{\kappa}\tanh \kappa h\right)},$$

$$u = \frac{d}{d\kappa}(\kappa v) = \tfrac{1}{2}v\,(1+2\kappa h/\sinh 2\kappa h)$$

Consequently we find

$$R = \tfrac{1}{4}wa^2\left(1-\frac{2\kappa h}{\sinh 2\kappa h}\right) \qquad (21)$$

As v increases from zero to $\sqrt{(gh)}$, R diminishes from $\frac{1}{4}wa^2$ to 0, provided the amplitude remains constant But, as Prof Lamb remarks,* the amplitude due to a disturbance of given character will also vary with the velocity. It is the variation of this factor that we have to examine in the manner used in the previous sections for deep water.

If a symmetrical line-pressure system $F(x)$, suitable for Fourier analysis, is moving uniformly with velocity v over the surface of water, the surface disturbance η is given by

$$\pi w\eta = \tfrac{1}{2}\int_0^\infty dt \int_0^\infty \kappa V\phi(\kappa)\sin\kappa\{x+(v-V)t\}\,d\kappa$$

$$-\tfrac{1}{2}\int_0^\infty dt \int_0^\infty \kappa V\phi(\kappa)\sin\kappa\{x+(v+V)t\}\,d\kappa, \qquad (22)$$

where $\phi(\kappa) = \int_{-\infty}^{\infty} F(\omega)\cos\kappa\omega\,d\omega.$

* H. Lamb, 'Hydrodynamics,' p. 391, 1906.

The method of evaluating these integrals approximately so as to give the regular wave-trains has been discussed in a previous paper and it is followed now in the case of finite depth.* We take, under certain limitations, the value of an integral such as

$$y = \int \phi(u) \sin\{g(u)\}\, du$$

to be the value of its principal group, viz.,

$$y_0 = \left\{\frac{2\pi}{g''(u_0)}\right\}^{\frac{1}{2}} \phi(u_0) \cos\{g(u_0) - \tfrac{1}{4}\pi\}, \qquad (22A)$$

where u_0 is such that $g'(u_0) = 0$.

Now in the integrals in (22) we have to find successively two principal groups, first with regard to κ and then in the variable t, and thus we may evaluate the amplitude factor in the resulting regular wave-trains

For water of depth h we may write

$$f(\kappa) = v - V = v - \sqrt{\left(\frac{g}{\kappa}\tanh \kappa h\right)}$$

The group with respect to κ gives a term proportional to

$$\cos\{t\kappa^2 f'(\kappa) + \tfrac{1}{4}\pi\},$$

where κ has the value given by

$$f(\kappa) + \kappa f'(\kappa) = -\frac{x}{t} \qquad (23)$$

From (22A), this introduces into the amplitude a factor

$$1/\sqrt{[t\{2f'(\kappa) + \kappa f''(\kappa)\}]} \qquad (24)$$

Further, the group with respect to t occurs for

$$\frac{d}{dt}\{t\kappa^2 f'(\kappa)\} = 0 \quad \text{or} \quad f(\kappa) = 0$$

Also we have in these circumstances

$$\frac{d^2}{dt^2}\{t\kappa^2 f'(\kappa)\} = \frac{d}{dt}\left\{\kappa^2 f'(\kappa) + \frac{\kappa x}{t}\right\} = \frac{d}{dt}\{-\kappa f(\kappa)\}$$

$$= -\frac{x}{t^2}\frac{f + \kappa f'}{2f' + \kappa f''} = \frac{1}{t}\frac{(f + \kappa f')^2}{2f' + \kappa f''} = \frac{(\kappa f')^2}{t(2f' + \kappa f'')}. \qquad (25)$$

Hence from (22A), (24), and (25) the selection of the two groups adds to the amplitude a factor $1/\kappa f'(\kappa)$, where

$$f(\kappa) = 0 = v - \sqrt{\left(\frac{g}{\kappa}\tanh \kappa h\right)}$$

* Havelock, 'Roy Soc Proc,' A, vol 81, p 411, 1908.

Also if u is the group-velocity for wave-length $2\pi/\kappa$ and wave-velocity V, we have, in this case,

$$u = \frac{d}{d\kappa}(\kappa V) = \frac{d}{d\kappa}\{\kappa v - \kappa f(\kappa)\} = v - \{f(\kappa) + \kappa f'(\kappa)\}$$

Hence, since in the final value $f(\kappa) = 0$, we have $\kappa f'(\kappa)$ equal to $v-u$ Thus if κ is the wave-length of the regular wave-trains in the rear of the disturbance, we find that they are given by

$$\eta = \text{const} \times \frac{\kappa v \phi(\kappa)}{v-u} \sin \kappa x, \tag{26}$$

where $$v = \sqrt{\left(\frac{g}{\kappa} \tanh \kappa h\right)}, \qquad u = \tfrac{1}{2} v \left(1 + \frac{2\kappa h}{\sinh 2\kappa h}\right)$$

Hence for the amplitude a we have

$$a = C\kappa\phi(\kappa) \Big/ \left(1 - \frac{2\kappa h}{\sinh 2\kappa h}\right)$$

Substituting now in (21) we obtain for the wave-making resistance, R proportional to

$$\kappa^3 \{\phi(\kappa)\}^2 \Big/ \left(1 - \frac{2\kappa h}{\sinh 2\kappa h}\right)$$

If we take the same distribution of pressure in the travelling disturbance, namely, $F(x) = P\alpha/\pi(\alpha^2 + x^2)$, we have $\phi(\kappa) = Pe^{-\alpha\kappa}$, further, we may again assume that the pressure P varies as v^4, so that we have the resistance in the form

$$R = A\kappa^3 v^4 e^{-2\alpha\kappa} \Big/ \left(1 - \frac{2\kappa h}{\sinh 2\kappa h}\right),$$

with $$\frac{\tanh \kappa h}{\kappa h} = \frac{v^2}{gh} \tag{27}$$

Considering R given as a function of v by these two equations, we see that R increases slowly at first and then rapidly up to a limiting value at the critical velocity $\sqrt{(gh)}$, after this point R is zero, for there is no value of κ satisfying the second equation with $v^2/gh > 1$

Further, the limiting value of R at the critical velocity is finite, for we have

$$\operatorname*{Lim}_{\kappa = 0} \frac{\kappa^3 h^3}{(1 - 2\kappa h/\sinh 2\kappa h)} = 1{\cdot}5.$$

We see that the R, v curve given by (27) is of the type sketched in fig 9 We may compare this with some of the curves given by Scott Russell for canal boats. The continuous curve in fig 10 is an experimental curve of

total resistance,* and the dotted curve is a parabolic curve inserted here to represent approximately the frictional resistance, the difference between the two curves represents the residuary resistance, and is clearly of the same type as the theoretical curve in fig. 9

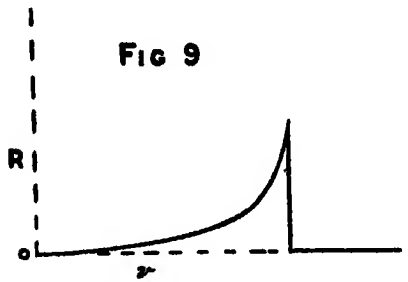


FIG 9

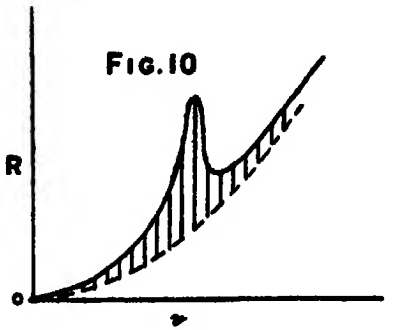


FIG. 10

We can obtain a better estimate of equation (27) by taking an experimental curve for a model in deep water, and then building up curves for different depths We must first put (27) into a form suitable for comparison with deep water results

Limiting the problem to one of transverse waves only, the formula (27) must reduce to $R = Ae^{-2\cdot53/c^2}$, for h infinite and $c =$ (speed in knots)/$\sqrt{}$(length in feet)

Writing v' for $v/\sqrt{(gh)}$ we find $c^2 = 11\cdot3v'^2h/L$; thus although the actual critical velocity does not depend upon the length of the ship but only on the depth of water, the speed-length ratio (c) has a critical value which is proportional to the square root of the ratio (depth of water)/(length of ship)

In (27) we cannot fix any value of v or c and then calculate R directly, we must work through the intermediate variable κh The equations may now be written as

$$R = A(\kappa h)^2 v'^4 e^{-\beta' \kappa h}/(1-2\kappa h/\sinh 2\kappa h), \qquad (28)$$

$$v'^2 = (\tanh \kappa h)/\kappa h, \quad \beta' = 0\cdot218L/h, \quad c^2 = 11\cdot3v'^2h/L.$$

With h infinite this reduces to the previous form for deep water with the same constant A, so that a direct comparison is possible. As the velocity v increases from 0 to $\sqrt{(gh)}$, κ diminishes from ∞ to 0, we select certain values of κh, calculate the values from tables of hyperbolic functions, and thus obtain the set of values in Table VII, writing m for

$$(\kappa h)^2 v'^4/(1-2\kappa h/\sinh 2\kappa h)$$

* J. Scott Russell, 'Edin. Phil. Trans.,' vol 14, p. 48, 1840.

Table VII

κh.	$v/\sqrt{(gh)}$	c^2L/h	m	$-\beta\kappa c^2$
∞	0	0	1 0	2 53
10	0 816	1 13	1·0	2 53
6	0 41	1 87	1 0	2 53
4	0 5	2 82	1 005	2 53
2	0 69	5 42	1·077	2 43
1	0 87	8 57	1 287	1 92
0	1 0	11 3	1 5	0

We consider now the experimental curve analysed in Case IV in the previous section, a model of 20 5 feet taken up to a value $c = 1\,8$ Assuming that the influence of finite depth was inappreciable in this range, we have for deep water

$$R = 2e^{-2\,53/c^2} + 82\,5\,\{1 - 0\,14\cos(10\,2/c^2)\}\,e^{-2\,53/c^2} \qquad (29)$$

We leave out of consideration at present the first term, which is supposed to represent the diverging waves, and we extend the calculations for R (transverse) from the rest of the formula up to $C = 3{\cdot}3$ taken at intervals of 0 1 for C, we obtain thus the lowest curve given in fig 11. With the help of Table VII, we calculate values of R for depths of about 5, 10, 12, 15, and 20 feet, taking in the formula (28) A equal to

$$82{\cdot}5\,\{1 - 0{\cdot}14\cos(10\,2/c^2)\}$$

so that the results apply to the same model at different depths An example of the calculations for one case may be sufficient, Table VIII shows the intermediate steps for $h = 12\,3$ ft., $L = 20\,5$.

Table VIII

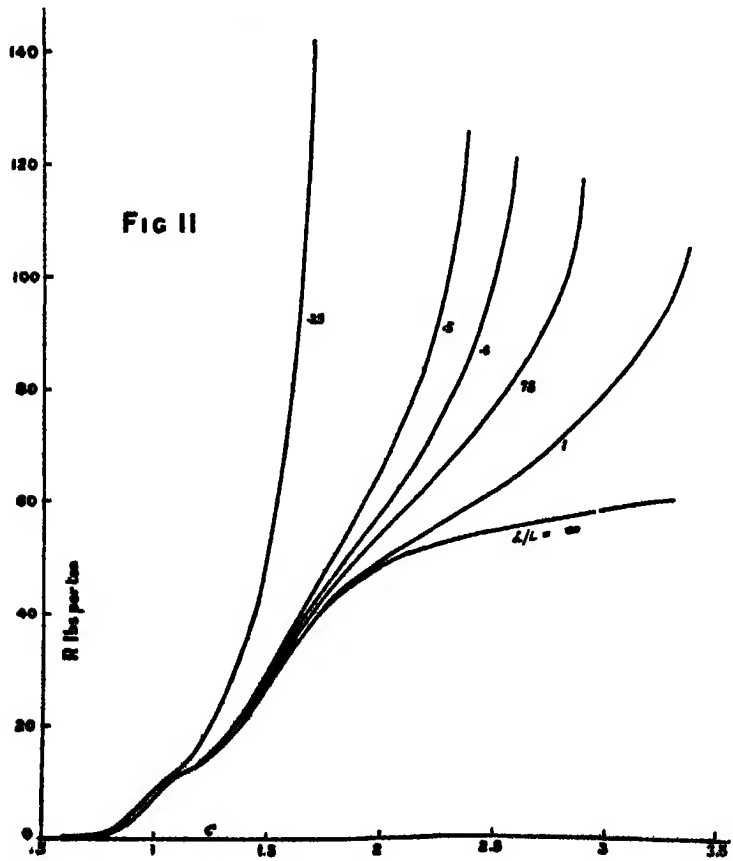
c^2	c	$-\beta\kappa$	R/A.	$e^{-2\,53/c^2}$
0 68	0 825	3 73	0 024	0 024
1 12	1·06	2 26	0 106	0 106
1 69	1 3	1 5	0·224	0 223
3 25	1 8	0 75	0 506	0 472
5·14	2 27	0 374	0 885	0·687
6 8	2·61	0	1 5	1

The results for the five values of h are given in Table IX, and from these the curves in fig 11 have been drawn.

The general character of the effect of finite depth is clear on inspection of the set of curves in fig 11. If it is required to go to high values of the speed-length ratio in a given tank, the ratio of the depth of water to the length of the model must be adjusted so that there is no appreciable effect in

Table IX

λ/L							
1	c	0 7	1 0	1·5	1·7	2 33	3
	R	0·7	7 5	27 5	39 2	59 2	80 5
0 75	c	0 7	1·0	1 2	1 4	2	2 54
	R	0 7	7 5	18	21 4	54 5	79 2
0 6	c	0 7	1·0	1 3	1 8	2 3	2·6
	R	0·7	7 5	17 9	47 7	78	122
0 5	c	0·7	1·0	1 2	1 65	2 1	2 38
	R	0 7	7 5	13 1	40	74	127
0·25	c	0·7	0 8	0 84	1 16	1 46	1 68
	R	0 7	1 7	3 3	14 1	43·6	142 5



the range of the experiments Since the curves given here are theoretical curves for transverse waves only, each of them ends abruptly at the critical

velocity—the resistance being zero after that point. In practice, we know that there are no such discontinuities in the resistance curves, and there are certain considerations which go to account for this difference First, as regards the transverse waves alone, the preceding formulæ show that the amplitude tends to become infinite at the critical velocity, although the corresponding resistance at uniform velocity remains finite, but, even apart from the effects of viscosity, there is a highest possible wave with a velocity depending partly upon the amplitude Secondly, we have left out of consideration the diverging waves, but these must become more important in the neighbourhood of the critical velocity, for we may regard the two systems as coalescing into one solitary wave in the limit as the critical velocity is reached After this point the diverging waves persist, so that the effect of these would be of the order of halving the drop in the resistance as the critical velocity is passed

Finally, we must consider the frictional resistance, which increases steadily with the velocity, so that the fall is finally a smaller percentage of the total resistance than might appear at first The curves given in fig 11 give an estimate of a maximum effect of this kind, considering only the transverse wave system

§7 *Further Types of Pressure Distribution*

The preceding formulæ have been built up on the effect of a travelling pressure disturbance of simple type, we consider now another type which we may use as an illustration

Let the pressure system be given by

$$p = f(x) = A(h^2-x^2)/(x^2+h^2)^2$$

The type of distribution is graphed in fig 12

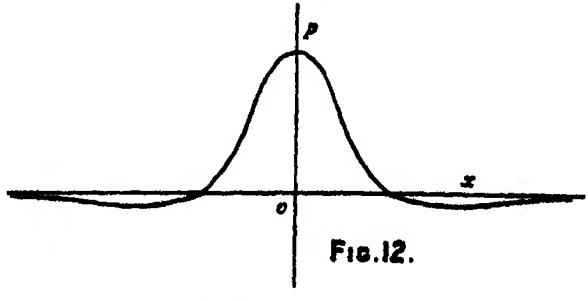


Fig. 12.

Proceeding as in §2, we have

$$\phi(\kappa) = 2A\int_0^\infty \frac{h^2-\omega^2}{(\omega^2+h^2)^2}\cos\kappa\omega\, d\omega = \pi A\kappa e^{-\kappa h}. \qquad (30)$$

Hence the amplitude of the regular wave-trains formed on deep water in the rear of this disturbance is proportional to $\kappa^2 A e^{-\kappa h}$, and the effective wave-making resistance is proportional to $\kappa^4 A^2 e^{-2\kappa h}$. We make the same assumption as before, viz., A proportional to v^2, and write $\kappa = g/v^2$, then the resistance is given by

$$R = C v^{-4} e^{-2gh/v^2}. \qquad (31)$$

We use this expression to show how R varies with the constant h of the pressure system. Let $v = 10$ ft/sec, and let $R = 1$ for $h = 0$, then we find the following relative values:

h	R
0	1·0
1	0·52
5	0·04
10	0·0016

R decreases very rapidly as h is increased. We have chosen this example for the following reason. Consider the motion of a thin infinite cylinder in an infinite perfect fluid; if we consider a plane parallel to the direction of motion and to the cylinder and at a distance h from it, we find that the distribution of excess or defect of pressure due to the motion is of the above type. Now, this is not the same as a cylinder moving in deep water at a depth h below the free surface, but it is suggested that as a first approximation the wave-forming effect is that of an equivalent diffused pressure system. The illustration shows how rapidly the wave-making resistance diminishes with the amount of diffusion, that is, with the depth h, this, of course, agrees with the experiments on the resistance to motion of submerged bodies, and, in fact, with the resistance of submarine vessels.

In the preceding work no attempt has been made to connect theoretically the constants in the pressure formula with those of the model; since the theory rests chiefly on the consideration of transverse waves only, this would presumably bring into question the length of entrance, run, and so forth. The consideration of any "transverse" constants, such as the beam, would need a fuller treatment of a diffused pressure system in two dimensions on the surface so as to give a more detailed investigation of both transverse and diverging wave systems.

Liberation of Helium from Radio-active Minerals by Grinding.

By J A Gray, B Sc (Melbourne)

(Communicated by the Hon R J Strutt, F.R S Received March 10,—
Read March 25, 1909)

It is almost certain that the helium in radio-active minerals is mechanically retained rather than chemically combined The question then arises, what is the size of the structure which retains each helium atom? Is this structure of molecular dimensions, or is it much coarser? The present investigation was undertaken in the hope of obtaining some information on this point.

Moss* has found that traces of helium can be liberated from pitchblende by grinding, and it seemed likely that, by pushing the process much further, a large percentage, if not the whole, of the helium present might be liberated If so, it would be legitimate to conclude that the structure in which the helium atoms were entangled was of the same order of magnitude as the particles of powder The experiments were carried out on thorianite from Ceylon, which has the advantage of containing a very large amount of helium To determine how much helium had been liberated by grinding, the amount of helium per gramme was found for coarse mineral, *i.e.* mineral containing the normal amount, and also for the ground mineral, the difference giving the amount of helium liberated

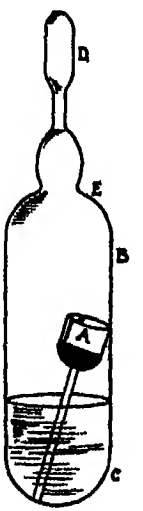


Fig. 1

To estimate the amount of helium per gramme of a certain specimen of mineral, a weighed quantity was placed in a small glass tube A (fig 1), with a glass rod joined to the closed end. This was placed in a large tube BC, in the bottom of which was some nitric acid. The glass rod was long enough to prevent the acid touching the mineral. A piece of glass tubing DE, $\frac{1}{4}$ inch bore, was joined to this larger tube and constricted at one part so that the tube could be easily sealed The tube was then placed in connection with a Topler pump by means of stout rubber tubing, which was fixed to the glass by copper wire to prevent air leaking into the pump The tube was evacuated and sealed off. Mineral and acid were well shaken together and the tube placed in a heating bath. Here the acid attacked the mineral and the contained helium was liberated.

The heating bath consisted of one box of asbestos, $\frac{3}{4}$ inch thick, inside

* 'Roy. Dublin Soc. Proc.,' vol. 8, Series 2, Part 12.

another box In the inner box were two electric lamps, by means of which a temperature of 100° C could be maintained At this temperature two and a-half hours were sufficient to complete the action. A small insoluble residue was left, and during the first experiments the tubes were placed in the bath again to see if the residue contained any helium, but with negative results. After the mineral had been digested, a nick was made with a file at the top of the tube, which was again placed in connection with the pump The air was drawn off from the pump and rubber tubing, and the glass tube was then broken at the nick, under the rubber The gases contained in the tube were drawn off and collected On their way through the pump they passed through a drying tube containing small pieces of potash, which also acted on any acid gases coming over The collected gases were mixed with oxygen and sparked in a eudiometer containing a little solid potash. The mixture was sparked continuously until all contraction had long ceased The remaining oxygen was burnt off by means of phosphorus, and the helium which remained measured in a tube divided to tenths of a cubic centimetre The measured volumes were corrected for temperature and pressure

In the mineral itself, helium is practically the only rare gas The only impurity of any consequence would be argon coming from air which might get into the collected gases, in the first place by not drawing off all the air the first time the tube was in connection with the pump, and again by air leaking into the pump when the helium and other gases are being collected In one experiment, 3 5 c c of gas were obtained at atmospheric temperature and pressure After sparking, this was reduced to 2 9 c c Thus there could be little gas other than helium given off from the mineral, and with care very little air could get in If an appreciable amount did get in, a correction could be made The gas remaining after the oxygen had been burnt off was usually mixed with oxygen again, and sparked to ensure that the sparking action had been complete.

To obtain the mineral in a finely divided state, the thorianite, which was in the form of small crystals, was first pounded in an iron mortar and the finer pounded mineral passed through a sieve of 0 3 mm mesh This finer mineral was well ground in small portions in an agate mortar and placed in a porcelain basin, where water was gently poured on it. The basin was shaken and the water made to rotate. The fine mineral that rose in the water was poured with the water into another basin, where it was allowed to settle, after which the water was poured off and the mineral dried over the heating bath, where the temperature was about 35° C. By this means, mineral ranging in size from 1 μ (or 0·001 mm) to 9 μ was obtained Experi-

ments on this and the mineral that would not pass through the sieve showed that 11 per cent of the helium had been liberated, the normal amount of helium being 8·9 c.c per gramme, that in the fine material being 7 9 c c per gramme at 0° C and 760 mm pressure To get the division finer, a second washing was made from the mineral containing 7 9 c c of helium per gramme, but a reduction of only 5 per cent. was effected This process could have been carried further, but was too uncertain and clumsy, and was discarded for the following method

Forty or fifty grammes of the thorianite were ground in small portions in the agate mortar This ground mineral was placed in a bottle and mixed with water The water and mineral were well shaken together and the mineral allowed to settle When it had been settling a definite time, 4 or 5 cm of the top layers of the water were syphoned off with the aid of a water pump Care was taken not to take off the mineral which floated on the surface of the water and was probably greasy The water syphoned off was transferred to a basin, where the suspended mineral was allowed to settle, the water then being poured off When enough mineral had been collected, it was dried and experimented on In this way mineral was obtained in which a further liberation of helium had been effected In the different experiments, the time of suspension varied from 25 minutes to 1 hour A microscopic examination was made of different samples of mineral, and by means of a camera lucida, drawings were made of magnified images of the particles under the object-glass By drawing the image of 1/100th mm scale, the size of the particles could be estimated All the above-mentioned experiments and some confirmatory ones are collected in the following table (see p 304)

From this table we can see that helium is liberated by grinding, and that the finer the mineral is ground the more helium is liberated However, experiments 12 to 16 show that there is a temporary limit to this liberation. Special care was taken with experiments 14 and 16 to get accurate results The other experiments are on rather too small quantities to be very dependable, but they serve to confirm experiments 14 to 16 A comparison of the experiments will be aided by the following diagrams, which are reproductions of those taken by the camera lucida referred to above They are for specimens of mineral containing respectively 7·9 c c per gramme (fig 2), 6 6 c c per gramme (25 minutes in suspension) (fig 3), 6 4 c c per gramme (one hour in suspension) (fig 4)

Comparing fig. 2 and fig. 3, the reason why the mineral represented by fig 2 contains more helium is probably because the larger particles of diameter 6 to 9 μ have lost very little helium. This would explain results

Table showing the Liberation of Helium by Grinding

Experiment.	Quantity experimented on in grammes.	Method of obtaining it	State of division of mineral. Size of particles	Helium contained (corrected volume)	Helium in c.c. per gramme	Per cent. liberation of helium.
				c.c		Per cent.
1	0·593	Mineral that would not pass through sieve	7·3 mm	5·32	9·0	
2	0·371	" " "	7·3 "	3·28	8·8	
3	0·468	" " "	7·3 "	4·16	8·9	
4	0·312	" " "	7·3 "	2·78	8·9	
5	0·31	Coarsest of same mineral, ground in agate mortar, the finer mineral having been completely washed off	0·01 to 0·04 mm	2·79	9·0	
6	0·584	Mineral well ground in agate mortar	1 to 20 or 30 μ	4·98	8·5	5
7	0·300	" "	—	2·57	8·6	
8	0·530	Obtained by washing from mineral ground in an agate mortar	1 to 9 μ	4·22	8·0	11
9	0·396	" " "	1 to 9 μ	3·10	7·8	
10	0·392	Washed from fine mineral as used in Experiments 8 and 9	1 to 8 μ	2·94	7·5	16
11	0·361	Ground with water between two flat agate surfaces from mineral as used in Experiment 6	1 to 8 μ	2·79	7·7	14
12	0·109	In suspension in 4 or 5 cm. of water for about 30 minutes	1 to 3 μ	0·70	6·4	
13	0·094	" " "	1 to 3 μ	0·60	6·4	
14	0·313	In suspension 25 minutes	1 to 4 μ	2·07	6·6	26
15	0·111	" 1 hour	1 to 2 μ	0·69	6·2	
16	0·259	" "	1 to 2 μ	1·67	6·4	28

NOTE.—Experiments 4, 7, 11 prove that the liberation of helium is due to the small size to which the mineral has been reduced and not to separation by the processes involved of mineral which initially contains less than the normal amount of helium.

Again, this liberation of helium must not be confused with the very small but prolonged escape of helium from radio-active minerals as studied by Prof Strutt ('Roy Soc. Proc.,' A, vol. 82, p 166, 1909) It is of quite a different order of magnitude and occurs as soon as the mineral is ground.

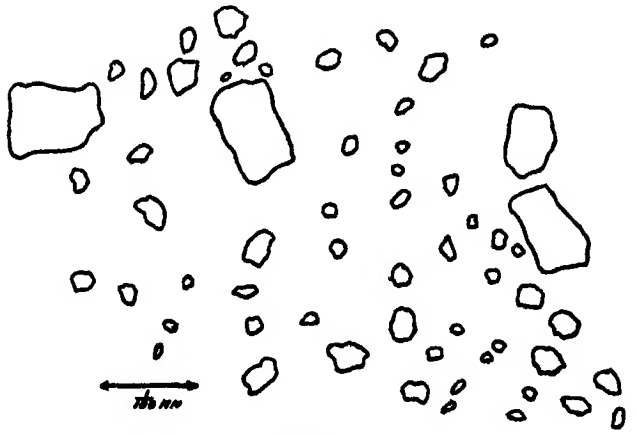


FIG 2

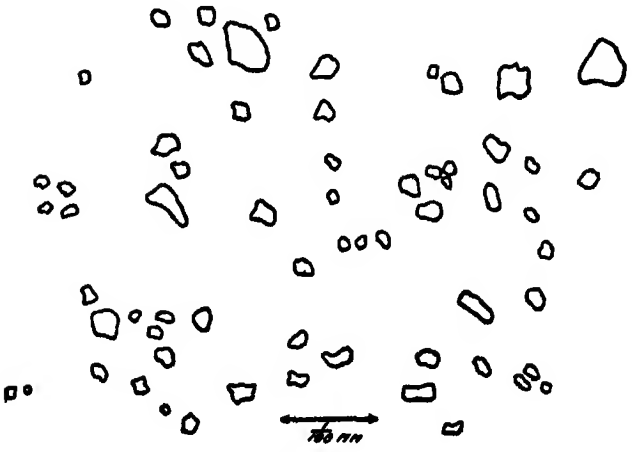


FIG 3

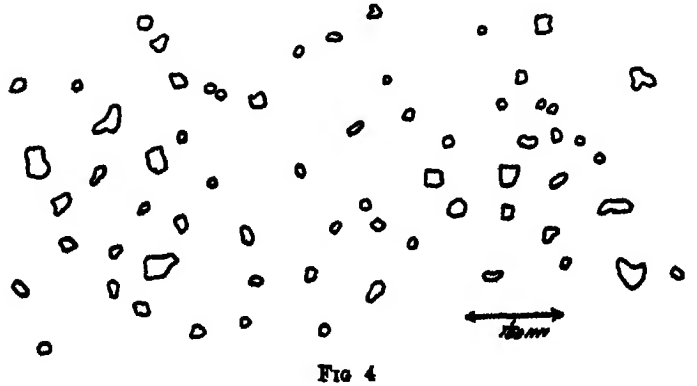


FIG 4

6 to 11, but when we come to compare figs 3 and 4, we see that, although fig 4 represents much the finer mineral, as is obvious also from the times of suspension, there is very little difference in the helium content We could change from fig. 3 to fig 4 by replacing the larger particles by particles of diameter 1 and 2 μ In other words, mineral, the particles of which were 3 and 4 μ in diameter, would not contain any more helium than particles 1 and 2 μ in diameter It seems, therefore, that as we come to particles, the diameter of which is in the neighbourhood of 3 μ, there is a temporary limit to the liberation of helium, and that 70 per cent of the helium is contained very firmly in the mineral The smallest of the ground particles is a little less than 1 μ in diameter, and it is impossible to say how much finer one would have to obtain the mineral for much of the remaining helium to be liberated Experiment 5 shows that, when the size of the particles is greater than 0 01 mm, scarcely any helium is liberated It is hard to say at what size the liberation of helium begins, but it is probably in the neighbourhood of 5 μ In any case it is clear that part at least of the helium in thorianite is retained in a structure which is very large compared with the molecular structure The experiments on which this conclusion is based may be summarised as follows —

1 Helium is liberated from thorianite, and a liberation of 28 per cent has been effected

2 The smaller the mineral is ground the more helium is liberated

3 This liberation has a temporary limit when the mineral is reduced to a size of about 3 μ

4 It is impossible to say how the remaining 72 per cent of helium is contained in the mineral and to how much finer than 1 μ the mineral would have to be reduced to liberate this helium

In conclusion, I wish to record my thanks to Prof Strutt, who suggested this research, for his great kindness and help throughout the work.

An Apparatus for Measurements of the Defining Power of Objectives.

By J DE GRAAFF HUNTER, M A., late Assistant at the National Physical Laboratory.

(Communicated by R. T Glazebrook, F R.S Received October 6, 1908,—Read February 18, 1909. Received in revised form April 5, 1909)

(From the National Physical Laboratory)

The apparatus here described was devised at the National Physical Laboratory for the purpose of obtaining a numerical estimate of the goodness of definition in the image given by an optical system. It has been employed also for determining the distribution of intensity, as affected by diffraction, in the images obtained through a small circular aperture, giving results in complete agreement with the usual theory.

As an object, the image of which is to be examined, is employed an "edge," forming the boundary line between a half bright, half dark, field The distribution of intensity along a line at right angles to the edge may be represented by the broken line ABEF in fig 1, ordinates representing

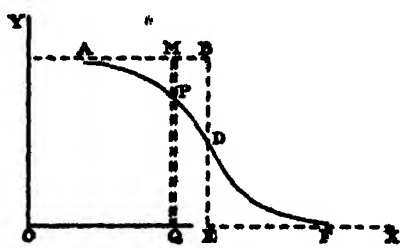


FIG 1

intensity, and abscissæ, measured from E, distances from the edge In the image, the intensity distribution will be given by some such curve as ADF, the light gradually falling off from full intensity at a point outside to zero at some point inside the geometrical image of the edge The object aimed at is to obtain a series of points on, and hence to draw, this intensity curve. The whole width we are concerned with in this blurred image of the edge is extremely small, and for the measurements magnification is necessary.

The optical arrangement which comes into consideration in what follows is shown diagrammatically in fig. 2. The "edge" is placed at the focus of a collimator, so that it may be regarded as virtually at an infinite distance from

the optical system, say a photographic lens, to be examined, the image formed by such lens is viewed by a microscope. In the focal plane of the

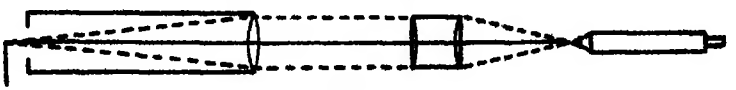


Fig 2

eyepiece of the microscope (Ramsden) is placed a fine slit, parallel to the "edge," through which alone the light passes. The slit is traversed across the field by a micrometer screw

The edge which serves as object is cut on the semicircumference of a metal disc (fig 3A). Only a very small portion of the edge is actually seen in the

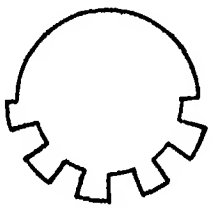


Fig 3A.

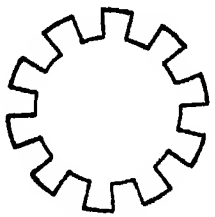


Fig 3B.

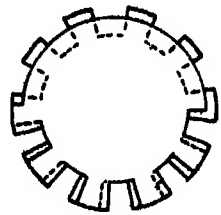


Fig 3C

image, and this is straight to the order of accuracy necessary Over the other half of the circumference are sectors, cut to such a depth that the inner radius is less than the radius of the edge The edge, at the part viewed, is uniformly illuminated from behind

Consider what is seen if this disc is rotated Over half the rotation there appears in the microscope field the blurred image of the edge, over the other half there is uniform illumination in the field, reduced only by 50 per cent. owing to the interposition of the sectors In general, at an appropriate speed of rotation, there will be a flicker as we pass from one half of the circumference to the other.

If, now, a second disc, shown in fig 3B, be placed behind the first, as in fig. 3C, the means of making a measurement is obtained If the two discs be rotated together, over the one half of the rotation is seen the blurred image of the edge, the illumination being everywhere reduced by one-half owing to the sectors, over the other half of the rotation there is uniform illumination, which, however, can be changed, in a measurable proportion, by adjustment of the second disc relative to the first. If the slit in the eyepiece be set in any definite position, MQ, parallel to the geometrical image of the edge (fig. 1), light of a definite intensity, PQ, will pass through it while the image of the edge travels across the microscope field, *i.e.* over one half of a rotation

of the discs, over the other half equal light intensity can be obtained by varying the position of the second disc Equality is determined by the disappearance of flicker

Actually, it is found more convenient to fix the relative position of the two discs, and to move the slit in the direction perpendicular to the edge The variation in the light intensity across the blurred image of the edge is very rapid except near the maximum or minimum intensity (see fig 1), and the position of the slit in which the flicker disappears can be determined to a high order of accuracy The position of the point Q corresponding to a sector setting giving a certain light intensity PQ can be found to within 2μ over the greater part of the intensity curve

It was found desirable to arrange that settings of the sectors, in a convenient series of positions, could be made automatically The positions chosen were those giving intensities 0, 1/6, 2/6, 3/6, 4/6, 5/6, and 6/6 of full intensity The mechanism which seemed most suitable for making these settings was a crown wheel escapement By this means the sectors could be moved relatively to one another by a known amount, and the observer could rely on the sectors coming accurately to the proper position without having to take his eye away from the microscope To allow for positions intermediate to those stated, it is only necessary to give the pallets of the escapement an adjustment over a small angle (θ) In the actual case, by making $\theta = 3°$, any relative position whatever of the sectors can be obtained, and the 19 possible settings at intervals of 1° (corresponding to 19 points on the intensity curve) can all be made with only one intermediate change of θ

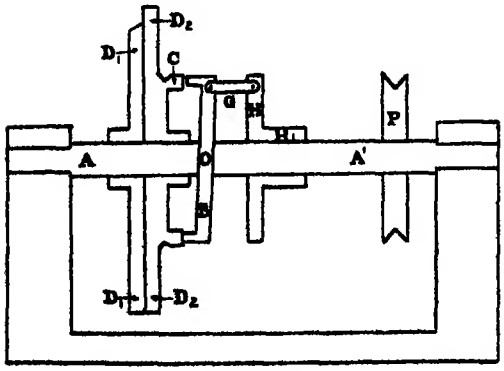


FIG. 4A.

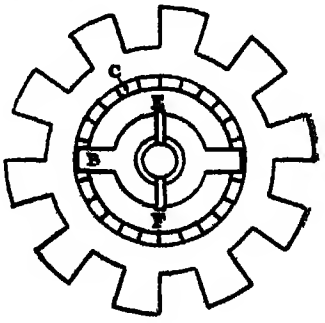


FIG 4B.

The second disc, bearing the crown wheel which forms the "scape wheel" of the escapement, is shown diagrammatically (fig. 4A section, 4B plan). A is the shaft, B an arm carrying the pallets of the escapement, i.e. the portions

which engage with the teeth of the crown wheel C. The arm B is free to rock about the axis EF on pins E, F screwed into the shaft A. To give an adjustment for the θ of the last paragraph, it is arranged that the pallets can swing on a sleeve on the arm B through an angle of about 5°. The arm B is connected by a short link G, with pin joints at each end, to the shouldered sleeve HH_1, which is free to slide a short distance along the main shaft AA'. In the portion marked H_1 the sleeve has a slot cut parallel to the length of the shaft AA'. This slot works over a pin fixed into the shaft, and so allows HH' a small motion along the shaft. This motion causes the arm B of the escapement to rock the requisite amount. It is given by means of pressure applied along two rollers, which run against the shoulder H, bearing at opposite ends of a diameter through AA'. The return motion is arranged for by means of suitably disposed springs. Bell crank levers are used to enable the observer at any distance (in this case upwards of 2 metres) to apply the pressure to the rollers. A brake string is placed in a V-groove cut at the back of the crown wheel C. The shaft is rotated by means of the pulley P, and this turns the first disc, and, through the escapement, the second disc also, against the brake string. When the escapement arm B is rocked over, the brake string holds the second disc back until the escapement again carries it on with the shaft. The tension of the brake string can be adjusted by means of a screw at one end and a spring at the other to the proper amount.

In examining the definition at points off the axis of the system, it is usually enough to find the definition or intensity curves for two azimuths, and the azimuths naturally chosen are those of the radial and transverse lines. On this account the whole apparatus is mounted on a stand free to turn about the proper axis (parallel to the shaft AA'), so that a simple rotation changes the portion of the first disc seen in the observing microscope from the radial line (when the top of the disc, tangent horizontal, is seen) to the transverse line (when the side of the disc, tangent vertical, is seen). A prop of suitable length is used to support one end of the stand, and by pushing this nearer to the axis of the stand the discs are raised to the proper position. It will be seen that this arrangement gives three-point support (the points being the prop and the two ends of the axis) to the stand, and also the proper horizontal and vertical motions, which exceed 5 cm., by one simple movement. Intermediate positions, necessary for definition curves of other azimuths, can also be given by combining a horizontal sliding of the whole arrangement with a rotation about the stand axis.

The apparatus has been used in conjunction with the Beck photographic lens-testing bench at the National Physical Laboratory. The lens for test is

held in the Beck bench so as to be free to turn about a vertical axis through its back nodal point, thus permitting the definition for oblique pencils to be readily examined.

The eyepiece of the microscope by which the image is examined is a Ramsden eyepiece of magnification ten In its front focal plane is a diaphragm in which is cut a diametral slit (as already explained) set tangentially to the image formed in the plane of the semicircular edge of the first disc Part of the diaphragm can be swung out of the way to enable a general view of the object to be obtained. This is necessary for setting up the object in the right position at the beginning, for the field of view through the slit—especially when a high-power microscope objective is used—is far too small to allow any part of the object to be recognised With the highest power used (1/6 inch) the image of the edge of the disc, even with a lens giving excellent definition, extended nearly across the entire microscope field The width of the slit in the eyepiece can be varied from 0 to 1 mm With a very steep definition curve it is desirable to work with as narrow a slit and as high a power as possible The whole eyepiece, with diaphragm and slit, can be turned so as to set the slit tangential to the image of the edge of the first disc

To read the position of any setting of the eyepiece, a scale consisting of fine slits cut in a diaphragm is fixed to the main tube of the microscope and coplanar with the eyepiece diaphragm, *i.e.* in the focal plane of the eyepiece, and the position of the eyepiece slit is read against this scale, which is lit up by the light from the object A little cover is arranged for these graduations, as the flickering occurring in them diverts attention from the observing slit, and so spoils the settings This arrangement allows readings to be taken without subjecting the eye to any outside light Greater accuracy of reading is obtained by using a micrometer screw, with a higher power objective.

To compensate partially for change of intensity of illumination at various parts of the curves so as to limit the range of intensities at which the photometric observations are made, a neutral-tinted darkened glass absorbing about 70 per cent of the light incident on it is fitted in front of the eyepiece. This can be removed when the intensity is small Neutral-tinted glasses are also placed between the disc and the source of light to make the intensity of illumination in the microscope convenient to the eye. These glasses are changed according to the power of the microscope objective and the aperture to which the lens is stopped down.

The automatic change of the position of the second disc gives rise to a peculiar order of change of intensity. As this order would be difficult to

remember, an indicator is arranged to show the position at any moment This is simply a ratchet wheel with 24 teeth, having the numbers indicating the position marked on it. The escapement is worked merely by pulling a string once and releasing, and a convenient way of doing this, so as to be certain that the pull has been drawn through far enough, is to attach the string to a crank, and give a complete turn of the crank to each movement of the escapement The same crank actuates the indicator.

With the arrangement described it is possible to make the settings at the rate of two per minute or so A whole series of readings for central and oblique definition at any aperture can be made in 10 minutes

Since the method employed is a flicker method it is clear that, unless monochromatic light is used, the various colours are weighted in proportion to the sensation they produce in the eye of the observer This, of course, is not the best weighting for photographic consideration. It is probable, however, in a lens which is fairly achromatic, that only a small difference would be found for the various colours The visual readings would apply practically without change if orthochromatic plates were to be used on the photographic objective

It is, of course, necessary that the circumference of the disc which forms the "edge" examined should be very accurately circular, and accurately centred on the shaft The error in the actual apparatus is exceedingly small, it can be examined and its effect allowed for, if necessary, by focussing the microscope directly on the edge without any intermediate optical system The departure from perfect definition due to the collimator and microscope is also involved in the measurements taken for the optical system under examination, it is, however, clear from the results obtained that for practical tests of definition this effect is negligibly small

The apparatus has been made up at the National Physical Laboratory by Mr. Murfitt, the instrument maker at the Laboratory Mr Murfitt gave very material assistance in the design of the various mechanisms, and the author is glad of this opportunity of recording his indebtedness for the help he has received

The theory of the distribution of intensity in the image formed by a lens with circular aperture, free from aberration, is well known The best account of it will be found in Lord Rayleigh's article on "The Wave Theory of Light" in the 'Encyclopædia Britannica'* The intensity curve in the image plane for a source consisting of an opaque straight edge with uniform illumination beyond the edge, as employed with this apparatus, has been

* 'Collected Papers,' vol 3

calculated by Struve,* and some of Struve's results are given in Lord Rayleigh's article above cited

The curves obtained with the apparatus for a certain photographic lens (denoted by No 2) at various aperture ratios are given in fig 5 These are

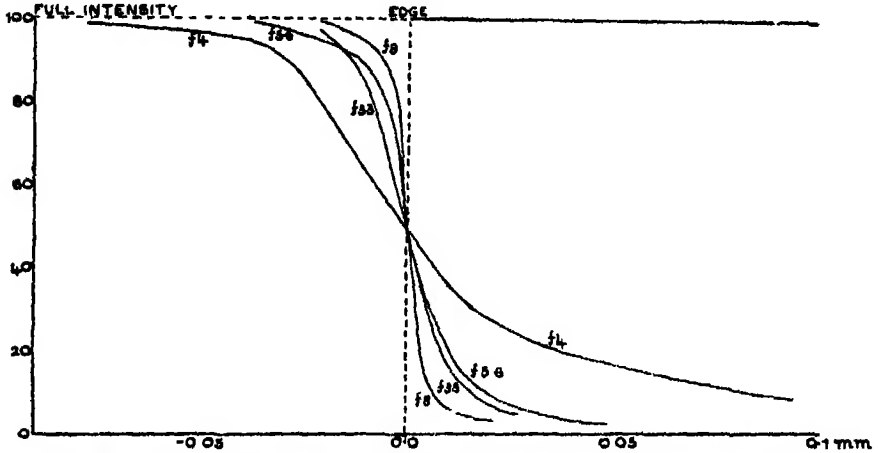


FIG. 5.—Intensity Curves for Lens No. 2 Central pencil, various apertures

for central definition, the axis of the lens intersecting the "edge." The curve for f 33 is practically identical with Struve's theoretical curve for this aperture It may be remarked that with most good photographic lenses it has been found that the best central definition is obtained at apertures between $f/8$ and $f/11$.

The apparatus may also readily be employed for the experimental determination of the intensity pattern in other cases of diffraction, and it is hoped that the results of such measurements made with it at the National Physical Laboratory may be published later It may also be suggested that some such contrivance, introducing a similar form of flicker photometry, might readily be applied to the measurement of spectrum line intensities or of the intensity of "visibility curves" which arise in the use of the Michelson interferometer. In this way greater accuracy might probably be obtained than by the method of estimation described in Michelson's 'Light Waves and their uses.' It is hardly necessary to say that this would lead to a more rigorous analysis of the molecular constitution of any substance examined by means of the interferometer. For such work only one source of illumination would be used (as in the apparatus described above), and the light would be partially allowed to pass through the interferometer and so

* 'Wiedemann's Annalen,' vol. 17, 1882, p. 1006.

to the photometer, while part would reach the photometer directly. The author hopes to have the opportunity of further considering this application.

In conclusion, he has great pleasure in acknowledging his indebtedness to Dr Glazebrook, the Director of the Laboratory, for his kindness in reading and revising the paper, and also to Mr. F J Selby for similar help. Many of the measurements were taken by Mr. T Smith, the author's successor at the laboratory, and to him also the author desires to express his thanks.

The Flight of a Rifled Projectile in Air

By Prof J B Henderson, D Sc, Royal Naval College, Greenwich.

(Communicated by W Burnside, F R.S. Received February 9,—Read February 25, 1909.)

The stability of a moving projectile has been treated by Sir George Greenhill as a practical application of a problem in hydrodynamics which he had previously solved, namely, the stability of a rotating spheroid moving slowly through a liquid * This hydrodynamical illustration, however, though in itself interesting, gives little assistance in the numerical treatment of the problem which is here discussed—the stability of a projectile after it is launched on its trajectory The shot is then moving faster than a wave of compression in air; for this reason, and on account of the eddying motion generally, the shot cannot be linked to the air in that closed kinematic chain which is assumed in all problems in hydrodynamics, and in which the velocity of every particle of fluid depends only on the velocity of the solid, the two varying together in a perfectly definite manner Only the air at a very short distance from the projectile can directly affect the motion of the latter, and in the following pages the problem is treated simply as that of a moving rotating body meeting with certain resistances.

This method of treating the problem seems natural and self-evident. It has been used by the writer in lecturing to naval gunnery lieutenants during the last four years, and by Mr. A Mallock, F.R.S, in a paper on "Ranges and Behaviour of Rifled Projectiles in Air,"† but, so far as the writer is aware, it has never before been carried to its ultimate conclusion—the synthesis of a trajectory in all its details, the initial conditions and the laws

* See 'The Engineer,' November—December, 1907.

† 'Roy. Soc. Proc.,' June 24, 1907.

of resistance alone being assumed Incidentally the causes of both horizontal and vertical "drift" are made manifest.

The projectile having left the gun with a certain initial velocity and a sufficient spin about its axis of figure to ensure stability about that axis, the direction of motion being inclined to the axis of figure, let us try to trace step by step the path followed by the centre of gravity of the shot and that traced out in space by the axis of figure

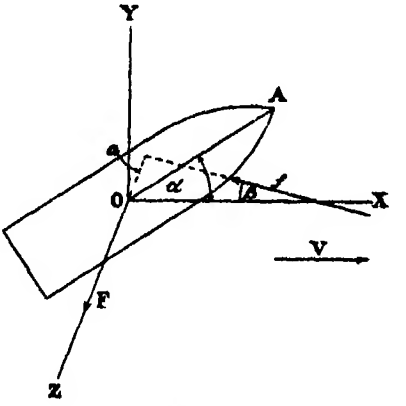


Fig 1.

Let OX be the direction of motion of the centre of gravity O (fig. 1), the axis of figure OA being deflected through a small angle α from OX, and suppose that, at the instant, OA is in the plane XY The air resistance is then represented by a force f which does not pass through the centre of gravity, but in modern projectiles intersects the axis in front of the centre of gravity. We thus have acting on the projectile a "tilting" couple fa, a force $f \cos \beta$ decelerating the speed and a force $f \sin \beta$ generating momentum perpendicular to OX in the plane AOX. The couple causes OA to precess round OX, the velocity of precession perpendicular to AOX being $\frac{\partial \psi}{\partial t} = \frac{fa}{I\varpi}$ (where I is the moment of inertia of the projectile about its axis of figure and ϖ its angular velocity of spin). Whence the period of precession round OA is

$$T = 2\pi \sin \alpha \Big/ \frac{\partial \psi}{\partial t} = \frac{2\pi I \varpi \sin \alpha}{fa}$$

The component force $f \sin \beta$ normal to OX gives a curvature to OX in the plane AOX, and since this component is always in the plane AOX and precesses with the shot round OX, the resulting curvature is constantly changing in direction and causes OX to assume a helical form

The curvature of the path is given by

$$\frac{d\theta}{dt} = \frac{f \sin \beta}{mV} \quad \text{and} \quad \frac{ds}{dt} = V, \quad \text{whence} \quad \frac{d\theta}{ds} = \frac{f \sin \beta}{mV^2},$$

where m is the mass and V the velocity of the projectile.

In practice, both α and β will be small angles, and the sines may be replaced by the angles themselves. Let us first of all assume the force of gravity to be annulled

In order to represent the changes in OA and OX by means of a diagram, since it is directions in space we are considering, let us represent angles by their traces on a spherical surface Let a huge sphere be circumscribed round the projectile of radius so large that the dimensions of the trajectory are negligible compared with it, let us say the celestial sphere, and let the diagram represent the paths traced out on the celestial sphere by the prolongation of OA and OX. To simplify the drawing of the diagram let us divide the period of precession of OA into 12 equal parts, and consider OX the direction of motion to be fixed during each interval, and at the end of the interval add an equivalent change of direction in OX This change of direction would be given by $\theta = \frac{f \sin \beta}{mV} \frac{T}{12}$, where T is the period of precession, and since α and β vary together we must, for the purposes of a diagram, make some assumption as to the connection between them Let us assume that β is simply proportional to α Then θ is simply proportional to α if T is constant. But $T = 2\pi I\omega \sin \alpha / fa$, and we may consider α proportional to $\sin \alpha$, and therefore T is constant, or at least independent of α.

In fig. 2, A_0 is the intersection of OA with the celestial sphere at a particular instant, and X_0 the simultaneous intersection of OX with the sphere The centre of the sphere is at O (not shown in the diagram), and the projectile is supposed to be approaching the observer along OX_0 perpendicular to the plane of the diagram. In one-twelfth of a period OA precesses round OX from OA_0 to OA_1, and simultaneously OX moves towards OA from OX_0 to OX_1, X_0X_1 being θ and being drawn in the mean direction of XA during the interval Similarly the successive positions $A_2, A_3 \ldots$ $X_2, X_3 \ldots$ are obtained. It is seen that both A and X have spiral traces on the celestial sphere, and that the angle AOX steadily decreases, also that, if the initial deflection of the axis of the shot is upwards, as supposed in the diagram, the direction of motion OX drifts to the right and simultaneously rises above the original direction OX_0. From this diagram and the forward velocity V it is easy to draw a diagram of the spiral surface traced out by the axis of the shot. This is shown in fig. 3 with the scale of lateral deviation greatly magnified. It

will be seen from the elevation that instead of the trajectory remaining along the original horizontal line OX_0 it rises above this line, and from the plan it will be observed that it deviates considerably to the right We shall see later that these effects have been greatly magnified in figs. 2 and 3, but it is important to notice that they are present in every trajectory, whether gravity

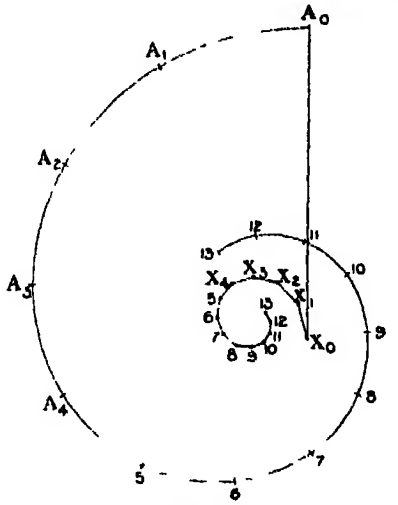
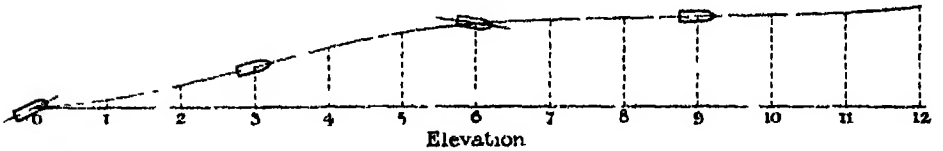
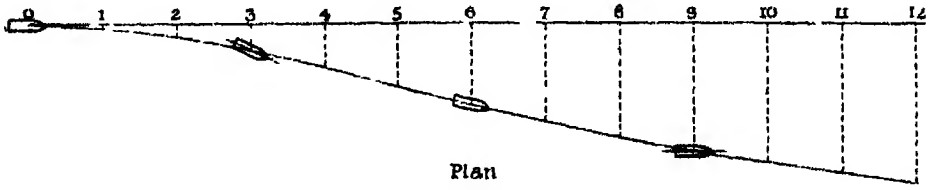


FIG 2



Elevation



Plan

FIG 3

is neglected or not One effect of magnifying the force $f \sin \beta$ is that the spiral motion almost disappears in one turn of the spiral. This point is discussed later.

The question may be asked: Have we any experimental evidence of this helical path? No accurate experiments have been made on the subject so far as the writer is aware. One hears of actual spiral paths being observed by persons stationed behind the gun, but the evidence on this point is very

conflicting Undoubtedly the projectile does occasionally leave the gun with its axis considerably inclined to the direction of motion, and in such a case the spiral motion will be exaggerated, but this is the exceptional case, and we shall see later that the dimensions of the spiral must, as a rule, be such as would be invisible to the eye by the time the shot first becomes visible—at, say, 100 yards distance The shot will as a rule leave the gun with its axis slightly inclined to the direction of motion. This inclination may be produced by the rush of gases past the shot at the moment of leaving, or by a parting kick from the gun in a radial direction just as the base of the shell is leaving the muzzle. Such a kick may arise from "whip" in the gun, or if a field gun is at all elevated, it will arise from any recoil of the gun carriage as a whole, which, being in the horizontal plane, is inclined to the direction of motion of the shot. In the interval of time between the point of the shell and the base of the shell leaving the gun, the gun must gain velocity of recoil, which it imparts to the base, hence the base gets a knock upwards and the axis of the shot immediately inclines to the left in consequence.

The direct evidence available which would throw light on this helical motion is confined to the distribution of shot marks on a target at short range, from a gun carefully sighted on the centre of the target This evidence would not be conclusive unless artificial arrangements were introduced for giving the shot an initial deflection to opposite sides, say by arranging the recoil horizontally and then firing at two targets, one at 10° elevation and the other at 10° depression, and then comparing the targets The radius of the helix must be of the order of 1 inch [but see addendum], consequently the holes pierced by a large shot in a series of equally spaced screens could not throw any light on the subject, owing to the difficulty of accurately surveying the holes If we consider the period of revolution in the spiral, that is the period of precession, the problem seems more hopeful. The shot in its path towards the target has at each moment maximum and minimum air compression on opposite sides, and these positions of maximum and minimum pressure are presented alternately to an observer stationed down the range at some distance laterally from the target, and since a compression is a seat of generation of atmospheric waves, the waves which reach the observer must have a slow pulsation of the periodicity equal to that of the precession in the spiral Whether these waves are audible or not the writer cannot say, but a sensitive manometric flame or a microphone diaphragm would probably disclose the period. They ought to be heard by an observer situated down the range in the interval between seeing the flash and hearing the report. It may be that the

shot is not large enough to throw the sound shadow on which the periodic pulsations would depend

A projectile having left the gun with its axis deflected from the direction of motion, let us now study the forces which reduce this deflection, and which tend in general to keep the axis of the projectile tangential to the trajectory These are the friction couple and the force already considered—the component of f normal to the direction of motion The former acts directly in altering the direction of the axis of the projectile so as to reduce the deviation, while the latter affects the direction of motion of the centre of gravity in such a manner as also to produce the same effect. We shall consider the friction couple first

Since the air pressure is unsymmetrically distributed round the projectile, the friction forces tending to stop the rotation will also be unsymmetrically distributed round the axis, for, if we consider the force f as being due to an excess cushion of air on one side of the projectile, the rotation of the shot will introduce an excess of friction in the peripheral direction in the neighbourhood of the excess cushion of air Thus we shall have a friction force q perpendicular to the plane AOX Or, considering a section of the projectile by a plane containing OX and perpendicular to the plane AOX, we get a figure somewhat like fig 4 There will be a dense wedge-shaped cushion of air M

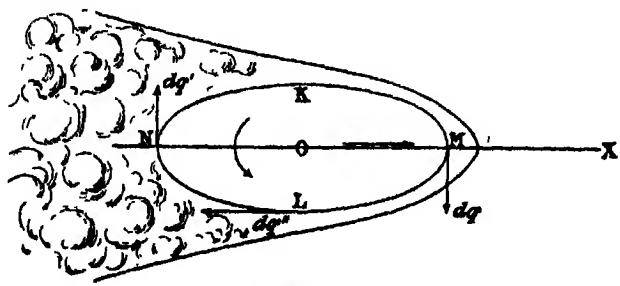


FIG 4.

on the advancing side, and a tail race N of eddies in the wake The rotation of the projectile is in the direction of the arrow, and this rotation imparts momentum to the cushion M and to the wake N in the peripheral direction, hence we have the equivalent of two forces, dq and dq', acting on the projectile at M and N. The relative velocities of the air and the projectile at K and L are different, hence we have an additional tangential force at L represented by dq''.

If the forward velocity of the projectile were small as in a golf ball, this extra retardation of the air on the side L would cause an accumulation of air on that side, and the cushion M would be moved slightly round on the

side L, the stream lines being modified accordingly There would then be a component pressure perpendicular to the plane AOX In golf the spin about a vertical axis on a "sliced" or "pulled" ball causes this displacement of the cushion of air on the front of the ball, and a drift to right or left is the result, the rate of drift increasing as the velocity falls off

In a projectile, however, the forward velocity is very great compared with the peripheral velocity due to any component spin about a vertical axis, and the rate at which the air in the cushion M is being renewed by the eddies effectually prevents any great deflection of the cushion towards the side L, the difference of the relative velocities of air and metal at K and L is negligible, and the force dq'' is zero Hence the resultant resistance f is approximately in the plane AOX as assumed in fig 1, and the drifts of a projectile and of a golf ball are due to different causes The effect of the forces dq and dq' over the whole projectile might be represented by two forces q and q' in fig 5 perpendicular to the plane AOX These are

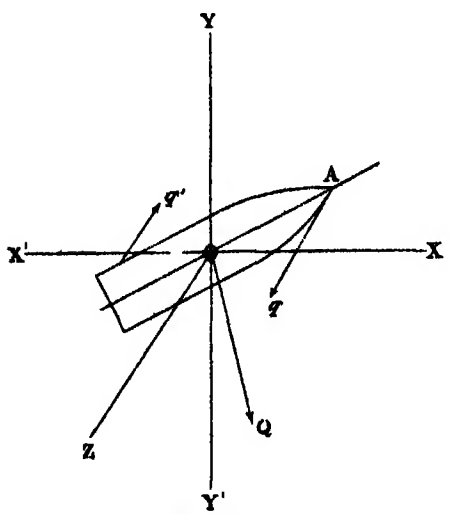


Fig. 5

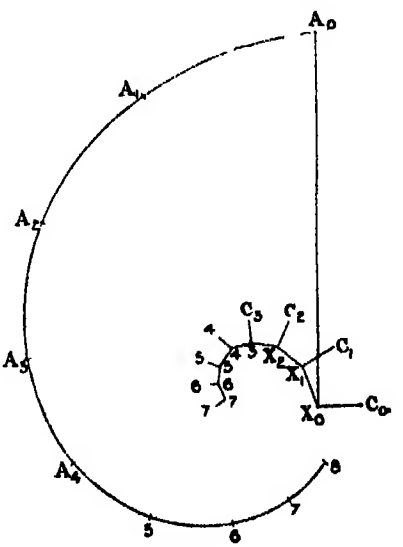


Fig 6.

equivalent to a force $q-q'$ through the centre of gravity and a couple having OQ as axis, OQ being in the plane AOX, or, in this figure, in the plane XY. The axis OA will therefore precess under the action of the friction couple so as to tend to coincide with OQ, and it will continue to do so until OX is reached—that is, until the axis OA is tangential to the trajectory, in which position the friction couple q vanishes, the friction then being symmetrical about OA.

The influence of the friction couple is shown in fig. 6, but not to scale. The f couple gives to OA a precession perpendicular to the plane AOX, and the q couple gives a precession in the plane AOX towards X. The forces of the f couple are great compared with the forces of the q couple, but the arm of the q couple is great compared with the arm of the f couple If we assume the f couple to be 10 times the q couple the centre about which A is turning is C_0, where C_0X_0 is $\frac{1}{10}A_0X_0$. The arc A_0A_1 is drawn with C_0 as centre The centripetal component $f \sin \beta$ will also be modified in direction by the small friction force $q-q'$ perpendicular to it. If the arms of the f and q couples were equal the resultant direction would be C_0A_0 in the initial position, hence X_0X_1 would be parallel to the mean direction of C_0A during the first interval, but whether we draw X_0X_1 parallel to the mean direction of C_0A or to the mean direction of X_0A makes little difference to the diagram The friction couple modifies the trace of A very considerably, as will be seen by comparing figs 2 and 6, the rate of diminution of the obliquity being considerably increased It will also be noticed that the drift is still to the right and upwards

Let us now try to get some idea of the magnitude of this helical motion by considering an actual example; let—

p = pitch of helix
I = moment of inertia of projectile about axis
ϖ = angular velocity of projectile about axis.
r = pitch of rifling
a = arm of f couple (fig 1).
α = inclination of axis to direction of motion
β = inclination of f to direction of motion.
T = period of precession in helix.
V = velocity of projectile

Then we have $$T = \frac{p}{V}, \qquad \varpi = \frac{2\pi}{r/V} = \frac{2\pi V}{r}$$

Precessional velocity of axis OA perpendicular to the plane AOX is

$$\frac{\partial \psi}{\partial t} = \frac{2\pi}{T} \sin \alpha = \frac{2\pi V}{p} \sin \alpha.$$

Tilting couple $= fa = I\varpi \frac{\partial \psi}{\partial t} = \frac{2\pi I \varpi}{T} \sin \alpha = \frac{4\pi^2 IV}{Tr} \sin \alpha$, therefore

$$T = \frac{4\pi^2 IV}{fr} \cdot \frac{\sin \alpha}{a}.$$

Now a depends on α, the connection between them depending on the shape of the ogival head. If we limit the problem to small deviations we

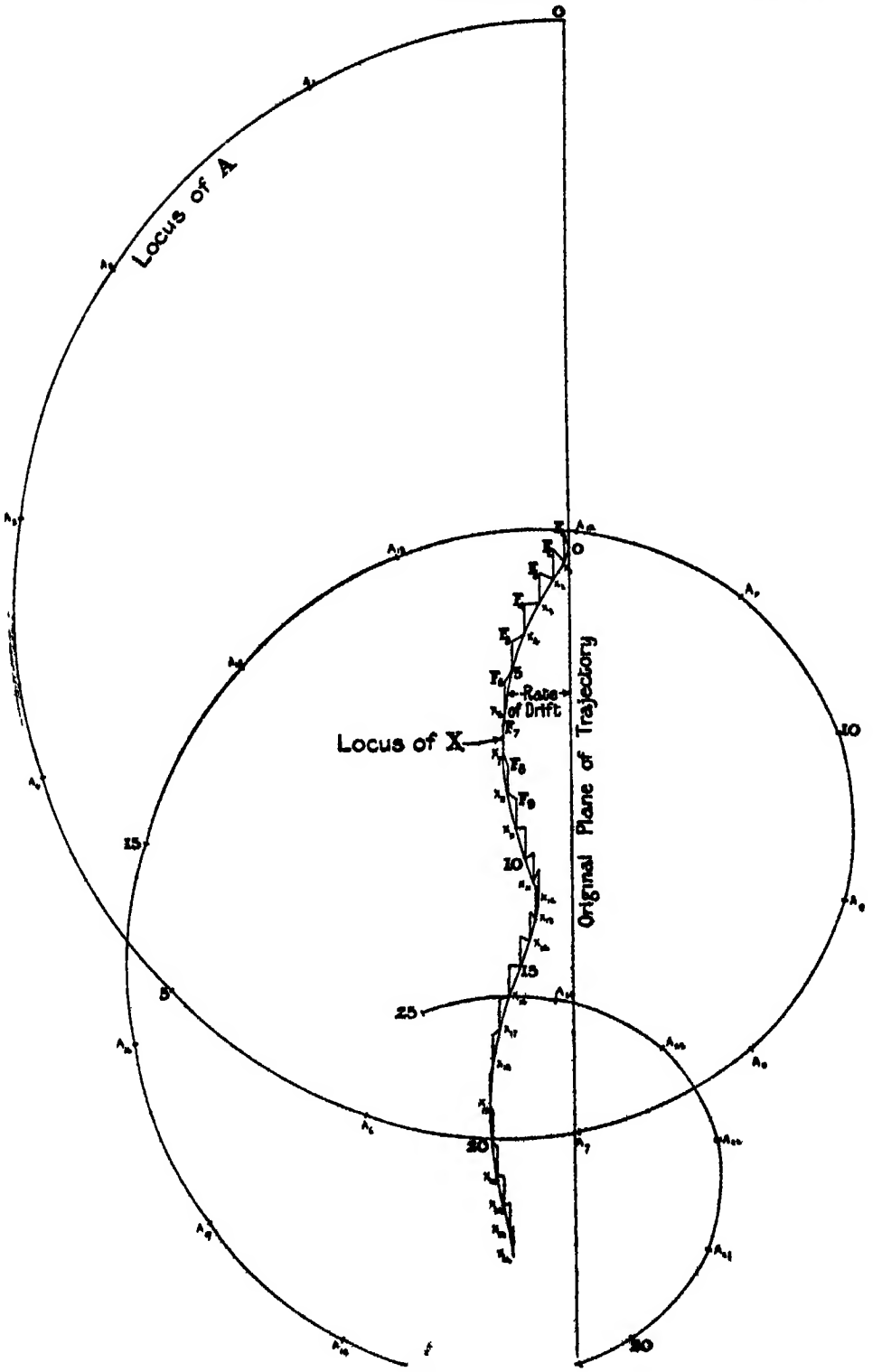

Locus of A
Locus of X
Rate of Drift
Original Plane of Trajectory

Fig 7

may write the angle for the sine of the angle, and by considering the shape of the head we see that β will at first increase much faster than α, since the pressure on one side of the ogive is increasing and on the other side decreasing as α increases. Taking β to be about 3α and α as one degree in a 12-inch shell, a will be of the order of 1 inch. The square of the radius of gyration may be taken as one-half the square of the radius of the projectile The resistance f at a velocity of 2500 feet per second is approximately four times the weight of the projectile, and r is 45 feet

$$\text{Thus} \qquad T = \frac{4\pi^2 m \times 0{\cdot}125 \text{ ft.}^2 \times 2500 \text{ ft/sec}}{4 \times m \times 32 \text{ ft./sec.}^2 \times 45 \text{ ft}} \times \frac{\frac{1}{60}}{\frac{1}{12} \text{ ft}}$$

$$= 0{\cdot}44 \text{ second}$$

There may be an error in the estimation of a of 100 per cent, so all we can say is that the period is of the order of 1 second.

Influence of Gravity on the Trajectory.

We are now in a position to discuss the effects of gravity on the trajectory The weight may be resolved into two components, one along and the other perpendicular to the direction of motion The first varies the magnitude of the velocity but not its direction, while the second varies the direction of motion We are only concerned at present with the second component The vertical plane of motion oscillates backwards and forwards in azimuth due to the helical motion, and in this plane, in every position it occupies, the normal component of the weight gives to the direction of motion a curvature downwards. Let us trace out on the celestial sphere the changes of direction of both the axis of the projectile and the direction of motion due to the weight In order not to complicate the diagram let us first of all neglect the friction couple, and let us choose the scales of the various magnitudes so as to make a clear diagram, but not to represent the physical facts These will be considered later. In fig 7, A_0 and X_0 are the initial intersections as before During the first twelfth of a period A precesses from A_0 to A_1, and due to the component $f \sin \beta$, if it acted alone, X_0 would move to F_1, but since the weight has also been acting during the interval it lowers X by the displacement F_1X_1. Hence, due to the combined action of f and W, X moves from X_0 to X_1 during the first twelfth of a period During the second twelfth A turns about X_1 from A_1 to A_2, and X moves simultaneously from X_1 to X_2, and so on.

Both the motion of A and the radial motion of X are proportional to the obliquity AOX, but the vertical motion of X is independent of the obliquity. Hence the vertical displacements F_1X_1, F_2X_2, etc., are all equal, while the

radial motions X_0F_1, X_1F_2, .. etc., are proportional respectively to A_0A_1, A_1A_2, etc., that is, to A_0OX_0, A_1OX_1, etc The traces of A and X are sketched in fig 7 for two whole periods of the helical motion. It will be noticed that the obliquity AOX gradually diminishes and that the direction of motion bears to the right. The horizontal ordinate of the locus of X is proportional to the rate of horizontal drift, and it would be easy to plot from this curve a curve of rate of drift as a function of time, and then by integrating it to get the linear drift at any moment. It will be noticed that the drift as drawn is always to the right, but suppose as a starting point for our curve we take, say, A_6, that is, we assume that the axis of the projectile is initially inclined downwards from the direction of motion, then the original plane of the trajectory is the vertical through X_6 and the drift is then to the left for almost the whole of the first complete period We see, therefore, that it must be impossible to obtain an analytical formula for drift which would apply to all guns or to all elevations of the same gun, since, as we have seen, the initial deviation of the axis of the shot is likely to vary with different guns, and with the same gun at different elevations The natural elevation of a field gun would cause an initial deflection in the direction corresponding to A_{10} if the carriage recoiled horizontally, and the initial plane of the trajectory would be a vertical through X_{10} Hence we should expect the shot to show drift to the left for short ranges and to the right for long ranges if the elevation were kept constant The elevation, however, is varied with the range, and in modern guns the recoil is parallel to the axis of the gun, hence the drift to the left at short ranges, which has been noticed in certain guns, must be caused by an initial deflection of the axis of the projectile to the left due to "whip" in the gun in the vertical plane, or to the rush of gases past the shot

Numerical Example

In order to get some idea of the relative magnitudes of the displacements of the direction of motion due to the weight and the normal component of the resistance, let us consider a 12″ shell moving at 2500 feet per second with an obliquity of axis of one degree, and suppose that the corresponding obliquity of the resistance f is three degrees The deceleration at this speed is about $4g$ The period of precession is, say, 0 48 second, and the intervals under consideration are 0 04 second The change of direction during 0·04 second due to the weight is given by

$$F_1X_1 = \frac{gt}{V} = \frac{32 \times 0{\cdot}04}{2500} \text{ radian} = 1\,8 \text{ minutes of arc.}$$

The normal acceleration due to $f \sin \beta$ is approximately β times the deceleration, or $\frac{1}{10} \times 4g = \frac{1}{25}g$ Hence

$$X_0F = \tfrac{1}{3}F_1X_1 = 0\,36 \text{ minute of arc}$$

Hence, if we take the distances X_0F_1, F_1X_1, and A_0X_0 in the ratio 1 · 5 180, we shall have a diagram approximating to an actual case Such a diagram is drawn in fig 8, but the friction couple has been omitted in order to show more clearly the effect of friction by a separate diagram. Assuming the friction couple to be one-tenth of the direct resistance or tilting couple, and redrawing the diagram as in fig 6, we get fig 9 This figure is important as including all the effects and showing a real trajectory approximately to scale so far as the angular deviations of the axis and the direction of motion are concerned. It will be noticed by comparing figs 8 and 9 that the friction couple is the principal agent in reducing the deviation of the axis of the shot from the direction of motion, that the effect of the normal component of the resistance is negligible in comparison, but that this normal component of the resistance is the active agent in producing drift

In fig 9 four complete periods are shown, and the deviation has been reduced from one degree to almost zero In order to study the deviation in the case when the axis is initially coincident with the direction of motion fig 10 is drawn, maintaining the same relative magnitudes as were used in fig. 9, but magnifying the scale of the whole diagram five times, so that a single loop at the foot of fig. 9 represents fig 10 It is seen that although the axis and direction of motion are initially coincident they are never again coincident, and that the axis keeps always to the right of the direction of motion In such a case drift to the left is of course impossible. Since the $f \sin \beta$ effects are negligible, the vertical scale becomes a scale of time, and the locus of X becomes a curve of rate of drift In order to show the shape of this curve the ordinates have been magnified 45 times, and the new curve so obtained is called a curve of rate of drift. The area of this curve if integrated gives the linear drift.

Fig 11 is drawn to show the effect of doubling the friction couple This effect, as will be seen by comparing figs 10 and 11, is very slight—if the initial deviation is zero—during the short interval of time considered in the diagram.

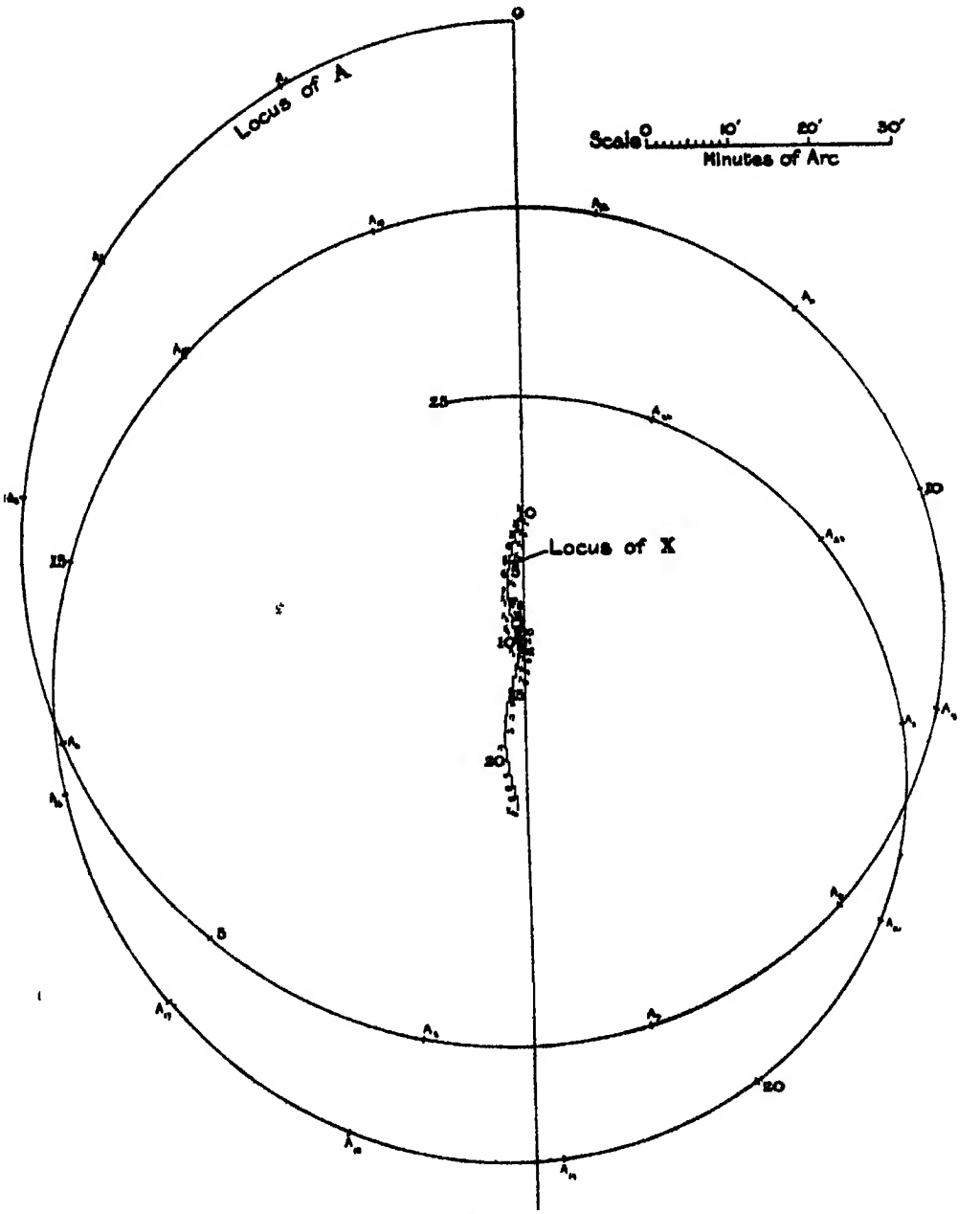


Fig 8.

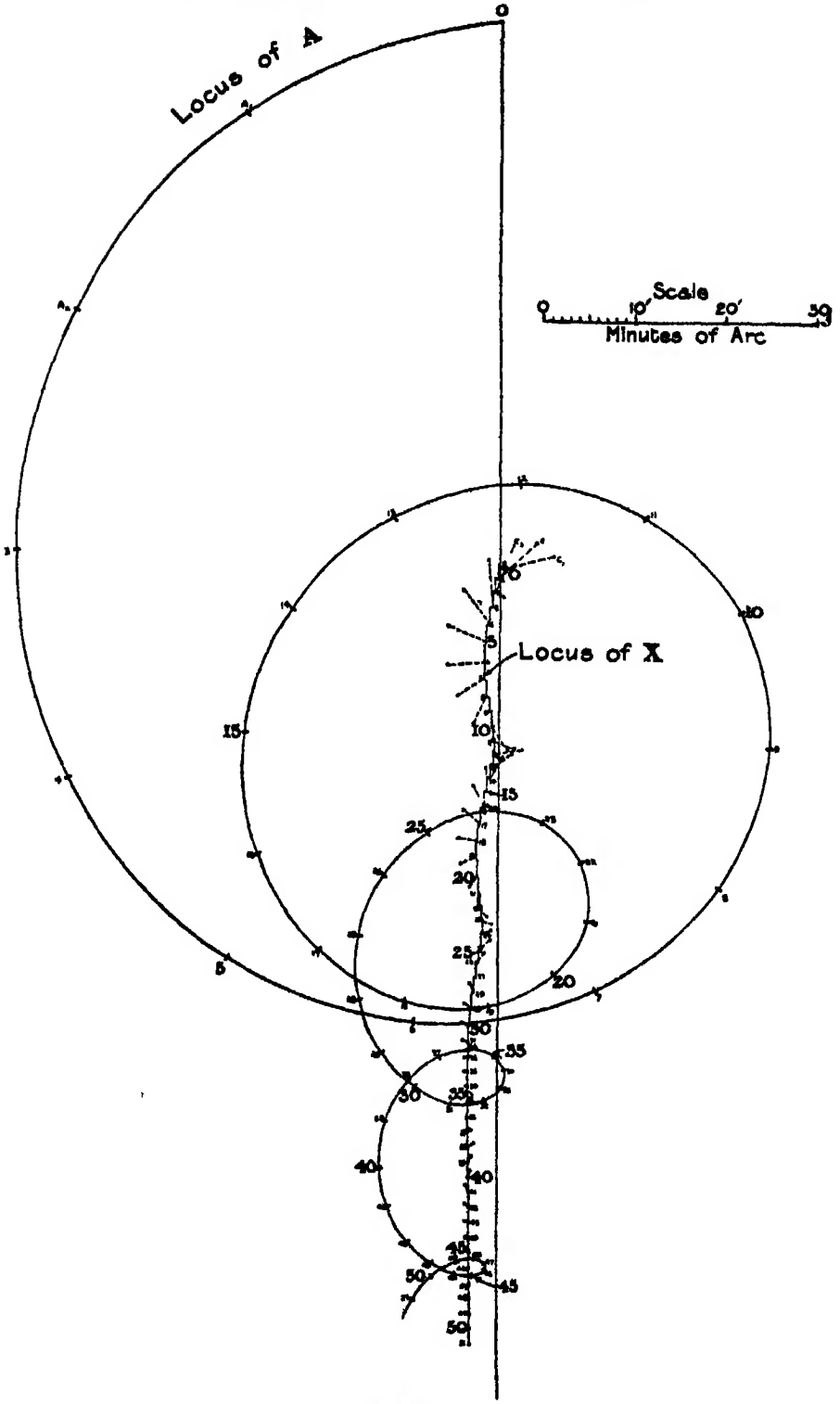


FIG. 9

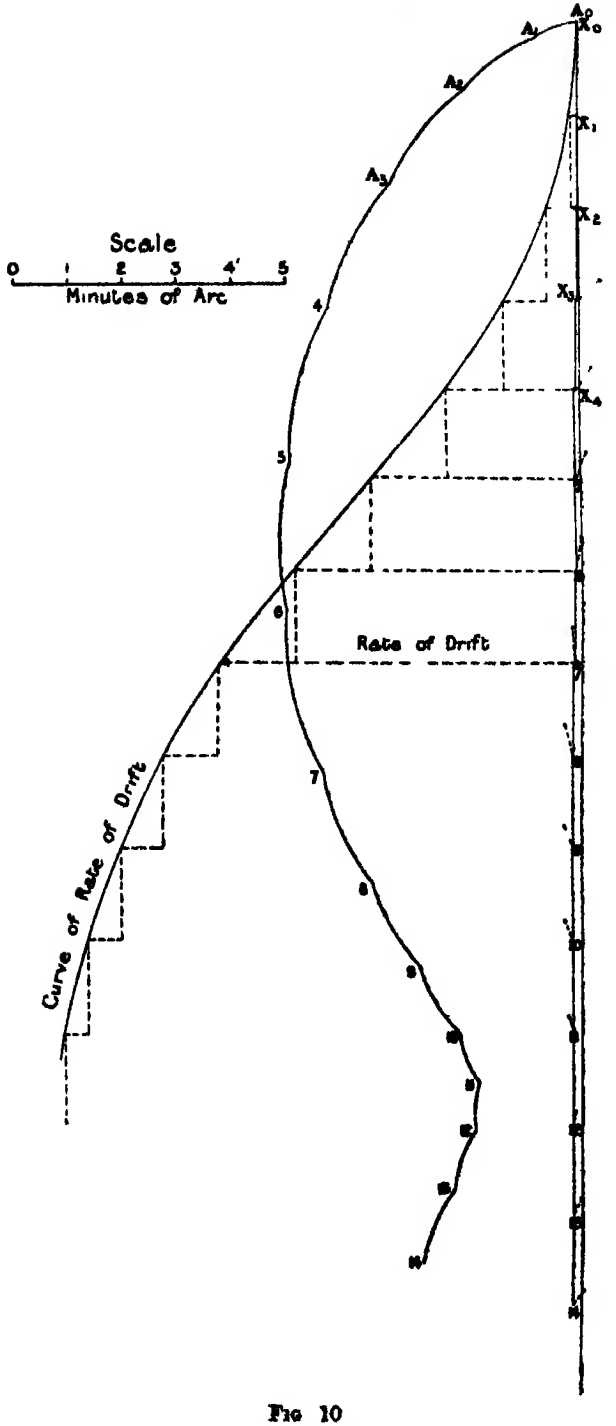

Scale
0 1 2 3 4' 5
Minutes of Arc
Rate of Drift
Curve of Rate of Drift

Fig 10

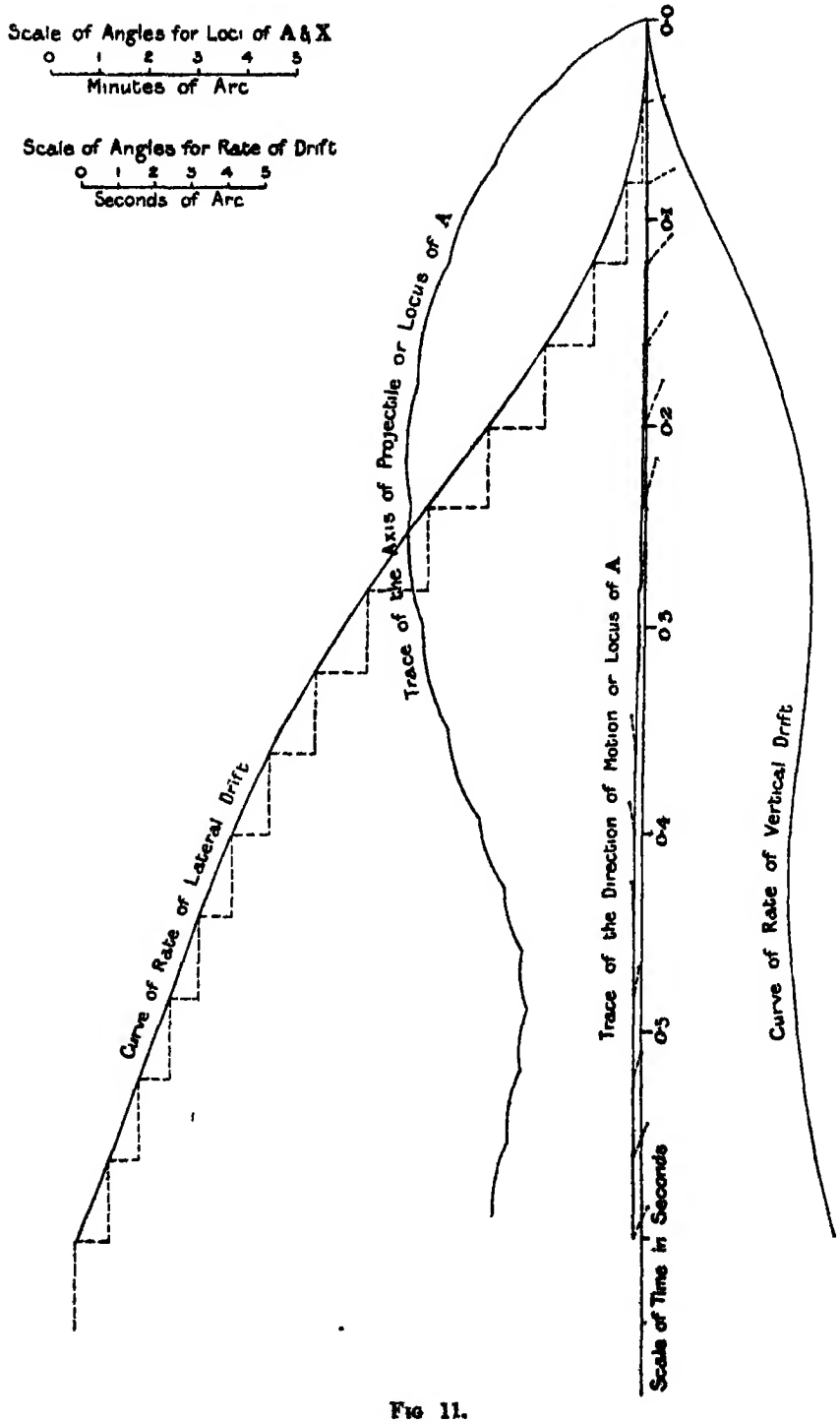

Scale of Angles for Loci of A & X
0 1 2 3 4 5
Minutes of Arc
Scale of Angles for Rate of Drift
0 1 2 3 4 5
Seconds of Arc
Curve of Rate of Lateral Drift
Trace of the Axis of Projectile or Locus of A
Trace of the Direction of Motion or Locus of A
Curve of Rate of Vertical Drift
0
0·1
0·2
0·3
0·4
0·5
Scale of Time in Seconds

Fig 11.

Vertical Drift

When neglecting the force of gravity we saw that the shot, in addition to drifting laterally, also became elevated above its initial direction of motion The horizontal drift of X is due to the horizontal component of F_1X_0 (fig 7), while the "vertical drift" of X, as distinguished from the vertical motion due to the weight, is due to the vertical component of F_1X_0 It is, of course, a very small quantity in practice, but in order to find its magnitude the two curves of horizontal and vertical rate of drift have been drawn in fig 11, and on integrating them over the interval represented on the diagram, 0 6 second, the lateral drift is found to be 0 06 foot and the vertical drift 0 027 foot

By dividing the time of flight of a projectile into periods of the precessional motion, and considering the velocity and deceleration as constant throughout each period but varying from period to period, it would be easy to trace out the complete history of the motions of the axis and of the path of flight for any given data regarding the shot and the resistances opposed to its motion

In conclusion, I desire to express my cordial thanks to my friend and colleague Prof W Burnside, F R S, for the interest he has shown in the problem discussed, and my indebtedness to him for his valuable criticism

[*Addendum, March* 3, 1909—At the meeting of the Society when this paper was read, the question was raised whether the drift could possibly be due to the helical motion, since no trace of helical motion has been observed on the trajectory of a rifle bullet as recorded by the holes pierced in a series of screens To settle this question, the curve of rate of drift for the first turn of the helical motion in fig 9 was redrawn on a scale 10 times as great as in fig 9, and was integrated The linear horizontal drifts at the ends of the successive quarter periods are 0 27, 1·27, 2 26, and 2 85 inches, which, if plotted at equal intervals on a time base, will be found to correspond with a spiral of 0 4 inch radius The deviation during this first turn of the spiral, it will be remembered, was taken as initially one degree, and is rapidly being decreased, hence 0·4 will be an excessive radius for the spiral path of a 12-inch shell. The cases on record where large projectiles have actually been seen travelling in spiral paths have probably been due to insufficient spin on the projectile, the driving band having slipped on the projectile or stripped in the gun

A radius of 0 4 inch would be quite observable on the screen records of a small bullet, but the following elementary considerations of dimensions show us that the radius of the helix will be proportional to the diameter of the

projectile if the velocities and deviations of the axes are equal. The mass of the shot, $\propto d^3$; moment of inertia $\propto d^5$, resistance $\propto d^2$, tilting couple $\propto d^3$, hence the velocity of precession, which varies as the tilting couple ÷ moment of inertia, $\propto d^{-2}$

The curvature of the path due to the normal component of the resistance $\propto$ resistance ÷ momentum of shot $\propto d^2/d^3 \propto d^{-1}$ Radius of the helix $\propto$ curvature of the path ÷ velocity of precession $\propto d^{-1} \times d^2 \propto d$ The radius of the helix of a 0·3-inch bullet would therefore be of the order of $0\,4''/40 = 0\,01$ inch, which would not be detectable on the screen records]

On the Graphical Determination of Fresnel's Integrals

By JOHN H SHAXBY, B Sc (Lond), University College of South Wales and Monmouthshire

(Communicated by Principal E H Griffiths, F R S Received March 9,— Read April 22, 1909)

The functions $\int_0^x \cos(\frac{1}{2}\pi x^2)\,dx$ and $\int_0^x \sin(\frac{1}{2}\pi x^2)\,dx$, known as Fresnel's integrals, have usually been evaluated by means of converging series, for instance those of Cauchy and Knochenhauer The graphical method of integration, by the summation of the areas contained between given ordinates, the arc of the curve they cut off and the x-axis, is readily applicable in this case, the quantity $\cos \frac{1}{2}\pi x^2$ (or $\sin \frac{1}{2}\pi x^2$) being plotted as ordinate against x as abscissa. The area can then be determined between any given limits by Simpson's Rule In practice, however, it is simpler and more accurate to apply Simpson's Rule directly to the calculated values of the ordinates, without plotting a curve

The curve $y = \cos \frac{1}{2}\pi x^2$ has its zero values at $x = 1, 3^{\frac{1}{2}}, 5^{\frac{1}{2}}$, etc , and is thus marked off into a series of loops of steadily decreasing area The simplest method of computation is to find the areas of these separate loops (the first being only half a loop) ranging, in values of x, from 0 to 1, 1 to $3^{\frac{1}{2}}$, $3^{\frac{1}{2}}$ to $5^{\frac{1}{2}}$, etc. The range for a particular loop will be termed its base-line.

In the subjoined tables the calculations have been made by dividing each base-line into 10 parts and calculating the ordinates corresponding to the points of division, *e g* , 0 1, 0·2, 0·3...0·9 for the first half-loop.

The area of any loop may then be written as kd, where d is the length of the base-line and k a factor depending upon the loop considered The

values of k are shown in Table I for the first six loops and for two other loops of higher order The table shows that k rapidly approaches a constant limiting value, 0·6366. Thus after a very few loops the area of a loop is at once given by the product of this constant and the length of the base-line The theoretical limit of k is discussed in Section 4.

The integration $\int_0^x \cos(\frac{1}{2}\pi x^2)\,dx$ to any upper limit $x = n^{\frac{1}{2}}$, where n is an odd integer, is thus readily performed by adding to the integral for the first few loops (say up to $11^{\frac{1}{2}}$) the value for the higher loops $k\Sigma d$, where Σd represents the quantity obtained by summing up the base-lines, attributing to them positive and negative values alternately (because of the alternating signs of the loop-areas), and writing $k = 0\,6366$.

Table I exhibits also the areas of the first six loops, and so provides the data for such a calculation

The values of $\int_0^x \sin(\frac{1}{2}\pi x^2)\,dx$ can similarly be calculated for the loop-areas of $y = \sin\frac{1}{2}\pi x^2$, where the base-lines range from 0 to $2^{\frac{1}{2}}$, $2^{\frac{1}{2}}$ to 2, 2 to $6^{\frac{1}{2}}$, etc, the values of x for which $\sin\frac{1}{2}\pi x^2 = 0$

These loop-areas and the corresponding values of k are shown in Table II, we again find the rapid approach of k to the constant value 0 6366

Table I —Constants and Areas of Loops of $y = \cos\frac{1}{2}\pi x^2$

Limits	Base line	Area	k
0—1	1·0000	0 7799	0 7799
1—$8^{\frac{1}{2}}$	0 7320	0 4589	0·6268
$3^{\frac{1}{2}}$—$5^{\frac{1}{2}}$	0 5040	0 3198	0·6344
$5^{\frac{1}{2}}$—$7^{\frac{1}{2}}$	0 4097	0 2608	0·6363
$7^{\frac{1}{2}}$—3	0 3542	0 2254	0·6362
3—$11^{\frac{1}{2}}$	0 3166	0 2015	0 6363
$99^{\frac{1}{2}}$—$101^{\frac{1}{2}}$	0 1000	—	0·6365
$999^{\frac{1}{2}}$—$1001^{\frac{1}{2}}$	0·0316	—	0·6366

Table II —Constants and Area of Loops of $y = \sin\frac{1}{2}\pi x^2$.

Limits	Base line.	Area.	k
0—$2^{\frac{1}{2}}$	1·4142	0·7145	0 5052
$2^{\frac{1}{2}}$—2	0·5858	0 3706	0·6327
2—$6^{\frac{1}{2}}$	0 4495	0·2856	0·6353
$6^{\frac{1}{2}}$—$8^{\frac{1}{2}}$	0·3789	0·2409	0·6357
$8^{\frac{1}{2}}$—$10^{\frac{1}{2}}$	0 3339	0·2124	0·6362
$10^{\frac{1}{2}}$—$12^{\frac{1}{2}}$	0 3018	0 1921	0·6368

2 Integration can of course be performed to any upper limit x by applying Simpson's Rule to the portion of a loop between n^2 and x, where n is the largest odd integer less than x^2 in the case of $\cos \frac{1}{2}\pi x^2$, the largest even integer less than x^2 in the case of $\sin \frac{1}{2}\pi x^2$. A simpler method is given in Section 5, for cases in which n is not small.

Table III gives the values of $\int_0^x \cos(\frac{1}{2}\pi x^2)\,dx$ for $x = 0{\cdot}1,\ 0{\cdot}2,$ etc., . . 1, calculated without further subdivision of the base-line of the first half loop, Gilbert's values being added for comparison. It is interesting to note, with a view to the availability of this procedure in cases where other methods may not be available, that such slight subdivision can give satisfactory results.

Table III

Upper limit x	$\int_0^x \cos(\frac{1}{2}\pi x^2)\,dx$	Gilbert's table.
0·1	0·1000	0·0999
0·2	0·1999	0·1999
0·3	0·2994	0·2994
0·4	0·3975	0·3975
0·5	0·4923	0·4923
0·6	0·5811	0·5811
0·7	0·6597	0·6597
0·8	0·7229	0·7230
0·9	0·7648	0·7648
1·0	0·7799	0·7799

3 The values of $\int_{x_1}^{x_2} \cos(\frac{1}{2}\pi x^2)\,dx$ and $\int_{x_1}^{x_2} \sin(\frac{1}{2}\pi x^2)\,dx$ in the special case when x_1 and x_2 are not small and differ but little from each other, can be directly found as follows —

$$d(x^2) = 2x\,dx \quad \text{and} \quad 2x = x_1 + x_2, \text{ approximately.}$$

So, approximately,

$$\cos(\tfrac{1}{2}\pi x^2)\,dx = \frac{1}{x_1 + x_2}\cos(\tfrac{1}{2}\pi x^2)\,d(x^2),$$

whence
$$\int_{x_1}^{x_2} \cos(\tfrac{1}{2}\pi x^2)\,dx = \frac{2}{\pi(x_1 + x_2)}(\sin \tfrac{1}{2}\pi x_2^2 - \sin \tfrac{1}{2}\pi x_1^2). \qquad (1)$$

Similarly,

$$\int_{x_1}^{x_2} \sin(\tfrac{1}{2}\pi x^2)\,dx = \frac{2}{\pi(x_1 + x_2)}(\cos \tfrac{1}{2}\pi x_1^2 - \cos \tfrac{1}{2}\pi x_2^2). \qquad (2)$$

4. Special cases.—(*a*) $\int_{x_1}^{x_2} \cos(\frac{1}{2}\pi x^2)\,dx$

In the case when $x_1^2 = 2n-1$, where n is any integer not too small for the approximation of Section 3, the integral becomes $\frac{2}{\pi(x_1+x_2)}(\sin\frac{1}{2}\pi x_2^2 \pm 1)$, where the positive sign refers to even values of n, the negative to odd values. If, in addition, $x_2^2 = 2n+1$, the value reduces to $\pm\frac{4}{\pi(x_1+x_2)}$, or, as in this case $x_2^2 - x_1^2 = 2$, to

$$\pm\frac{2(x_2-x_1)}{\pi} = \pm\,0\,6366\,(x_2-x_1) \qquad (3)$$

This expression therefore gives the area of the loop of the curve comprised between the limits $x_1 = (2n-1)^{\frac{1}{2}}$ and $x_2 = (2n+1)^{\frac{1}{2}}$, and is numerically equal to 0 6366 of the base-line of the loop

(*b*) $\int_{x_1}^{x_2} \sin(\frac{1}{2}\pi x^2)\,dx$ A similar investigation shows that if $x_1^2 = 2n$, the value becomes $\frac{2}{\pi(x_1+x_2)}(\pm 1 - \cos\frac{1}{2}\pi x_2^2)$, where the positive sign is to be associated as before with even values of n

If we consider one loop, *i.e.* if $x_2 = 2n+2$, the integral becomes again $2(x_2-x_1)/\pi$, showing that the limiting value of k is again 0 6366

5 $\int_0^x \cos(\frac{1}{2}\pi x^2)\,dx = \int_0^{\sqrt{n}} \cos(\frac{1}{2}\pi x^2)\,dx + \int_{\sqrt{n}}^x \cos(\frac{1}{2}\pi x^2)\,dx$, where n is the greatest odd integer less than x^2

From equations (1) and (3) computations may now be made rapidly, thus, for example —

$$\int_0^{3\,1} \cos(\tfrac{1}{2}\pi x^2)\,dx = \int_0^3 \cos(\tfrac{1}{2}\pi x^2)\,dx + \int_3^{3\,1} \cos(\tfrac{1}{2}\pi x^2)\,dx$$

$$= 0\,7799 - 0\,4589 + 0\,3198 - 0{\cdot}2603 + 0\,2254 + \frac{2(\sin\frac{1}{2}(3\,1)^2\pi - 1)}{\pi(3\,0+3\,1)}.$$

The last quantity $= -0{\cdot}0444$, so the integral is 0 5615.

$\int_0^x \sin(\frac{1}{2}\pi x^2)\,dx$ can be computed in a similar way, the integer corresponding to n being even instead of odd.

Table IV shows the values of the integral $\int_0^x \cos(\frac{1}{2}\pi x^2)\,dx$ calculated by the above method for upper limits ranging from 3·0 to 3·5 by steps of 0 1.

Table IV

Upper limit x	$\int_0^x \cos(\frac{1}{2}\pi x^2)dx$	Gilbert's table
3·0	0·6059	0·6057
3·1	0 5615	0 5616
3 2	0 4662	0 4663
3 3	0 4058	0 4057
3 4	0·4386	0 4385
3·5	0 5827	0 5826

6 The method is also capable of indicating the values (0 5) of the integrals between the limits 0 and ∞ Thus

$$\int_0^\infty \sin(\tfrac{1}{2}\pi w^2)\,dx = \int_0^{\sqrt{(n_0-2)}} ydx + \int_{\sqrt{(n_0-2)}}^\infty ydx, \qquad (4)$$

where $n_0 = 4N$, N being any integer

$$= \int_0^{\sqrt{(n_0-2)}} ydx + k[\{-\sqrt{n_0}+\sqrt{(n_0-2)}+\sqrt{(n_0+2)}-\sqrt{n_0}\}+\{-\sqrt{(n_0+4)}+\text{etc}\}$$
$$+\text{etc}\ ]$$
$$= \int_0^{\sqrt{(n_0-2)}} ydx - k\,\{n_0^{-\frac{1}{2}}+(n_0+4)^{-\frac{1}{2}}+\ .\}$$
$$= \int_0^{\sqrt{(n_0-2)}} ydx - \frac{k}{8}\{N^{-\frac{1}{2}}+(N+1)^{-\frac{1}{2}}+\ .\}$$
$$= \int_0^{\sqrt{(n_0-2)}} ydx - \frac{k}{8}\sum_N^\infty n^{-\frac{1}{2}}$$
$$= \int_0^{\sqrt{(n_0-2)}} ydx - \frac{k}{8}\left(\sum_1^\infty n^{-\frac{1}{2}} - \sum_1^{N-1} n^{-\frac{1}{2}}\right).$$

If $n_0 = 28$, $N = 7$, $(n-2) = 26$, $\int_0^{\sqrt{26}} ydx = 0{\cdot}5622$ and $\sum_1^6 n^{-\frac{1}{2}} = 1\,83$

$\sum_1^\infty n^{-\frac{1}{2}}$ can be estimated as 2 61 by the method of approximate summation of series by differential coefficients *

Hence $$\int_0^\infty ydx = 0\,5622 - \tfrac{1}{8}\times 0\,6366\,(2\,61-1\,83) = 0{\cdot}500.$$

* Boole, 'Finite Differences,' p 90.

A Phenomenon connected with the Discharge of Electricity from Pointed Conductors

By H. T Barnes, Macdonald Professor of Physics, and A N Shaw, Demonstrator of Physics, McGill University, Montreal, with a Note by John Zeleny, Professor of Physics, University of Minnesota

(Communicated by Prof E Rutherford, F R.S Received March 30,—Read April 29, 1909.)

The discharge of electricity from pointed conductors has been the subject of many important investigations, notably by J J Thomson, Chattock, Warburg, Zeleny, and others The effect of moisture on the discharge has recently been studied in detail by Zeleny,* who was the first to observe the formation of a coloured deposit on steel needle points when observed under a microscope He describes this deposit as having a reddish-brown colour, resembling ordinary rust, which is attached to the point in irregular pieces which extend outward some little distance, and finds it produced even in fairly dry air A marked difference in the volume of the deposit was observed, depending on the direction of the discharge to the point As anode the amount was much in excess of that produced when the point was made the cathode As Prof Zeleny passes over this phenomenon with but a brief mention for the steel points he used, we considered it of interest to study this deposit more in detail, using pointed conductors of various metals, and to observe what effect the presence of the deposit had on the discharge It appeared to us probable that the deposit was connected with the presence of moisture in the air surrounding the point, and that it could be eliminated altogether by having absolutely dry air. We have found this to be the case, and in consequence one of the disturbing factors connected with point discharge may be eliminated only by working with air from which every trace of moisture has been removed.

Most metals react easily with oxygen in the presence of moisture, but especially when made the anode in an electric circuit It has been shown by one of us, working with Mr G. W. Shearer,† that the metals aluminium, magnesium, and zinc, form, as the first stage of their oxidation, hydrogen peroxide, when immersed in water containing dissolved air or oxygen. The amount of peroxide formed was observed to be considerably increased when the metal was made an anode Other metals, such as iron, appear at once

* 'Phys. Rev,' vol 25, p. 305, 1907, 'Phys. Rev,' vol. 26, p. 448, 1908

† 'Journ. Phys. Chem.,' vol. 12, p. 155, 1908, vol. 12, p. 468, 1908

to decompose any peroxide produced as fast as it is formed It seems highly probable that the first three metals form over their surface a protective film which, in the case of aluminium,* is exceedingly resistant The non-formation of this surface film allows free action between the hydrogen peroxide produced and the metal, resulting in the decomposition of the former and further oxidation of the latter The formation of the surface film on aluminium has been frequently studied, especially in connection with the asymmetric conductivity of that metal The readiness with which the surface film may be broken down by the current, when travelling from the metal as cathode, indicates, we think, the incompleteness of the oxidation producing the film

When a considerable amount of clean and pure aluminium foil is immersed in water containing dissolved air, the amount of which is kept up by bubbling or agitation, the amount of peroxide is considerably increased The passage of a current with the foil as anode and a small wire cathode still further augments the yield This was also observed in the case of zinc, and to a less extent for magnesium An attempt at quantitative estimations of the amount of peroxide produced was made by colorimetric tests, using starch and potassium iodide, and comparing with a very dilute solution of standard hydrogen peroxide With aluminium and zinc it was not possible to increase the yield by more than 1 part in 50,000, while for magnesium this was found much smaller, *i e* 1 part in about 600,000

The photographic effect of metals, which was thought at one time to be due to the emission of an emanation or of rays, has recently been shown by S Saeland† to be due probably to the formation on a clean metal surface of hydrogen peroxide produced by reaction with the moisture in the air The metals which were found to be most active were magnesium, aluminium, and zinc, which are the three we have found to yield measurable quantities of peroxide in water with dissolved oxygen Saeland produced the effects of the so-called metal rays by small traces of peroxide, and no trace of photographic action was observed from a clean metal surface in perfectly dry air or in hydrogen In the light of all these results we considered it highly probable that the deposit on the steel points obtained by Zeleny was due to moisture in the air surrounding the electrode which, under the influence of the electrified gas, was condensed around the negative oxygen ions and swept into the metal anode point Townsend has shown‡ that an electrified gas has the power of condensing moisture, even in an unsaturated

* 'Am. Electrochem. Soc. Trans.,' vol 13, p. 169, 1908.
† 'Ann. d. Phys.,' vol 26, p 899, 1908.
‡ 'Camb. Phil. Soc. Proc.,' vol. 9, p 249, 1897

atmosphere How minute a trace of water vapour in the air can be condensed on the negative ions has been indicated in some recent experiments of Prof Townsend.* The production of minute droplets in a gas containing only a small amount of moisture would probably not be observable by transmitted light unless viewed through a great thickness of gas Such a tenuous cloud is probably the cause of the haze often observed in the atmosphere† at low values of the relative humidity.

Our experiments appear to show that even a minute trace of water vapour is condensed around, and thus carried to the metal by the negative oxygen ions which cause the formation of the deposit, and that these ions alone are not able to cause the oxidation of the point even under a powerful electric field.

Before describing the apparatus used in this work, which was essentially the same as that employed by Zeleny, we wish to describe and, as far as possible, classify the various deposits observed on the different metal points

In every case the deposit was examined under the microscope after the point was removed from the apparatus. No observations could be conveniently made while the discharge was passing

Appearance of the Deposit.

It was found, at the outset, that a low-power microscope failed to reveal the character of the deposit, and merely indicated the presence of some foreign material at the point This, to the unaided eye, was in the majority of cases absolutely invisible A microscope of high power was, therefore, used to study the nature of the growths which we have classified under the different heads as follows —

I. Granular Formation
II Tubular or Icicle Formation
III Smooth Formation.
IV. Thin-film Formation

The four types are all probably connected with each other, but in appearance they are quite distinct

I *The Granular Formation.*

This formation appears in irregular growths which, under certain conditions, will extend to a distance of about a tenth of a millimetre from the surface of the point, but is usually of a far smaller order The shape

* 'Roy Soc. Proc.,' A, vol 81, p. 464, 1908
† Frank W Proctor, 'Monthly Weather Review,' vol 35, p 22, 1907

of these growths (see figs 1, 2, and 3) is very similar to that of ordinary rust, as seen under the same microscope They are composed of minute particles resembling powdered crystals, it is for this reason that we have called the deposit "granular" Thin portions show a translucence usually colourless, the thicker portions are almost opaque and vary from dark reddish brown (for steel) to grey black (in the case of copper).

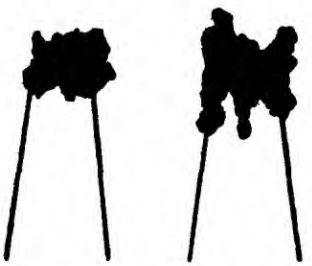


Fig 1.—Steel Anode Points.

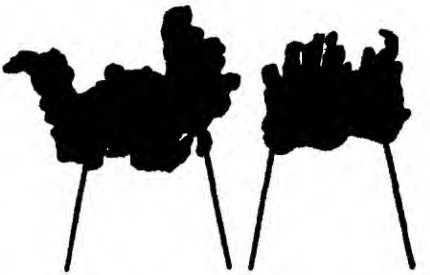


Fig 2.—Aluminium Anode Points.

This form of deposit is the most prevalent one, and may be considered as the final stage. It probably passes through types (II) and (III) before becoming granular In fig 4 is shown the amount of deposit on the same point discharging negatively

Fig 3.—Detached Granular Deposit.

Fig 4.—Steel Cathode Point.

II *The Tubular or Icicle Formation*

These formations are most interesting in appearance (see figs. 5 and 6) Varying from yellow to colourless they have almost the exact appearance of

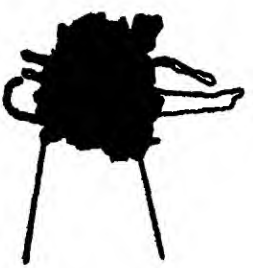


Fig. 5.—Anode Deposit with Five Tubular Growths.

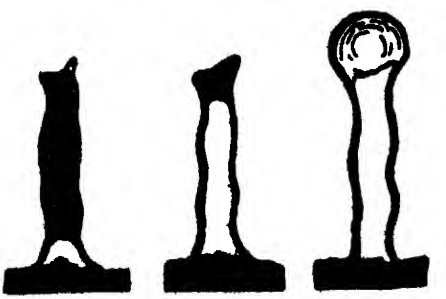


Fig 6.—Tubular Growth showing Water Core at Different Levels.

the common icicle A similar formation can be obtained by dipping aluminium into HNO_3 and then into Hg. It was observed on one occasion that a piece of freshly scraped nickel developed a few very small icicle formations without electric discharge.

The icicles are to be seen in three forms:—

(*a*) A growing form with liquid core.
(*b*) A permanent form with liquid core.
(*c*) A permanent form with hollow core

All three forms consist of a thin tube whose sides are apparently composed of the more opaque granular formations In (*a*) the tube is filled with a very mobile fluid which rises and falls rapidly with any change in the immediately surrounding humidity If there is sufficient change the fluid will rise beyond the extremity of the icicle, and form a large bubble which may burst. The film of this bubble is very thin and will sometimes show very pretty coloured interference effects In receding back into the tube, under the influence of dry air, the fluid leaves the icicle extended and further developed. In this condition the granular formation of the walls of the icicle becomes visible, and a large part of the transparency is lost, the tube appears to dry up, and the outlines of each granule stand out. If the fluid rises again it appears to soak into the sides, and the whole once more becomes translucent. These different phases are illustrated in fig 6

(*b*) The permanent form with liquid core:—The mobile fluid in the bubble often becomes thick, and eventually viscous, before receding into the core of the icicle. In this case the whole hardens into a permanent shape of a dark but even colour, in which case the granular formation cannot be distinguished.

(*c*) The permanent form with hollow core —This is merely the dry tube mentioned above in the special case when the fluid has receded first and then hardened, leaving behind a permanent granular icicle

III *The Smooth Formation*

This formation consists of the hardened fluid already described, existing by itself in flat spheroidal drops (see fig. 7) There has been no building up of granular tubes previous to the hardening of the surface. In colour this deposit is usually a dark yellow or brown.

IV. *The Thin-film Formation*

The thickness of this formation cannot be observed, it is perfectly transparent, and can only be seen as a definite band of interference colours across

the surface of the untarnished metal, just below the point (see fig 8). These zones of colouring are, however, always bordered on the side nearest the point by a very narrow black band, which seems to either merge into the region of the granular deposit, if there is any, or stop suddenly if there is

Fig. 7.—Smooth Formation.

Fig. 8.—Thin Film Formation.

none, and leave the point itself quite clean The black band, like the film, has no appreciable thickness. It is, however, often visible to the naked eye, and under long treatment will develop into a smooth black tarnish.

Production of the Deposit.

A *The Apparatus used.*—To permit the use of dry air, oxygen, etc., the point and plate were placed in a closed brass cylinder The cylinder used is shown in fig 9

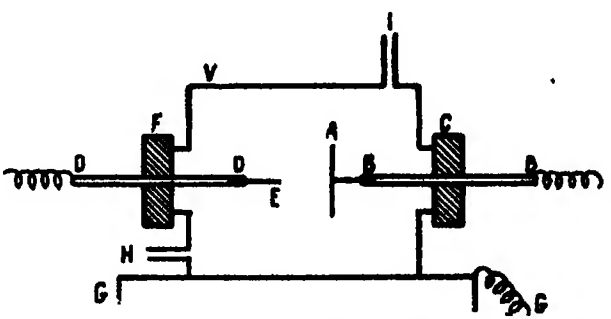


Fig 9.—Discharge Chamber

The figure is a vertical section The brass cylindrical vessel V is 13 cm high and 15 cm. in diameter It is kept connected to earth in order to prevent the apparatus becoming charged The plate A is a flat smooth brass disc, 6 cm. in diameter, which fits into a metal holder BB, connected to one terminal of the electric machine Besides the plate A, another plate, 1 8 cm. in diameter, two rings, and several rounded and plane wire ends were tried, but in most cases the plate A was found to be the best. The holder BB is insulated from the vessel by means of a hard rubber plug C, into which it fits tightly. In a similar manner a metal rod DD, connected to the other

terminal of the electric machine, holds the point E and fits into the insulating plug F The rods DD and BB are graduated, and can be moved to adjust the distance between the point and the plate The vessel V is made comparatively air-tight with the detached bottom G by means of tap-grease or vaseline There was a small removable window in the side of the vessel, which is not shown in the figure H and I are tubes, by means of which dry air, oxygen, etc, can be introduced and passed through the apparatus.

To generate the current a Wimshurst machine was used. Since the humidity of the summer weather, during which these experiments were performed, made it impossible for this open type of machine to be operated under ordinary conditions, it was mounted over a large electric heater, which thoroughly dried both it and the surrounding air The machine could be turned rapidly by means of a small motor. This enabled us to get a potential difference between point and plate when 2 cm. apart of over 20,000 volts

The air or oxygen was dried by passing first through a calcium chloride tower, then through sulphuric acid, and finally through cotton wool. In addition, a dish of H_2SO_4 was required within the vessel V, when perfect dryness was desired.

The points used were made of various metals (mentioned elsewhere) by filing as finely as possible with a revolving machine file, except in the case of steel, when ordinary needles of sizes 10, 8, and 5 were used

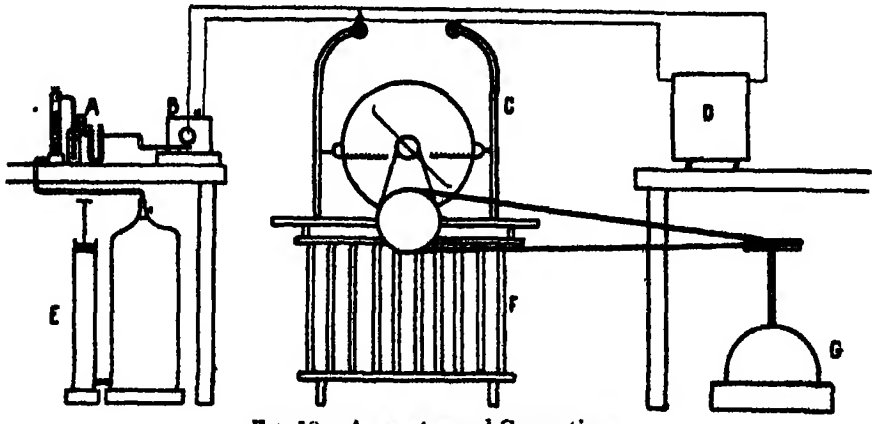


Fig 10.—Apparatus and Connections.

Fig 10 represents a diagram of the general connections:—

A is the drying apparatus.

B is the vessel V of fig. 9

C is the Wimshurst machine.

D is a Kelvin electrostatic voltmeter measuring up to 20,000 volts.

E is the air tank and pump—or else a gasometer for oxygen.
F is the electric heater
G is the motor

For the experiments using air under pressure, a specially air-tight cylinder and a mercury gauge were substituted for the vessel V In this case the cylinder is 20 5 cm long and 11 5 cm in diameter The plate used is the same as A, which is the plate used before The point and plate are supported and insulated in a similar manner to the former apparatus, but in this case the metal holders have to be waxed where they fit the insulating plugs, otherwise the vessel would leak under pressure The distance between the point and plane was adjusted by means of a metal sleeve

B *Methods and Conditions* —Metals used —The deposit can be produced on various metals The following list will give an idea of the relative amounts obtained on the more common metals when discharging as anode ·—

(1) Aluminium, (2) zinc, (3) steel, (4) cadmium—gave the largest deposits in order

(5) Tin, (6) magnesium, (7) silver, (8) brass, (9) copper—gave medium to very small deposits

(10) Platinum—gave fine tarnish after 40 minutes' treatment

(11) Lead, (12) nickel, (13) palladium—probably no deposit, but doubtful

(14) Gold—no deposit

In the case of the first five metals of the above list it was found that the points always gave a much larger quantity of deposit when they were positive electrodes than when negative The quantities formed on the remaining metals were so small that, with one exception, it was impossible to form a comparison The exception was silver, where the granular deposit appeared appreciably only when the electrode was discharging negatively, and the thin-film deposit only when discharging positively The deposit formed on any of the negative electrodes was always more evenly distributed over the point, it appeared, also, to be in a more finely divided state It was noticed that the potential difference for a given distance was always less (from 5 to 40 per cent, according to point) for negative discharge than for positive The shape of the two glows also differed considerably, and the positive was brighter at the surface of the metal.

Effect of Distance and Potential —Throughout the work the supply of current was kept about the same, hence the potentials always varied with the distance used, and it is therefore necessary to discuss the two together. Their effect on the amount of deposit is a considerable one, the table below gives the average results of several observations for each distance

	Distance.	Average potentials	Amount of deposit.
	cm	volts	
Using steel points	0·1	200	Small
	0·2	2,000	,,
	0·5	3,000	,,
	1·5	8,800	Medium
	2·0	10,300	Very large
	3·0	?	Medium
	5·0	18,000	Small
Using aluminium points . .	0·5	4,800	,,
	1·0	6,400	,,
	2·0	11,000	Very large
	2·5	13,000	Large
	3·0	16,000	Medium

(The first three results were affected by sparking, which always renders the reading of the potential difficult, owing to the erratic jumping of the voltmeter It also blows away the deposit from the point)

Fresh points were used each time, for we found that the number of times a needle has been used has an important effect on the amount of deposit produced.

It will be seen from the table that for both aluminium and steel the distance for maximum deposit is 2 cm. This distance also gave the best deposit for zinc.

Classification of Effects—In regard to these various influences on the production of deposits, we can classify them as having effects of the "First" or "Second Order." The sign of the current and the distance have effects of the first order, but time, within certain limits given below, has an effect of the second order

The shape of the needle has a close connection with the potential We found that very sharp points considerably increased the deposit, and blunt points developed only very small growths, but for various degrees of sharpness within these extremes there was not very much variation The effect must, however, be classed as one of the first order.

The time of discharge is not, as might be supposed, a very important factor. For the first 10 minutes when the deposit is commencing to form, the amount increases perceptibly with the time, and up to this period time may be called a factor of the first order, but after the first 15 minutes the deposit grows so very slowly that for all periods of increase less than half an hour one may say that time is a factor of the second order. This statement refers only to cases when the ordinary air is used.

The number of times a needle has been used has an effect on the amount

of deposit which is often of the first order. If a needle has been used very often, the deposit is much slower in growth, and consequently in a given time we get less deposit than with a new needle, but, on the other hand, there seems to be an increased tendency to grow, because with points that have been frequently used we get the deposit continuing to form in the open air after the discharge has ceased This apparently contradictory condition may be explained as follows —Upon close observation of many points, it was found that the electric discharge had a very strong disintegrating, or we might say "deforming," effect on sharp points (see fig 11) There is a con-

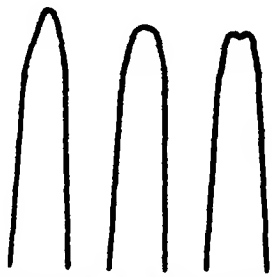


Fig 11 —Magnified Needle Points before and after a Short and Long Discharge.

siderable blunting of the point, and, in some cases, a small crater is actually produced at the extremity This can be observed in metals not producing deposits, as gold We can therefore deduce—(1) that the deposit is actually decreased because the point is blunted, but (2) that the real tendency of continual usage is to make a given part of a surface which remains unaltered in shape *more* capable of producing deposit instead of less, as the observation at first sight suggested We should really expect this to be the case, because another action of the current is to slightly roughen the surface, hence there is more surface exposed and we should have more deposit In all these cases of continued usage the needle has, of course, been repeatedly cleaned

Long enough usage of almost any shape of discharging terminal will ultimately tarnish the surface. Wire ends, sparking points of induction coils, and even the brass plates, all exhibit a considerable tarnish of the same nature as our deposit, provided the time of actual use can be measured in hours and days.

The question of the influence of the amount of moisture in the air is the most important determination of all.

So far the results were obtained by using the ordinary air of the laboratory. When the discharge was performed through air saturated with steam, we found that the amount of deposit was greatly augmented. The potential was, of course, greatly diminished for any given distance. It was also noticed that

the damp days were the most favourable for the production of large formations, especially of the icicle variety.

Using different degrees of dry air, we found that the amount of deposit varied accordingly, and, finally, if extreme precautions were taken for the use of as dry air as possible, there is complete absence of deposit In order to obtain this dryness it was necessary to place a large dish of H_2SO_4 in the apparatus, and to pass dry air through the vessel for at least 25 minutes previous to discharging This condition holds also for pure dry oxygen, which is a much more powerful deposit producer than air The use of calcium chloride bulbs is not sufficient alone

Decrease of humidity perceptibly decreases the deposit, but when we approach absolute dryness there is a sudden jump from a considerable amount of deposit to no deposit. It was found that we could repeatedly eliminate the deposit in perfectly dry air or oxygen, but if the least possible trace of moisture was introduced through the exit tube I by a very slight puff of damp air at the beginning of the discharge without stopping the flow of dry air, there was obtained an amount of deposit which was, in every case, at least half of the maximum amount obtainable with steam. This result shows that moisture must act as a catalytic agent in the production of these formations, and the fact that the addition of a minute trace of moisture to dry air raises the amount of deposit from zero to between 50 and 80 per cent. of its maximum value suggests that although aqueous vapour is necessary to start the formation, it is not necessary for the continuation of its growth when once started, but if present it will certainly accelerate the growth by forming fresh nuclei of action and thus extend the surface attacked.

Carbon dioxide does not apparently have any influence on the formation of deposit It was found that there was no difference between using perfectly dry air containing CO_2 and perfectly dry pure oxygen

The use of moist oxygen gave the largest deposits obtained for every metal The "moist" oxygen or "moist" air was obtained by allowing the gas to bubble through clean water before entering the apparatus It was found that almost dry oxygen gave a larger deposit than air under ordinary conditions. Moist oxygen increased the amount over that obtained with moist air by about 50 per cent.

By first allowing a spark discharge for a few minutes and then inserting the point, the presence of an appreciable amount of ozone was insured. This had the effect of slightly diminishing the deposit We do not, however consider this as a proof that ozone is a deterrent in the formation of deposit because in order to insure the non-escape of ozone it was necessary to close the apparatus and thus lessen the amount of available moisture and oxygen.

It was found that a faint glow was the most suitable kind of discharge If sparks crossed, the deposit was largely blown away If a smaller brass plate or if rings were used for the opposite electrode, the discharging surface was lessened and the deposit decreased

It was found that the mechanical circulation of new air or oxygen around the point greatly increased the deposit The air was blown on to the point with different velocities It was evident that the faster the motion of the air the greater the deposit, this fact held till the draught of air was strong enough to displace particles clinging to the point. In these experiments time was not of the second order in effect until the growth had continued for at least an hour The best results we always obtained by arranging that the current of air was at right angles to the electric current.

In regard to the liquid core of icicle formations, we found that there was an equilibrium point between the quantity of fluid in the tube and the immediately surrounding air If fresh supplies of air were rapidly brought into contact with the icicle while it was being observed under the microscope, the fluid rose in the tube, if moist air was used it rose and expanded into the large bubble described before If dry air was directed on to the formation, the fluid receded and left behind the dry tube extended and enlarged The phenomenon may be one of condensation and evaporation through the thin surface film

By keeping moist air blowing very strongly on a platinum point while discharging for more than forty minutes, we were able to obtain a faint black tarnish, showing that even this comparatively inert metal is influenced by the electric discharge

We made a large number of observations on the effect of pressure, but were unable to obtain very definite results.

At first sight these results appeared more contradictory than they really were. An examination of our observations shows that the following conclusions may be drawn —

1 With new and consequently very sharp points, pressure increases the deposit

2 With points that are not new and which have, consequently, their surfaces affected and their points blunted, increase of pressure decreases the deposit.

3 For ranges of pressure less than one atmosphere, the effect of change of pressure is of the second order and may act in either direction according to the condition of the point

It was found that the smooth formation could generally be produced by placing a point coated with icicle formations in very moist air, or better, in

moist oxygen, and discharging at a shorter distance than usual. The icicles are apparently broken down into the granular formation, and the fluid adheres in spheroidal drops These may again grow icicles.

Points coated with tap-grease yielded a small but very compact deposit. The insulating grease is evidently easily pierced.

Conclusions

With the possible exception of platinum, in which case the formation may be platinum black, we think it safe to assume that the deposits are the oxides of the metals This is supported by the fact that the growth is much greater when the point is positive than when negative In regard to this circumstance, Prof Zeleny says, in his paper, the reason why the oxide forms so much more readily during the positive discharge than it does during the negative may be that in this case negative ions of oxygen are carried from the surrounding air to the metal surface, and forming as they do the negative parts of the iron oxide molecules, they unite more readily with the iron than do the positive ions which are carried to the metal surface during the negative discharge.

Since it seems certain, from our experiments, that moisture is essential to the production of the deposits, it appears probable that in comparatively dry air the negative ions possess greater aptitude for collecting water vapour than do positive ions, just as in the case of a supersaturated atmosphere

The appearance of the icicle or tubular deposit, which appears to be the beginning of the more permanent deposit on the point, looks as though it formed around a minute droplet of water, probably hydrogen peroxide. Each droplet is probably formed by the coalescing of the moisture condensed around several oxygen ions As soon as this wets the metal surface, a ring of oxide is formed and appears to be drawn by surface tension, in a thin film, around the drop. As the film thickens it builds up from the base a hard compact tube, leaving the surface film stretched across the end. This film separates the liquid from its vapour and appears to permit the passage of the vapour Hence in a dry atmosphere the liquid rapidly evaporates and leaves the tube hard and compact, in an atmosphere containing an appreciable amount of water vapour it remains in equilibrium; while in a moist atmosphere the vapour is condensed through the film and the volume of liquid increased. Thus, as we have already described, when a tube is blown upon under the microscope with very moist air, the liquid appears to rise and swell out the end in the form of a bubble that may burst the film coating When dry air is used to blow over the tube, the liquid appears to recede into the bone-like tube and disappear altogether.

The air in our laboratory is fairly moist in summer, with a humidity ranging from 60 to 90 per cent, which was the case when these measurements were made. Hence, it was fairly easy to examine the rising and falling of the liquid core of the tubes under the microscope. In winter, however, the humidity of our heated laboratory is low, ranging from 3 to 20 per cent., depending on the outside air temperature.* Hence, at this time, any tubular forms that are produced in our apparatus with added moisture dry up before they can be removed to the microscope, only the dried-up hollow tube being observed. If the droplet is increased when the sides have hardened we get an extension of the surface which slowly hardens after the manner of the natural water icicle Hence, tubes will grow by themselves on points removed from the discharging chamber if the atmosphere is very damp

If the droplet does not grow, the hardening of the surface produces the smooth formation In regard to the order of development of these different deposits, we can regard the tubes or icicles and smooth formations as first produced. The mechanical agitation of the air by the discharge will break most of the more delicate tubes immediately they are formed into the common and irregular granular deposit.

How far oxygen becomes occluded in the metal under electric pressure is uncertain, but it looks as though it might be in considerable quantities, and thus accelerate the growth of deposits after the discharge ceases The influence of moisture is very marked on the potential required to start a spark, as observed by J J Thomson, who describes a dry gas as being in an unstable state as far as many of its electrical properties are concerned. On the application of a potential greater than is required to start the spark, the "lag" in a perfectly dry gas is very much greater than in one containing only a trace of moisture Hence the moisture acts as a kind of lubricant, either as a surface effect or in hastening the ionisation of the medium.

In conclusion, we add a note kindly supplied by Prof Zeleny, to whom we submitted the manuscript The work is closely connected with the valuable study Prof Zeleny has been making of Point Discharge, and we are especially gratified to be allowed to include his views here

Note by JOHN ZELENY, Professor of Physics, University of Minnesota, Minneapolis

The fact that the presence of water is always necessary for the formation of oxides has been explained by supposing that the chemical union can take

* Compare "Deficient Humidity of the Atmosphere," by T A. Starkey and H. T Barnes, 'Roy Soc. Can. Trans.,' (2), vol 12, p. 203, 1906.

place only in the presence of an electrolyte The contact of the two elements with the water introduces ions of both into the liquid, where they unite to form the oxide The results of Prof Barnes and Mr Shaw show that water is likewise essential to the formation of these deposits (evidently oxides) on discharging points The difference in this case is that the oxygen comes to the metal in the ionic state, and since no union takes place when dry gas is used, we may conclude that both the oxygen and the metal must be in the form of ions before the two can unite chemically

It is very probable, as the authors assume, that most of the water is carried to the positive points by the ions themselves In measuring the velocities of the ions which are produced by Rontgen rays, I found that while the values obtained for the positive ions in air and oxygen were practically the same whether the gases were dry or moist, the velocities of the negative ions were considerably smaller when the gases were moist This indicates that the negative ions become larger in the moist gases by attracting to themselves one or more molecules of water A simple calculation shows that the accumulation of this water on the point is sufficient to account for the observed drops of liquid.

The existence of drops of liquid on the discharge point indicates that the average temperature there is not as high as the luminosity present in the adjacent gas might lead us to expect An attempt was made to determine the rise in temperature of a point during discharge by using a compound discharge point made of two sharply-pointed wires of dissimilar metals held in contact at an acute angle at their very tips The wires were included in a galvanometer circuit so that the compound point formed one of the junctions of a thermo-electric circuit, which was grounded near the point to prevent the discharge current from affecting the galvanometer The discharge from the point was induced by a charged plate placed opposite to it. The observed rise in temperature was 10° C for a discharge current of about 10^{-6} ampere The actual rise in temperature at the discharging surface itself must be considerably greater than that observed, for the heat has to be conducted from the discharging surface to the surface of contact of the two metals, and the loss to the larger metal parts back of the point must be a large one

On the Effect of Temperature on Ionisation.

By J A Crowther, M.A., Mackinnon Student of the Royal Society, Fellow of St John's College, Cambridge

(Communicated by Prof Sir J J Thomson, F.R.S Received April 1,— Read April 29, 1909)

The effect of temperature on the ionisation produced in a gas by Rontgen rays was first investigated by Perrin,* who, using air, concluded that the total ionisation in a gas was independent of the temperature if the pressure was kept constant McClung,† however, who repeated these experiments later with air, carbon dioxide, and hydrogen, found that the ionisation in a gas was independent of the temperature if the density of the gas is kept constant, that is, if it is heated at constant volume

Although no source of error could be indicated in Perrin's work, there was little doubt that the later experiments of McClung were correct, and that between the limits of his experiments (15° C to 272° C), and for the gases used, the ionisation produced by Rontgen rays was independent of the temperature when the gas was kept at constant density

It is well known that the ionisation produced by rays of given intensity in certain gases and vapours, for example methyl iodide, ethyl bromide, or carbon tetrachloride, is much greater than that in air, or carbon dioxide. The present investigation was made to discover—

(i) If the effect of temperature on the ionisation produced in these gases and vapours was the same as for air

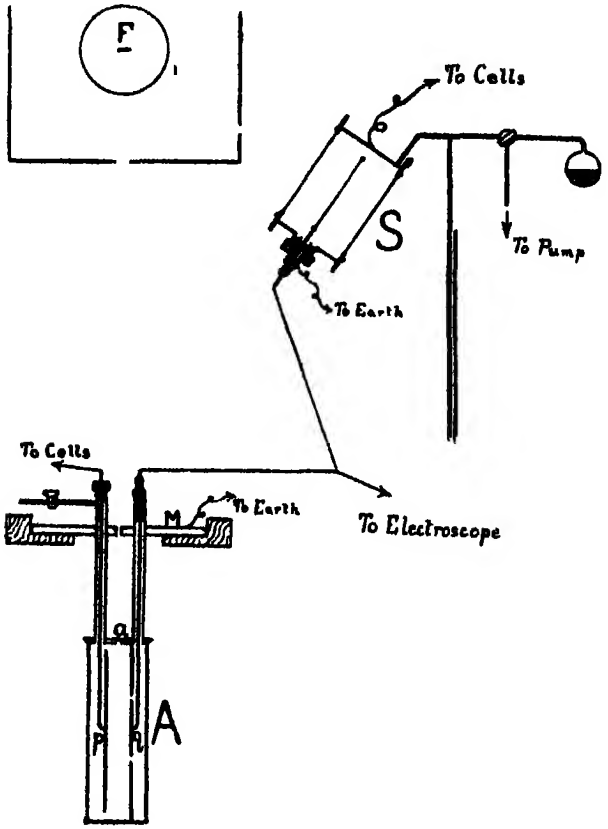
(ii) If cooling down air to a temperature near its condensation point caused any appreciable alteration in the ionisation produced in it by rays of given intensity.

As it is almost impossible to clean out completely a vessel which has once contained organic vapours, the second experiment was performed first

In order to perform the experiments at low temperatures, the ionisation chamber had to be constructed so that it could easily be inserted in an ordinary Dewar tube. Its dimensions, therefore, were kept quite small At the same time it was necessary that the insulations should be sufficiently far from the liquid air to enable them to be kept practically at the temperature of the room. Any considerable fall in temperature of the insulations invariably led to the deposition of moisture upon them, and a consequent

* 'Annales de Chim. et de Phys.,' [7], vol 11, p. 496, 1897.
† McClung, 'Phil Mag.,' [6], vol 7, p. 81, 1904.

breakdown of their insulating properties The form finally decided upon is shown in the figure.



The body of the ionisation chamber A is a brass tube 10 cm. long and 3 cm in diameter. The electrode q, which is to be connected to the electroscope, is a small aluminium plate $2\frac{1}{2} \times 1$ cm., surrounded by an earthed guard ring and insulated with quartz The high-potential electrode p is a larger aluminium plate, placed parallel to q, and about 1 cm from it. The electrodes pass out through two brass tubes each 8 cm. long M is a heavy brass plate which serves the double purpose of maintaining the insulations at an equable temperature, and of acting as a support to the can A. When A is in liquid air, M can be kept warm by small heating spirals, and during the heating experiments it can be kept cool by ice. As the insulations are thus shielded from temperature changes, the junctions here can be made air-tight with sealing wax.

The Röntgen rays enter the chamber A by a circular hole a, 5 mm. in

diameter, closed with very thin copper foil. As A is intended to be heated up, it is obviously impracticable to have a sealing wax joint at *a*. By substituting copper for aluminium as the material of the window, it is possible to make the joint air-tight by soldering. The electrode *q* is sufficiently far from the window *a* for all the soft secondary rays emitted by the latter to be absorbed by the gas before reaching it, on account of the thinness of the foil used the penetrating secondary rays were not sufficient in quantity to have any appreciable effect on the ionisation. The vessel A is adjusted with respect to the X-ray bulb, so that the rays pass through the chamber without striking the electrodes This adjustment is made by the use of a fluorescent screen before soldering on the bottom of the vessel, and the absence of any soft secondary radiation is tested for by drawing the ionisation-pressure curve for the vessel

In a temperature chamber intended for work with gases at constant density, it is necessary to ensure that practically the whole of the gas shall be at the same temperature, otherwise, since the pressure is the same throughout, there will be a transference of gas from the hotter to the colder parts of the vessel, and the density of the gas between the electrodes will change For this reason the chamber A was cut off from the gauges when readings were being made. For the same reason the tubes through which the electrodes pass out of the chamber were made as narrow as possible, about 4 mm internal diameter In this way the volume of gas under experiment not actually within the heating furnace did not exceed 3 per cent of the whole

In order to eliminate the effects of any alteration in the intensity or hardness of the rays, and also in order to be able to detect with more certainty any small alteration in the ionisation in A, a "balance" method was used The standard chamber S was of the usual flat box type in which the case forms the high-potential electrode, the other electrode consisting of an aluminium leaf stretched over a wire ring. The flat sides of the chamber are of aluminium foil to allow of the entrance of the beam of rays. The insulation, as in the other chamber, was of quartz. The two chambers were connected to the same electroscope, and their high-potential electrodes were charged, by small storage cells, to equal and opposite potentials The ionisation currents were, therefore, in opposite directions in the two chambers.

It is well known that the relative amounts of ionisation produced by Rontgen rays in different gases vary with the nature of the rays. In order to avoid, as far as possible, any disturbance in the balance between the two chambers, due to alterations in the hardness of the rays, the standard

chamber S was always filled with the gas which was under observation at the time in the chamber A. In making an experiment, the two chambers were filled with gas from the same source to some suitable pressure, and the tap connecting A to the reservoir was then closed The chamber A then contained a definite mass of gas, which remained the same throughout the experiment The pressure in S was then altered until the magnitude of the current through the two chambers, when the rays were passing, was the same, as shown by the leaf of the electroscope remaining stationary The temperature of A was then altered, and a balance again obtained With air in the chambers it was found quite possible to balance to within 3 per cent. of the total ionisation in the chamber, with ethyl bromide and methyl iodide, which were subsequently employed, an even greater degree of accuracy could be obtained Quite a small change in the ionisation in A, due to alteration of temperature, should, therefore, have been perceptible

Experiments on Air

The two ionisation chambers were pumped out repeatedly by means of a water pump, and filled with air which had been freed from carbon dioxide and water vapour by passing first through tubes containing caustic potash and calcium chloride, then through a long tube filled with a mixture of phosphorous pentoxide and asbestos (the asbestos being used to prevent the pentoxide from caking, and clogging the tube), and finally through a plug of glass wool The vessel A was closed, and a balance obtained by altering the pressure in S A Dewar tube filled with liquid air was then brought underneath A and gradually raised until the level of the liquid air was just below the upper rim of the vessel Violent ebullition at first took place, but after a few minutes a steady state was reached On again turning on the Rontgen rays it was found that the balance was quite undisturbed, the ionisation in A being apparently unaffected by cooling it down in liquid air. A change of as much as 3 per cent. in the ionisation would have been quite appreciable, but no such change was discovered

The experiment was repeated many times, but always with the same result. In some of the experiments a capillary gauge of fine bore was connected to the vessel A so that the pressure in A could be read at any stage in the experiment When the liquid air was applied the pressure fell rapidly at first, but finally became stationary at about 470 mm. of mercury, indicating a mean temperature for the air in A of below $-170°$. The conduction of heat along the metal tubes leading from the vessel was probably sufficient to prevent the air in it from falling quite to the temperature of the liquid air. As mentioned before, the electrodes were kept

warm by small heating coils wound round the tubes. The flow of heat along the tubes was therefore considerable.

Taken in conjunction with the experiments of McClung, these experiments may be regarded as showing that between the limits $-170°$ C and $+273°$ C the ionisation produced by Rontgen rays in a given volume of air at constant density is independent of the temperature of the gas

Experiments on Vapours

The experiments on vapours were made not by lowering but by raising the temperature of the ionisation chamber A For this purpose a small electric furnace was wound round the outside of A, a thermometer being inserted between the furnace and the walls of A to indicate the temperature A very convenient method of making an electric heater for a purpose such as this is to thread a German silver wire through an asbestos tube and wind the two evenly round the outside of the vessel A few layers of asbestos cloth are wrapped round the outside, and the lower end of the can is covered with a packing of loose asbestos. The upper end, with the window *a*, has, of course, to be left uncovered in order to admit the beam of Rontgen rays With this device there is no danger of the different turns of wire slipping, when they expand on heating, and so causing short circuits in the furnace; while, on the other hand, when repairs or alterations have to be made in the chamber A the whole furnace can be taken to pieces and rewound in a few minutes

Methyl iodide and ethyl bromide were selected for experiment, these two vapours differing widely in their behaviour under the action of Rontgen rays both from air and from each other. It may be mentioned in passing that at high temperatures both these substances pass readily through indiarubber The connection between the vessel A and the gauges was therefore made by slipping a slightly wider glass tube over the brass exit tube of A, the joint being rendered air-tight with sealing wax The whole of the apparatus for passing the vapours into the vessels and for measuring their pressure was of glass.

Several observations were made with each of the two chosen vapours They all presented precisely similar features. A sample set of readings for each substance is given in the appended table

As a balance method was employed, the readings in the columns headed "Ionisation" give the pressure in S of ethyl bromide or methyl iodide, as the case may be, necessary to balance the ionisation current through the same substance in A at the stated temperatures.

It will be noticed that in both series of observations given in the table,

and in fact in every series made with these vapours, there is an initial rise in the ionisation in the chamber A on first raising the temperature. At a temperature of about 70° this increase ceases, and the ionisation in A remains constant within the limits of experimental error up to the highest temperatures reached

Table

Ethyl bromide	Pressure, 146 mm	Methyl iodide	Pressure, 110 mm
Temperature	Ionisation	Temperature	Ionisation
14°	62·0	15°	61 7
89	79 2	72	67·0
136	79·0	103	67 5
173	79·0	146	67·0
184	79 4	184	67 2

This initial increase in ionisation is not of the same magnitude in different series of experiments made with the same vapour It varies, in fact, with the length of time during which the vapour has been standing in the chamber before the readings commence Thus, in a set of readings in which ethyl bromide was admitted into the chamber A less than an hour before the readings began, the ionisation on first heating increased from 70 to 75, remaining constant at the latter value when the temperature was still further increased In the experiments recorded in the table, the chamber had been filled with ethyl bromide vapour overnight, and the initial increase is proportionately much bigger

These results, and the fact that from a temperature of about 70° upwards the ionisation at constant density is independent of the temperature, show that this initial increase in ionisation is not a genuine alteration in the ionising power of the rays with alteration in temperature. It must, in fact, be ascribed to the vapour which condenses on the walls of the chamber at ordinary temperatures being driven off as the temperature is raised, and so increasing the effective amount of gas in the chamber.

It is easy to show that very appreciable amounts of vapour are so condensed, even when the vapour is considerably removed from its saturation point Thus in one experiment, when the vessel was filled with ethyl bromide and allowed to stand, the pressure fell from 260 mm. of mercury to 253 mm in the first half hour, and finally, after standing all night, to 214 mm When methyl iodide vapour was similarly left to stand in the vessel overnight the pressure fell from 215 mm. of mercury to 181 mm.

It will be seen that these effects are of the same order of magnitude as the initial increase in ionisation on heating the chamber A There can be little doubt, therefore, that this initial increase is due to the vapour being given off from the walls of the vessel as the temperature is raised

We must conclude, therefore, that for the very ionisable vapours such as ethyl bromide and methyl iodide, as well as for air, the ionisation by Rontgen rays, when the density of the gas remains constant, is independent of the temperature of the gas.

Summary

The ionisation produced by Röntgen rays has been measured in air at the temperature of liquid air, and in ethyl bromide and methyl iodide at various temperatures up to 184° C. It was found that in every case the amount of ionisation produced was independent of the temperature of the gas, if the density of the gas remained constant

I have much pleasure, in conclusion, in expressing my best thanks to Prof Sir J J Thomson for his constant and kindly interest throughout the course of the present experiments

The Ionisation in Various Gases by Secondary γ-Rays.

By R D Kleeman, D Sc (Adelaide), B A (Cantab), Research Student of Emmanuel College, Cambridge

(Communicated by Prof Sir J J Thomson, F R.S Received April 3,—Read April 29, 1909)

In a paper published in the 'Philosophical Magazine,'* the writer has described some experiments carried out with the secondary γ-rays emitted by various substances exposed to the primary γ-rays of radium It was found that the secondary rays are, on the whole, softer than the primary, thus, for example, the coefficient of absorption of lead for the secondary rays from zinc was found to be about six times that obtained for the primary rays It was thought, therefore, that the ionisation of gases by secondary γ-rays would probably differ in many features from that obtained with the primary rays A set of measurements on the ionisation produced in various gases by secondary γ-rays was accordingly undertaken. No experiments, as far as the writer is aware, have yet been made on this subject

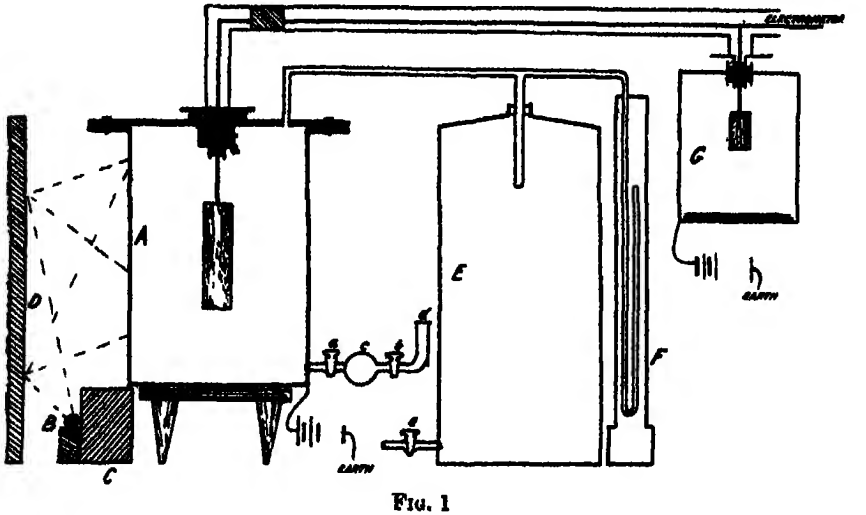


Fig. 1

Fig 1 is a diagram of the apparatus used in these experiments A is a cylindrical ionisation chamber of brass, 17 cm. high and 12 5 cm. in diameter, the walls of the chamber being about 3 mm. thick. B is a glass tube (surrounded by lead sheeting 2 mm. thick), which contained about 30 milli-

* p. 637, May, 1908

grammes of radium bromide The chamber A was approximately screened from the radium by the lead block C, 5 5 cm. thick The γ-radiation from the radium fell in part on the radiator D, which consequently became a source of secondary γ-rays. Some of the secondary rays traversed the chamber A, ionising the gas it contained

The radiator D was held in position by means of an arrangement shown in fig 2 It consisted of three fixed blocks of wood provided with slits, whose width was slightly greater than the thickness of the radiator The radiator was held to one of the sides of each slit by means of a wedge The radiator therefore always occupied its previous position if removed and again replaced.

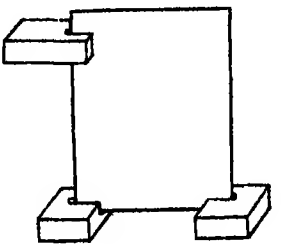


Fig 2.

The chamber A and the glass flask E, which were connected with one another by a rubber tube, could be exhausted to a pressure of about 0 5 cm of mercury by means of a Fleuss pump The pressure in the chambers was indicated by a mercury pressure gauge F

The part *a*, *b*, *c*, *d* of the apparatus was used to introduce the vapour of a liquid into the chamber This was done by first closing the tap *b* and opening the tap *a*, and exhausting the chamber down to as low a pressure as it was possible to obtain with the pump, and then closing the tap *e* Some of the liquid to be used was then poured into the funnel *d*, and by opening the tap *b* part of the liquid was allowed to run into the bulb *c* Evaporation of the liquid into the chamber then took place, and was allowed to go on until some convenient pressure of the vapour was reached, the tap *a* being then closed The residual air in the chamber A (which could not be removed by the pump) was swept by the vapour into the chamber E

G is an ionisation chamber in which the air was ionised by a layer of uranium oxide The electrode of this chamber was connected with that of the chamber A, and its leak used to compensate in part for that in the latter chamber, by keeping the chambers at potentials of opposite sign The apparent leak was kept by means of this arrangement down to a workable amount. The chamber A was connected to a positive potential of 200 volts, while the chamber G was connected to a negative potential of 200 volts

The ionisation per second due to the secondary γ-rays from the radiator D was obtained by measuring the leak with and without the radiator, the difference between the leaks giving the ionisation in the chamber A due to the secondary γ-rays. The leak per second was in each case obtained by observing the time it took to obtain approximately a given convenient

deflection of the electrometer needle, the current being then broken and the first and second swing of the needle read The position of rest of the needle was then calculated by a simple formula given by the writer * By using approximately equal deflections, the error due to change of capacity of the electrometer with the deflection of needle was avoided Moreover, the loss of leak due to leakage over insulation, etc., was eliminated in this case when the difference between the leaks with and without the radiator was taken ,† this loss was, however, comparatively small in these experiments

Ionisation and Pressure.

The ionisations in different gases can only be compared with one another if the ionisation in each case is proportional to the pressure, the ionisation being then proportional to the intensity of the ionising agent This is realised if only a small fraction of the energy of the ionising agent is absorbed by the gas, for then the ionisation is proportional to the mass of the gas, and consequently proportional to the pressure It has been shown by several observers that the ionisation in a gas by the γ-rays of radium and the β-rays of uranium is in each case proportional to the pressure up to pressures greater than those necessary to determine the ionisation with accuracy.‡ Since the penetrating power of the secondary γ-rays is greater than that of the β-rays, and the percentage absorption therefore smaller in the former than in the latter case, this should also be true for the former rays It was thought better, however, to make some experiments to test this point, since it was not known to what extent the radiation from the walls of the vessel affected the ionisation

The gases used in these experiments were air and ethyl bromide The ionisations obtained with different pressures are plotted against the pressures in fig 3 It will be seen that two approximately straight lines are obtained which pass through the origin. The ionisation is therefore proportional to the pressure between the limits of pressure used The density of the ethyl bromide vapour for the largest pressure used was greater than the density of any of the gases in the determination of the ionisations, and therefore the fraction of the energy of the ionising agent absorbed was greater than it was in any case in the latter experiments. It was therefore thought unnecessary to make any further experiments with different gases in order to show that the ionisation varied as the pressure in each case

* 'Phil. Mag.,' p. 276, October, 1906
† See 'Phil. Mag.,' p. 642, May, 1906
‡ Strutt, 'Phil Trans.,' A, vol. 196, p. 507, 1901, etc

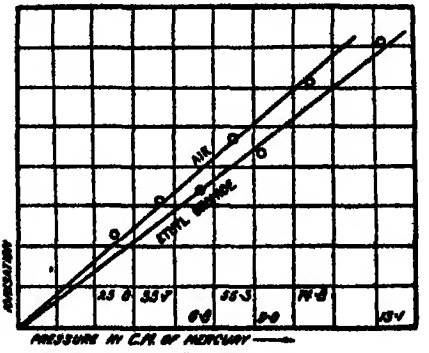


FIG 3

Nature and Causes of Ionisation in the Chamber

The ionisation in the chamber was partly caused by the penetrating cathode radiation given off by the walls of the chamber under the action of the γ-rays This is objectionable, but it seemed unavoidable The method that has been used in the case of X-rays in confining the beam of rays to the centre of the chamber so that it touches the walls only where it enters and leaves did not appear practicable in this case And this for two reasons firstly, it is impossible to obtain a well-defined beam of γ-rays by means of a metal stop, on account of the great penetrating power of the rays, and also on account of the necessarily large surface of the aperture becoming a source of secondary γ-radiation ;* secondly, the gas *itself*, under the influence of the γ-rays, gives off penetrating cathode rays which would cross the screened part of the chamber, and thus produce secondary radiation from its walls. A correction would therefore have to be applied as to the magnitude of which nothing would be known, so that nothing would be gained by using such an arrangement It was therefore thought better to use the simpler though somewhat undesirable form of apparatus described.

The ionisation in a chamber of the form used may consist of three parts. One part may consist of ions ejected by the γ-rays from the gas molecules with a velocity which is so small that they are unable to produce any further ions themselves. One of the other parts we know consists of ions made by the secondary cathode rays from the walls of the chamber, and the third part of ions made by the cathode rays of high velocity ejected from the gas molecules. The first of these parts is proportional to the mass of the gas and therefore proportional to the pressure, and this is also true for

* This secondary radiation further complicates matters on account of its nature not being the same as that of the primary; the larger part produced would enter the chamber because it is initially projected in the direction of the exciting rays.

the second part, since the ionisation of a gas by β-rays has been shown to be proportional to the pressure Since the number of electrons ejected from the gas is proportional to the pressure, and the number of ions each produces proportional to the pressure, the third part is proportional to the square of the pressure. The ionisation in the chamber may, therefore, be written $(A+B)p+Cp^2$, where p denotes the pressure.

Laby and Kaye* have shown that the ionisation in air and carbon dioxide is proportional to the pressure over a wide range of pressures The term Cp^2 is, therefore, small within this range Experiments on the variation of the ionisation with pressure will, however, not give any information as to the relative values of A and B, since both the ionisation produced by the secondary cathode radiation from the walls of the chamber and that due to the ejection of slow-moving ions from the molecules of the gas by the γ-rays vary as the pressure Attempts have been made to obtain an estimate of the amount of ionisation in a chamber which is not due to the radiation from the walls All these estimates depend on some calculations based on certain assumptions, generally involving the absorption of the cathode rays produced by the γ-rays. Now the γ-rays produce cathode rays whose penetrating powers vary very considerably, and these calculations are therefore not satisfactory, they may easily be very considerably out In fact, McLennan† deduced that the ionisation in a chamber not due to the secondary radiation from the walls is equal to about one-half of the total ionisation in the chamber, while Wilson‡ makes it equal to one-sixth of the total ionisation, it does not appear certain, therefore, that the total ionisation is not due entirely to the secondary radiation from the walls The writer, therefore, made some direct experiments to test this point

Several slightly different arrangements were used, each of which involved the deflection of the secondary radiation from the apparatus by means of a magnetic field. A diagram of the apparatus first used is shown in fig 4 A is an ionisation chamber on a lead block B, 5 cm thick, the chamber being placed symmetrically with respect to an aperture a in the block Thirty milligrammes of radium were placed at C underneath the aperture. A magnet, whose poles measured 5 5 cm by 5 5 cm., was placed so that the chamber was between its poles Since the electrons produced by γ-rays are ejected in the direction of propagation of the rays, there was a diffuse pencil of β-rays projected into the chamber from its lower side If the ionisation is produced by the β-rays from the walls of the chamber, the

* 'Phil. Mag,' p 879, December, 1908.
† 'Phil Mag.,' December, 1907.
‡ 'Phil. Mag.,' p. 216, January, 1909.

ionisation ought to be principally due to this beam, and should therefore decrease considerably when a strong magnetic field is applied It was found, however, that a magnetic field produces little change in the amount of ionisation. Thus, in a particular case, the ionisation in a sheet lead chamber 5·5 cm. high, 5·5 cm. broad, and 7·5 cm long, was 1895 in arbitrary units. When a field of over 2000 units was applied, which was sufficient to bend the rays having the same velocity as the penetrating β-rays from radium into a circle of radius less than 8 mm, the ionisation current decreased to 1645, or about 12 per cent. When a chamber 7 cm high, 4 cm long, and 3 cm. deep was used, the current was decreased from 1560 to 1475 when the magnetic field was applied, or about 6 per cent If all the ionisation in the gas was produced by the β-rays from the walls of the chambers, the

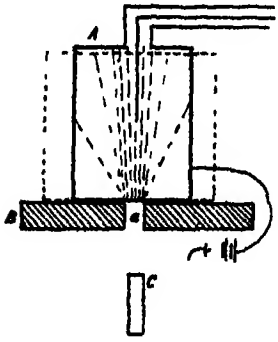


Fig 4.

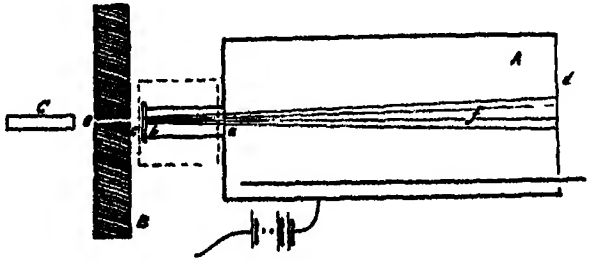


Fig 5.

ionisation should have decreased to a greater extent in each case, although a part of the path of each ray when deflected was still contained by the chamber, and consequently produced ionisation

Since part of the leak in this experiment was always due to the β-rays from the walls of the ionisation chamber, it was thought desirable to carry out the experiment in a somewhat different form Fig. 5 gives a diagram of the apparatus used. A is a cylindrical ionisation chamber, 19 cm. long and 8 cm. in diameter, to which the tube *ab*, 6·5 cm. long and 3·5 cm in diameter, was co-axially attached The chamber was placed so that the tube *ab* was between the poles of an electro-magnet The end *a* of the tube was closed in one set of measurements by a sheet of thin aluminium leaf equivalent in mass to a layer of air 0·7 cm thick, the end *b* of the tube being closed with a metal plate *c* The end *d* of the chamber A was closed with a thin sheet of zinc B is a lead block, 3 cm thick, through which a circular hole *e* was drilled about 1 cm. in diameter. The axis of this hole and the

glass tube *c* containing 30 milligrammes of radium (which was surrounded by sheet lead 2 mm thick) were placed co-axially with the tube *ab* and the ionisation chamber A The ionisation in the chamber was produced principally in the cone *f* by the γ-rays from the radium and the secondary β-rays from the plate *c*, the secondary β-rays being initially projected in the direction of propagation of the γ-rays * The secondary β-rays produced in a plate are to some extent scattered by the plate, but the larger part of the radiation proceeds approximately in the direction of propagation of the γ-rays,† and the larger part of the radiation from the plate *c* therefore entered the chamber

The measurements were carried out by first placing a lead plug into the hole *e* of the lead block B and measuring the leak in the chamber This gave the leak in the chamber due to imperfect screening of the lead block B The leak was then measured with the plug removed. The difference between this leak and the former gave the ionisation in the cone *f* due directly to the γ-rays and the secondary β-rays from the plate *c* A magnetic field of sufficient strength to bend the β-rays from *c* so that they did not enter the chamber A was then applied, and the leak again measured. The difference between this leak and the first gave the leak in the cone approximately due to the direct action of the γ-rays

Whether the strength of the magnetic field was sufficient to bend the β-rays from the plate *c* so that they do not enter the chamber A was tested as follows The plate *c* and the lead sheeting surrounding the radium was removed so that a pencil of β-rays from the radium now penetrated into the chamber When the current used in these experiments was switched on to the magnet, the ionisation was decreased to about 20 per cent. of its original amount, showing that practically all the β-rays were prevented from entering the chamber by the magnetic field

Firstly, some measurements were made with no aluminium leaf placed at *a* It was found that using a lead plate at *c* 2 mm. thick, the ionisation in the cone *f* decreased 15 per cent. when the current was switched on to the magnet, when the plate was of aluminium 3 mm. thick the decrease was 23 per cent

An aluminium leaf was then placed at *a*, the ionisation in the tube *ab* being now excluded from the leak The decrease obtained with the lead radiator when the magnetic field was applied was 23 per cent, and 31 per cent. with the aluminium radiator.

These figures show that when the leak due to the β-rays from the plate *c*

* Bragg and Madsen, 'Roy Soc. South Australia Trans.,' vol. 32, January, 1908

† Kleeman, 'Roy. Soc. Proc.,' A, vol. 82, 1909

is eliminated, there still remains a large proportion of the total leak, and a considerable part of the ionisation in the cone *f* is therefore due to the direct action of the γ-rays on the gas

Some further measurements were carried out with a slight modification of the foregoing experiment A diagram of the modified apparatus is shown in fig 6 A is an ionisation chamber 10·5 cm long, 10·4 cm broad, and 7 cm deep, of which the upper and lower side consisted of thin, tightly stretched tissue paper, equivalent in mass to a layer of air 1 cm thick The chamber was placed on the poles B_1 and B_2 of an electro-magnet, which were resting on a lead block C, 5 cm thick This lead block had an aperture *a*, 3 cm by 3·2 cm., which was placed in a symmetrical position with respect to the poles of the electro-magnet and the ionisation chamber D is the tube containing the radium, placed at a distance of 10 cm from the lead block The electrode of the chamber consisted of a wire bent into the form of a square, so that the principal stream of γ-rays through the aperture did not impinge upon it The ionisation in the chamber was, as before, principally due to the direct ionisation of the gas by the γ-rays and the secondary β-rays from the plate *b* placed over the aperture *a* The plate *b* was of aluminium 4 mm thick

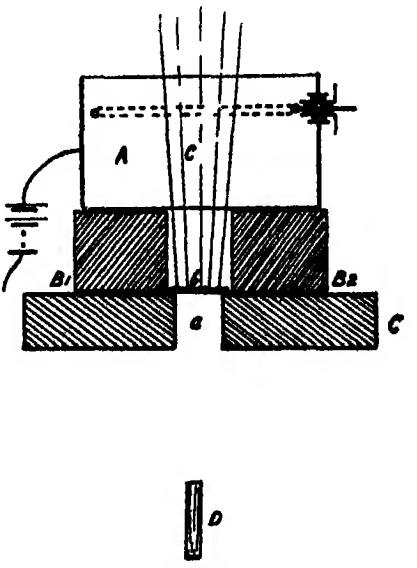


Fig 6

The readings were carried out in exactly the same way as in the foregoing experiment. The ionisation in the cone *c* decreased to about 55 per cent. of its original value, when a magnetic field of sufficient strength to prevent the β-rays from the plate *b* entering the chamber was applied Part of the ionisation in the cone *c* is thus, as obtained before, due to the direct action of the γ-rays on the gas

It appears, then, from these experiments that the ionisation in a chamber is due in part to the direct action of the γ-rays on the gas it contains. And since Laby and Kaye have shown that the amount of ionisation produced by the secondary β- and γ-radiation from the gas is small, this ionisation consists of slow-moving β-rays ejected by the γ-rays, which have not sufficient velocity to produce any further ionisation themselves. The ratio

of this part of the ionisation in a chamber to that produced by the radiation from its walls will depend on the ratio of the total inside surface of the walls of the chamber to its volume, and other conditions. The numbers obtained in the experiments just described suggest, however, that in most cases this ratio will probably be greater than one-half, or the ionisation produced directly by the primary γ-rays is probably in most cases greater than 50 per cent of the total ionisation

The ionisation produced directly by secondary rays in an ionisation chamber is probably a percentage of the same order as the above of the total ionisation, it is more likely larger than smaller. An experiment on this point would be difficult to carry out It seems, however, unnecessary, as this quantity depends on a number of other things besides the nature of the rays, such as the size of chamber, etc.

Source of Vapours and Gases

The vapours of the liquids $CHCl_3$, $C_4H_{10}O$, CS_2, CCl_4, C_5H_{12}, CH_3I, C_2H_5I, C_2H_5Br were obtained by letting small quantities of these liquids evaporate into the ionisation chamber in the way already explained The preparations used were those of Kahlbaum

Mr Jones, of the Chemical Laboratory, Cambridge, kindly supplied the writer with a small quantity of $Ni(CO)_4$ The compound is a liquid at ordinary temperature, and could therefore be dealt with in the same way as the above liquids.

The compounds C_2H_5Cl, CH_3Br are in the form of vapour at the ordinary room temperature and atmospheric pressure, and are therefore sold in sealed glass flasks, in which they are in the liquid state under high pressure. The flasks to be used were put into melting ice for half an hour and then opened, the pressure in each case being then less than 1 atmosphere A quantity of liquid from one of the flasks could be evaporated into the chamber by connecting it and the flask by a rubber tube These preparations were also those of Kahlbaum

The gases O_2, CO_2, NH_3, N_2O were obtained of commercial purity from steel cylinders containing these gases under high pressure supplied by various manufacturers.

The H_2 was prepared by means of a Kipp's apparatus, sulphuric acid and zinc being used. It was purified and dried by bubbling it through a solution of potassium and permanganate of potassium in water, and then through strong sulphuric acid.

The SO_2 was prepared by allowing strong sulphuric acid to drop on sodium

sulphite, the SO_2 evolved being dried by passing it through strong sulphuric acid.

The C_2H_2 was prepared by allowing water to drop on commercial calcium carbide The gas evolved was purified by passing it through a bleaching solution and then through water, and then dried by passing it through a tube containing calcium chloride.

Experimental Results

The experimental results obtained with a number of gases are given in Table I, the ionisations being expressed in terms of air as unity Radiators of zinc, carbon, and lead were used The surface of each radiating plate measured 24 by 28 cm, and their thicknesses were respectively 0 65, 2, and 0 5 cm Each value in the table is the mean of several determinations made on different days, and each determination is the mean of at least eight electrometer readings A fair agreement between the different determinations was usually obtained

The values obtained must depend to some extent on the thickness of the radiator used, since the nature of the γ-rays emitted by a radiator must depend to a certain extent on its thickness They must also be somewhat influenced by the shape and the size of the chamber The values obtained therefore apply more particularly to the conditions realised in these experiments. This should not, however, violate the general conclusions that can be drawn from the results, as will appear when these are discussed

The ionisations obtained with the primary γ-rays, the β-rays of uranium, and the α-particle from radium for a number of gases are also placed in the table for comparison They have been taken from a previous paper by the writer * The values for the α-particle, with the exception of NH_3 and SO_2, are due to Prof Bragg,† whose results were incorporated in the paper.

Table II contains the results obtained with soft and hard primary X-rays by McClung,‡ Strutt,§ Eve,‖ and Crowther ¶ The most recent results are those by Crowther The "hard rays" used by him were obtained with the X-ray bulb as hard as it was possible to work with, using a Rudge induction coil worked by a turbine mercury interrupter, while the "soft rays" were the softest rays that would produce an appreciable amount of ionisation Crowther confined his beam to the middle of the ionisation chamber so that

* 'Roy Soc. Proc.,' A, vol 79, 1907

† 'Roy Soc South Australia Trans.,' October, 1906.

‡ 'Phil. Mag.,' [6], vol. 8, p. 357, 1904

§ 'Roy Soc. Proc.,' vol. 72, p. 209, 1903.

‖ 'Phil. Mag.,' [6], vol 8, p 610, 1904.

¶ 'Camb Phil Soc Proc.,' vol 15, Part I, p. 38, 1908

it touched the walls only where it entered and left the chamber, thereby introducing a minimum of secondary radiation from the walls The considerable differences between his results and those of the other observers for the same gases cannot be accounted for by differences in the penetrating

Table I.

Name of gas.	Ionisations by the primary γ-rays of radium	Ionisations by secondary γ rays			Ionisation by the β rays of uranium	Ionisation by the α rays of radium
		Secondary rays from zinc	Secondary rays from carbon	Secondary rays from lead		
Air	1 00	1 ·00	1 00	1 00	1 00	1 00
O_2	1 16	1 11	1 18	1 ·07	1 17	1 15
N_2O	1 55	1 34	—	—	1 55	1 53
CO_2	1 58	1 53	1 58	1 55	1 60	1 59
NH_3	0 898	0 914	0 ·921	—	0 888	0 81
C_2H_2	—	1 20	1 24	—	—	1 40
$C_4H_{10}O$	4 29	4 29	4 35	4 34	4 39	4 40
C_5H_{12}	4 53	4 36	—	4 17	4 55	4 35
C_2H_4O	2 17	2 16	—	—	2 12	2 14
CS_2	3 66	3 40	3 43	3 31	3 62	2 99
SO_2	2 27	2 17	2 49	—	2 25	2 ·01
C_2H_3Cl	3 19	3 39	—	—	3 24	3 12
$CHCl_3$	4 ·98	5 20	5 ·08	—	4 94	4 08
CCl_4	6 38	6 35	6 ·00	6 16	6 28	5 28
$Ni(CO)_4$	5 ·98	6 60	—	—	—	—
CH_2Br	3 81	6 15	6 47	5 ·83	3 ·73	2 ·75
C_2H_5Br	4 ·63	6 05	6 30	—	4 41	—
CH_3I	5 37	12 ·07	15 19	10 36	5 11	3 43
C_2H_5I	6 47	12 46	15 ·60	—	5 90	4 ·00
H_2	0 ·160	0 0882	0 0838	0 139	0 165	0 ·24

Table II

Name of gas	Soft X-rays.			Hard X rays		
	McClung	Strutt	Crowther.	McClung	Eve.	Crowther.
Air	1 00	1 00	1 ·00	1 ·00	1 00	1 ·00
H_2	0 105	0 114	0 ·01	0 177	0 42	0 ·18
O_2	1 3	1 39	—	1 17	—	—
CO_2	1 46	1 ·60	1 57	1 88	—	1 ·49
SO_2	11 ·05	7 ·97	—	4 ·79	2 3	—
$CHCl_3$	—	31 9	—	—	4 6	—
$C_2H_5O_2$	—	—	4 ·95	—	—	3 ·90
C_2H_5Cl	—	—	18 ·0	—	—	17 ·3
CCl_4	—	45 3	67 ·0	—	4 ·9	71 ·0
C_2H_5Br	—	—	72 ·0	—	—	118 ·0
CH_3I	—	72 0	145 ·0	—	13 ·5	13 ·5

power of the X-rays used. They are very probably due to the fact that these observers allowed the X-rays to fall on the walls of the ionisation chamber The chamber was in that case crossed and recrossed by the radiation successively reflected between opposite walls, becoming softer and softer during the process, so that the nature of the rays as a whole which produced the ionisation was very different from the rays given out by the X-ray bulb The general trend of the values is, however, the same.

Deductions from the Results

The ionisation values for the primary and secondary γ-rays can be divided into three sets, which are separated from one another in Table I by horizontal lines. The first set consists of gases whose molecules are composed of atoms of H, C, N, O, S, Cl It will be seen that the ionisation values of these gases are practically the same for the primary and secondary rays in the case of each of the radiators used In the case of gases containing atoms of higher atomic weight than chlorine, however, there are considerable differences with primary and secondary rays, which depend on the nature of the radiator Thus the ionisation in the first set of gases, and probably in all gases except H_2 composed of atoms of low atomic weight, is practically independent of the penetrating power of the γ-rays.

An inspection of the results obtained by Crowther with hard and soft X-rays shows that the ionisation in these gases with X-rays depends comparatively little on the softness and hardness of the rays But on the whole his values are from 6 to 12 times larger than those which the writer obtained with γ-rays.

A comparison of the ionisation values of the gases of the first set obtained with the α- and β-rays, and the primary and secondary γ-rays, shows that they are approximately the same for each kind of rays The process of ionisation in these gases thus appears to be the same for each ionising agent.

The second set in the table consists of gases whose molecules contain at least one atom which is of greater mass than the chlorine atom. It will be seen that the ionisation is greater in these gases with the secondary rays than with the primary Thus the ionisation in C_2H_5I with a carbon radiator is nearly three times that obtained with the primary rays. The difference in ionisation increases with the atomic weight of the heavy atom in the gas, and is greatest with the carbon radiator and smallest with the lead radiator.

In the case of X-rays, the ionisation in the gases of this set also increases with the softness of the rays, except in the case of ethyl bromide. The ionisation in ethyl bromide has been shown by Crowther to decrease very

considerably with the softness of the rays. This property of ethyl bromide is very remarkable, and some interesting deductions can be made from this result and that obtained by the writer with γ-rays

Thus if the γ- and X-rays are the same in nature there should be no discontinuity between them, but rays of all conceivable penetrating powers should exist, including the hardest γ-rays and the softest X-rays The curve connecting ionisation and penetrating power of rays for ethyl bromide would therefore very probably be of the form shown in fig 7 The regions of the

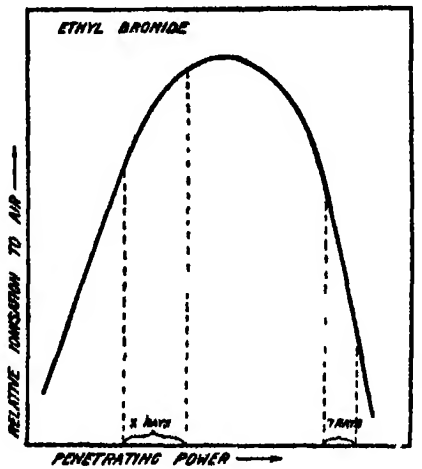


FIG 7

curve suggested by the experiments of Crowther and the writer are marked off in the figure. The curve suggests that it might be possible by increasing the hardness of the X-rays from a bulb to reach a point at which a further increase in the hardness of the rays would produce a decrease instead of an increase in the ionisation Or, by successively reflecting γ-rays from a number of radiators one might obtain rays of such softness that the ionisation produced would decrease instead of increase with the softness of the rays

When the ionisations obtained with the α-, β-, and γ-rays for the gases of the second set are compared with one another, it will be seen that on the whole they vary considerably with the nature and quality of the ionising agent, and in this respect form a marked contrast to the ionisations in the gases of the first set.

The third set in the table contains the gas H_2. The ionisation in this gas was found to be smaller with secondary than with primary rays. The ionisation thus increases with the penetrating power of the rays. The decrease was greatest with the carbon radiator and smallest with the lead

radiator. With the heavy gases we have seen the increase of the ionisation is greatest with the carbon radiator and smallest with the lead radiator. Thus the cause which produces a decrease of the ionisation in H_2 produces an increase in a heavy gas. The nature of this cause is probably related in some way to the penetrating power of the rays But it is impossible to trace a definite connection between this effect and the penetrating power of the rays from carbon and lead as measured by the absorption produced by thin sheets of metal, because the relative penetrating powers obtained depend on the metal used in their determination This point is further discussed under "Ionisation and Absorption of Rays."

The behaviour of H_2 with γ-rays is analogous to that with X-rays, as an inspection of Table II will show This suggests that the curve connecting the ionisation in H_2 and the penetrating power of the ionising agent, assuming that the γ- and X-rays are of the same nature, is probably of the form shown in fig 8 The regions whose form is suggested by the experiments with X-rays and γ-rays are, as before, marked off in the figure It is not improbable, as already suggested in the case of ethyl-bromide, that it would be possible to obtain some experimental evidence, by using very hard X-rays or very soft γ-rays, of the connection between ionisation and penetrating power indicated by the middle portion of the curve.

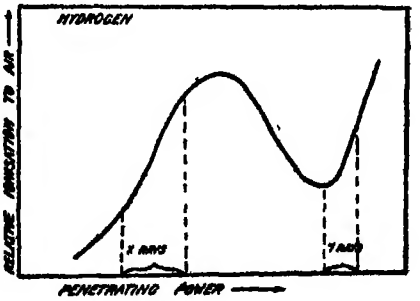


Fig 8

The ionisation in the various gases, omitting H_2 for the present, is approximately an additive quantity This is shown by Table III, which contains the ionisations by the secondary rays from a zinc radiator, and the quantities called "atomic ionisations" by means of which the values under "calculated ionisations" in the table were calculated. The experimental and calculated results obtained previously by the writer with the primary γ-rays are also placed in the table. The values of the atomic ionisations have been so selected that there is roughly the same percentage error between each calculated and experimental value.

Table III.

Atomic ionisations Primary γ-rays		Name of gas	Experimental ionisation Primary γ-rays	Calculated ionisation Primary γ-rays	Experimental ionisation Secondary γ rays from zinc	Calculated ionisation. Secondary γ-rays from zinc
H	0 18	Air	1 00	—	—	—
C	0 46	O_2	1 16	1 16	1 11	1 14
N	0·45	N_2O	1 55	1 48	1 34	1 39
O	0 58	CO_2	1 58	1 62	1 53	1 58
S	1 60	NH_3	0·898	0 99	0·914	0 95
Cl	1 44	C_2H_2	—	—	1·20	1 24
Ni	1 82	$C_4H_{10}O$	4 29	4 22	4 29	4 13
Br	2 81	C_5H_{12}	4 53	4 46	4 36	4 36
I	4 50	C_2H_4O	2 17	2 22	2 16	2 17
Atomic ionisation Secondary γ-rays		CS_2	3 66	3 66	3 40	3 40
		SO_2	2 27	2 76	2 17	2 62
H	0 18	C_2H_3Cl	3 19	3 26	3 39	3 28
C	0 44	$CHCl_3$	4 93	4·96	5 20	5 12
N	0 41	CCl_4	6 33	6 22	6 35	6 44
O	0·57	$Ni(CO)_4$	5·98	5·98	6 60	6 60
S	1 48	CH_2Br	3 81	3 81	6·15	5 69
Cl	1 50	C_2H_3Br	4·63	4 63	6·05	6 49
Ni	2 56	CH_2I	5 37	5 50	12 07	11 86
Br	4·71	C_2H_5I	6 47	6 32	12 46	12 66
I	10 88					

In obtaining the values of the atomic ionisations the gas SO_2 was not considered, because, if included, a value for the atomic ionisation of S is obtained which makes the calculated values of SO_2 and CS_2 differ by a much greater percentage from the experimental values than is the case with any of the other gases SO_2 and CS_2 thus evidently do not fit in very well with the other gases with respect to the additive law It is remarkable that the ionisation values obtained with the primary γ-rays, and the α- and β-rays, show similar deviations The calculated value for SO_2, if we take the atomic ionisation for S obtained from CS_2, is in each case greater than the experimental (see Table V).

The agreement between calculation and experiment for the remaining gases in Table III is not quite so good with the secondary rays as with the primary. This is probably in some measure due to the errors of experiment being greater in the former case. The departure of C_2H_5Br, however—its ionisation is smaller than that of CH_2Br, although it contains an additional C and two H atoms—seems to be real.

The atomic ionisations for the primary and secondary rays are plotted

against the atomic weight in fig. 9 The curves are separated somewhat in the direction of the ordinate in order to prevent overlapping near the origin. It will be seen that the upward slope of the curve for the secondary rays increases with increase of atomic weight The curve for the primary rays, on the other hand, does not show any increase in slope The sensitiveness of an atom to the penetrating power of γ-rays thus increases with the atomic weight.

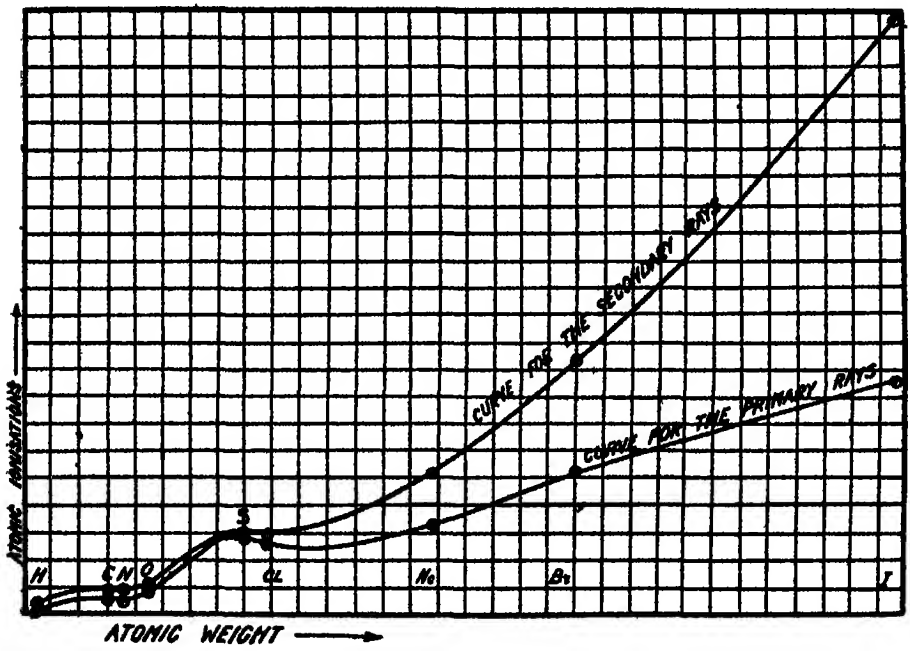


Fig 9

The atomic ionisations for the primary γ-rays and the α- and β-rays taken from the paper by the writer already quoted, and for the secondary γ-rays, are given in Table IV. It will be seen that the atomic ionisations for the atoms H, C, N, O, S, Cl, are approximately the same with each of the ionising agents, as we would expect from an inspection of the ionisation values of the gases in the first set of Table I.

In Table V are given the experimental and calculated ionisations in H_2 and SO_2, for the α- and β-, and the primary and secondary γ-rays. It will be seen that in each case the calculated values are larger than the experimental, especially in the case of H_2. The ionisation in H_2 does not fit in even approximately with the other gases in respect to the additive law

The writer has shown that in the case of the α-particle if both the

Table IV.—Atomic Ionisations.

Name of atom	Primary γ-rays.	Secondary γ rays from zinc	β rays from uranium	α-particles from radium
H	0·18	0 18	0·18	0 175
C	0 46	0 44	0 46	0 51
N	0 45	0 41	0 475	0 47
O	0 58	0 57	0 58	0 55
S	1·60	1 48	1·60	1 24
Cl	1 44	1 50	1 44	1 16
N_1	1 82	2 56	—	—
Br	2 81	4 71	2·67	1·72
I	4 50	10 88	4 10	2 26

Table V.

Name of gas.	Experimental ionisations.			
	Primary γ rays	Secondary γ-rays from zinc.	β rays from uranium.	α-particles from radium
H_2	0 160	0·0682	0 165	0 24
SO_2	2 27	2 17	2·25	2·01
	Ionisations calculated by means of the atomic ionisations given in Table IV			
H_2	0·36	0·36	0·36	0 35
SO_2	2·76	2·62	2 76	2 84

stopping powers and ionisations for a number of gases follow an additive law, the energy required to make an ion from an atom is independent of the nature of its chemical combination in these gases * Further, if there is in addition a gas whose ionisation does not fit in with the other gases according to the additive law, but whose stopping power is normal, the energy expended per ion made from one of its atoms is not the same as that expended when the atom occurs in one of the other gases. It is greater or less than that in the latter case, accordingly as the ionisation of the gas by the additive law is greater or less than the experimental. It was thus shown that the energy spent by the α-particle on an ion made from an atom

* 'Roy. Soc. Proc.,' A, vol. 79, 1907.

of H in the gas H_2 is greater than when the atom occurs as a constituent in any of the other gases. It is very probable that this is also true for the other ionising agents besides the α-particle The decrease of the ionisation in H_2 with increase in softness of the γ- and X-rays may then be due to an increase in the expenditure of energy in the production of an ion.

On the whole it is evident that the chemical bond which holds two atoms of hydrogen together differs in nature from that which holds it in combination with the atoms in some other gases. If an atom consists of an assemblage of electrons, we would expect that some of the electrons would be under a constraint when the atom is combined with other atoms, and that this constraint might be the same for some combinations while different for others The electrons under constraint would probably be more difficult to pull out of an atom than the other electrons, and the departures from the additive law accounted for by departures in the nature of the constraint The departure of H_2 from the additive law, and the increase of this departure with the increase in the softness of the ionising rays, adds another anomaly to the list for which the gas is already famous. It is probable that these anomalies are due to the same cause, which is probably a departure in the arrangement of the forces of constraint holding the two atoms of the molecule together from the arrangement in the case when hydrogen is combined with other atoms. If that is so, the forces outside a H_2 molecule should differ considerably from those outside any other molecule, and the scattering of moving electrons by H_2 should therefore be anomalous. Now this has been found to be the case.* The scattering of electrons by the gases air, argon, and carbonic acid is about what we should expect if we suppose that the charge on the electron does not influence its mean free path, the absorption by hydrogen is, however, considerably greater than the value got on this supposition, showing that the hydrogen molecule is surrounded by a field of force which is practically absent in the case of the molecules of the other gases.

Dependence of Ionisation on the Direction of Propagation of the Secondary Rays with respect to that of the Primary.

The experiments described were made with secondary rays propagated more or less in the opposite direction to the primary rays. Some experiments were also made with the secondary rays propagated in the same direction as the primary. The arrangement used is shown diagrammatically in fig. 10. A denotes the ionisation chamber, B the radiator, and C the

* See Prof. J. J. Thomson's 'Conduction of Electricity through Gases,' 2nd edition, p. 382.

radium Readings were taken with and without the radiator in the same way as before

It was found that the ionisation in CH_3I, with a zinc radiator, was 8·42 instead of 12 07, as was obtained with the previous arrangement The nature of the secondary radiation from a plate thus depends on its direction of propagation with respect to that of the primary rays

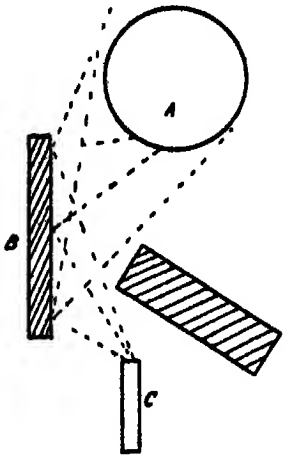


Fig 10

Since the ionisation with the primary rays is less than with the secondary, which are more absorbable, we infer that the secondary rays, which are propagated in the same direction as the primary, are of a more penetrating character than those which are propagated in the opposite direction

This result falls into line with some experiments by Madsen,* who showed that the secondary γ-rays which proceed from the side of a radiating plate where the γ-rays emerge are of a more penetrating character than those from the side where the γ-rays enter Madsen measured the penetrating power of the rays by observing the absorption produced by sheets of metal of different thicknesses placed in their path. The reason for the difference in the penetrating power of the rays is that the secondary γ-rays are initially propagated in the same direction as the primary, but during their further course are scattered to a certain extent by the material, and softened in the process The returned secondary rays have undergone the maximum amount of scattering, and are therefore softer than those which are propagated approximately in the same direction as the primary

Connection between Ionisation and Absorption

The amount of ionisation produced by the electrons ejected from the molecules of a gas with sufficient velocity to produce ions by collision is very small, as we have seen at the beginning of this paper The ionisation in a gas measured in a chamber whose walls radiate no secondary rays is therefore proportional to that fraction of the energy of the γ-rays absorbed which is converted into energy of electrons of slow velocity, or δ-rays.

The amount of energy absorbed of a beam of γ-rays by a sheet of metal placed in its path is a measure of the energy absorbed in the production of cathode rays of high speed as well as slow-moving ones. And since the

* 'Roy. Soc. South Australia Trans.,' vol 32, July, 1908.

energy represented by an electron of high speed ejected by γ-rays may be several thousand times that represented by a slow-moving electron, or δ-ray, the proportion of energy absorbed which is converted into kinetic energy of electrons of high speed may be quite large

It appears, therefore, that we cannot compare the ionisations in gases with the absorption of solid materials unless we know how the absorbed energy is distributed between the electrons of high and slow speed The results on the ionisations in gases by secondary γ-rays described in this paper, and those on the absorption of secondary rays by different materials obtained by the writer,* cannot therefore be compared with one another. Some idea of the distribution of the absorbed energy between the electrons of high and slow speed might be obtained by comparing the ionisation in the vapour of a liquid by a given source of rays, and the absorption of energy by the liquid for equal masses

It is a pleasure to me, in conclusion, to acknowledge the keen interest and helpful encouragement of Prof Sir J. J Thomson during this research.

* 'Phil Mag,' p 618, November, 1907.

Note on a Curious Property of Neon

By J Norman Collie, F R.S

(Received April 8,—Read May 6, 1909)

During some work with specially pure neon, belonging to Sir W Ramsay, it was noticed that as the gas escaped at atmospheric pressure from a Topler pump, up through the mercury into a inverted test-tube, each bubble glowed with a fire-red glow. Further experiments showed that if neon were sealed up in a glass tube with mercury and the tube shaken, the glow was very apparent.

This glowing of gases when shaken with mercury in a tube is well known, but neon shows this peculiarity, even at atmospheric pressures, in such a marked manner that it was thought worth while to further investigate the phenomenon A tube, for instance, filled with helium at 100 mm pressure only gives a very faint bluish glow when shaken The amount of neon available, however, was only 5 c c, and further experiments would have been impossible without much larger quantities of the gas. Fortunately M Claude, of Paris, most kindly sent Sir W Ramsay a large quantity of residual gases obtained during the liquefaction of air. These consisted of a mixture of helium, neon, and nitrogen, the nitrogen being present to the extent of about 50 per cent., the remainder being the helium and neon From this mixture nearly pure neon can be obtained by fractional absorption of the gases by charcoal cooled in liquid air

In every case, before filling the tubes with neon, they were thoroughly cleaned with sulphuric acid and chromic acid Some mercury was then put into the tubes and they were sealed on to the Topler pump and pumped out, the mercury in the tubes was then boiled and the pump washed out with some neon; the tubes were again pumped out and finally pure neon admitted, the pressures in the different tubes varied from 120 to 200 mm, as it was found that the glow was as bright at these as at ordinary pressures, and a saving of neon was thus made

At first it was expected that the glow would always be produced when the tube containing the mercury and neon was shaken, but this was found not to be so, as the following experiments will show:—

1. A straight tube rounded at both ends was filled with neon (probably containing a little helium) at about 200 mm pressure. After shaking intermittently for two or three hours the glow became very feeble and remained so. After two days, as there did not seem to be any improvement, the ends

of the tube were enclosed in lead foil and sparks from an induction coil were passed through it. At once the brilliancy returned, but it was brighter at one end (the positive terminal) than at the other. The same result can be obtained by merely holding the tube in the hand and moving it up and down on either of the terminals of the induction coil, whilst a spark is passing between the terminals.

Exactly the same result was obtained from several other tubes that had lost their original brilliant glow, and had remained in this condition, some of them for as much as three weeks. In most cases, after the tube had been revivified in this manner, shaking had less effect on the brilliancy of the glow than when the tube had just been made.

2. Another tube, directly after it was made, that glowed brightly when shaken, on being brought near the terminals of the induction coil, at once lost most of its brilliancy, moreover, further sparking from the coil did not improve it.

3. Another tube had a thick platinum wire sealed through one of its ends. On shaking, the glow in this tube soon became feeble, and sparks from the induction coil were passed through it in various ways without improving it. It remained in this state for some days, finally, by merely rolling the mercury up and down, the full original brilliancy returned.

4. The inside of a tube was strongly etched with hydrofluoric acid. This tube, after filling with neon, glowed just as brightly as the others.

5. A tube that was glowing brightly had its lower end immersed in liquid air. The mercury was frozen solid, and the upper end was then heated to well above 400° C. Any mercury that might have condensed on the glass by this treatment would condense in the lower portion of the tube. On allowing the tube to return to ordinary temperatures, and shaking, it was found that the end that had been heated glowed even more brilliantly than the one that had been cooled.

The tube in experiment (2), that glowed only feebly, was then heated at one end. The heated end was found to have regained the property of glowing brightly when the tube was shaken.

Many other tubes were made and obtained in different degrees of efficiency, by passing mild or violent discharges through them. Some were produced that glowed at both ends but not in the middle, others would glow only in the middle, and some at only one end. This abnormal state seems fairly permanent if no further electric charges are brought near the tubes.

If the neon has the slightest trace of moisture in it, no glow can be obtained; minute traces of carbon monoxide also diminish the power of glowing in a marked manner, but spectroscopic traces of hydrogen do not

seem to have much effect. There is no doubt, however, that the purer the neon the more brilliant is the glow, and at the same time it is more difficult to destroy the glow by shaking or electrifying the tube

An experiment was made in a silica tube with the neon at atmospheric pressure, the glow was very much brighter than in a glass tube under the same circumstances

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Note on the Results of Cooling certain Hydrated Platin-Cyanides in Liquid Air

By J Emerson Reynolds, M D, Sc D, F R S.

(Received April 19,—Read April 29, 1909)

In the course of Sir James Dewar's important low-temperature researches he made an interesting and significant observation with a salt which had been supplied to the Laboratory of the Royal Institution as "Lithium Platinocyanide"* When this nearly white crystallised substance was cooled in liquid air it assumed a distinct *red* colour, which did not persist at ordinary temperatures, the material resuming its usual appearance Sir James was so good as to give the writer a portion of the salt for examination, as it seemed desirable to seek for some explanation of the remarkable colour change observed.

On repeating the above-mentioned experiment several times with one and the same portion of Sir J. Dewar's specimen it was subsequently found that the substance gradually lost the property of becoming red in liquid air, and assumed instead a marked *yellow* colour, which was retained at ordinary temperatures. This additional phenomenon has also to be explained, as it is presumably connected with that first observed.

Chemical examination of the Royal Institution specimen led to the conclusion that it was a mixture of the hydrated chloride, cyanide and sulphate of lithium with a platin-cyanogen salt of lithium, and that the proportion of the latter compound present was small.

The percentages of platinum and of lithium were directly determined in the R.I specimen and found to be

Platinum	1 82
Lithium	9 56

* See p. 667, 'Proceedings of the Royal Institution' for 1895.

Hence the percentage of platinum *compound* present could not exceed 5 per cent of the mixture of salts When this specimen was examined under a microscope some minute red specks were seen, and these minute particles deepened much in tint when the material was cooled in liquid air The general red coloration of the mass at the same time indicated that the platinum colour-producing compound was also diffused through the salts in a state of solid solution

The separation and identification of a small amount of a platin-cyanide in much saline material is not very satisfactory, hence the method of comparison was adopted It seemed highly probable, having regard to all the circumstances, that the question to be decided was whether the compound present was a platinocyanide or one of the much less known platinicyanides —the presumption being, of course, rather in favour of the former

With a view to this comparison I prepared afresh some pure lithium platinocyanide and obtained the salt in fine grass-green crystals when fully hydrated On completely analysing these crystals they gave data agreeing well with the formula

$$Li_2Pt(CN)_4,5H_2O$$

When cooled in liquid air this salt did *not* alter materially in colour—its green tint simply became paler after prolonged immersion Even when previously diffused through hydrated lithium chloride, and the mixture cooled as before, it merely assumed a somewhat more yellow shade, but neither by cold nor heat did the pure material, or the mixture, become *red* on dehydration It was, therefore, evident that the platinum compound present in the R.I. specimen was not a platinocyanide of lithium, and was probably a platinicyanide of the same base—the latter differing from the former in containing one more cyanogen group

Happening to have in my collection a finely crystallised specimen of lithium platinicyanide, its exact composition was then ascertained by direct analysis, and was found to be represented by the formula

$$Li_2Pt(CN)_6,2H_2O$$

Hence the material was ready to hand for carrying out the further comparison. This compound is, however, of a full *orange red colour*, at ordinary temperatures, and when cooled in liquid air becomes a magnificent *ruby-red* which does not alter on prolonged cooling. These observations did not, at first, seem to help much toward the end in view, but a careful study of the variations in hydration of the platinicyanide cleared away all further difficulty.

The orange-red di-hydrated crystals easily dissolve in water and form

a colourless solution. When this solution is cautiously evaporated at 40° to 50° to the crystallising point and then quickly cooled to 15°, long *colourless* needle-like crystals separate which exhibit a slight lavender fluorescence These crystals, when collected and quickly dried by pressure, were found to include $3H_2O$, *i.e* one more molecule of water of crystallisation than the red salt. This colourless tri-hydrate easily parts with one molecule of water and *becomes the red di-hydrate* either by heat or when the colourless crystals are cooled in liquid air In the latter case, very rapid cooling always gave some *yellow* material in addition to the red substance, but when the reduction in temperature was carried out very slowly the red compound only was produced

Further, when the orange-red crystals of the di-hydrated salt were very carefully heated until *one* of the remaining molecules of water was driven off, a *yellow* substance remained, which latter, if exposed to moist air, speedily resumed water and became red again I found, however, that a persistent yellow mono-hydrate could be obtained by adding to a colourless aqueous solution of the tri-hydrate a small proportion of an indifferent but highly hydrated salt—sodium sulphate—then evaporating to dryness and gently heating the residue The red stage of dehydration was quickly passed, and a persistent pure yellow-coloured product remained, recalling in appearance the yellow substance which results from the quick cooling of the pure substance, as noted above, and also the product of the repeated cooling and thawing of the R I specimen, as mentioned at the beginning of this note

Finally, when the pure platinicyanide was sufficiently heated, the last molecule of water of crystallisation was driven off, and a *white* anhydrous substance remained It is, therefore, comparatively easy to obtain the following compounds by the means above indicated —

$Li_2Pt(CN)_5$*	White
$Li_2Pt(CN)_5,H_2O$ . .	Yellow.
$Li_2Pt(CN)_5,2H_2O$	Orange-red
$Li_2Pt(CN)_5,3H_2O$	Colourless.

These variations of colour with degrees of hydration are doubtless to be connected with the differences in arrangement of the water molecules in the greater crystalline molecules, and their consequent effects on light.

The study of these hydrates evidently supplies the interpretation of the phenomena observed on cooling the Royal Institution specimen repeatedly

* These formulæ should probably be doubled, but it is unnecessary to do so here, as the simpler expressions serve equally to represent the essential variations.

in liquid air. The mixture of hydrated chloride, cyanide and sulphate of lithium used included rather less than 5 per cent of lithium platinicyanide, which was chiefly in the tri-hydrated colourless condition When the temperature of the mixture was reduced in liquid air, one molecule of water quickly separated, and the *red* di-hydrated salt was formed, but on warming up to the ordinary temperature, the colourless tri-hydrate was reproduced The other hydrated lithium salts present are doubtless simultaneously dehydrated at the low temperatures reached, although these changes cannot be directly recognised as they are unaccompanied by colour alteration In rapid cooling of the mixed (or even of the pure) material in liquid air a little of the yellow mono-hydrate is always formed and, as already noted, this rehydration of the yellow substance is singularly inhibited when neutral salts are present which are themselves avid of water, so that frequent alternations of cooling and warming gradually lead to the complete conversion of the platinum compound into the persistent yellow mono-hydrate.

The facts observed regarding the chemical changes of lithium platinicyanide hydrates not only serve to explain the phenomena noted on cooling the R.I. specimen to temperatures between $-180°$ and $-200°$, but also indicate that the study of graduated dehydration of coloured salts at low temperatures may present considerable advantages, as compared with that of similar salts under the more completely disintegrating effects of heat.

Some Results in the Theory of Elimination.

By A L Dixon, M A, Fellow of Merton College, Oxford

(Communicated by Prof Love, F R S Received May 1,—Read May 27, 1909.)

In two recent papers,* I have given expressions, in the form of determinants, for the eliminants (i) of two quantics in one variable and (ii) of three quantics in two variables My present purpose is to show how from these expressions may be derived others which give the eliminants as a sum of terms.

In the case of two quantics $\phi(x)$, $\psi(x)$, each of the nth degree in one variable, I obtained the result

$$\Delta = ARE,$$

where E is the pure eliminant, Δ is a determinant of order n, whose elements are denoted by (a_s, r_t), where (a, r) is written for $[\phi(a)\psi(r) - \phi(r)\psi(a)]/(a-r)$, $a_1, a_2, \ldots a_n$, $r_1 r_2, \ldots r_n$ being two sets of arbitrary quantities, and A and R denote respectively the determinants whose sth rows are

$$| \; 1, \; a_s, \; a_s^2, \; a_s^3, \; \ldots \; a_s^{n-1} \; |$$

and

$$| \; 1, \; r_s, \; r_s^2, \; r_s^3, \; \ldots \; r_s^{n-1} \; |$$

This is easy to prove, for if we write $(a, r) = \Sigma C_{st} a^s r^t$, it is well known that, according to Cayley's statement of Bézout's method, one form of E is the determinant whose elements are C_{st}, and if we multiply this determinant successively by A and R we get the determinant Δ.

For three quantics in two variables (and for n quantics in $n-1$ variables) an expression analogous to Bézout's determinant may be obtained for the eliminant of $\phi(x, y)$, $\psi(x, y)$, $\chi(x, y)$, if these are all of the form $\Sigma A_{rs} x^r y^s (r \leqq n, s \leqq m)$, where n, m are the same for all three quantics, ϕ, ψ, χ. This form I denote by saying that ϕ, ψ, χ are of degree n in x, and m in y For, consider the function $F(x, y, a, b)$ given by

$$(x-a)(y-b)F(x, y, a, b) = \begin{vmatrix} \phi(x, y), & \phi(x, b), & \phi(a, b) \\ \psi(x, y), & \psi(x, b), & \psi(a, b) \\ \chi(x, y), & \chi(x, b), & \chi(a, b) \end{vmatrix}$$

The determinant on the right-hand side is obviously divisible by $x-a$ and $y-b$, so that F is of degree $2n-1$ in x, and $m-1$ in y, and of degree $n-1$ in a, and $2m-1$ in b.

Also F obviously vanishes if (x, y) is a common root of $\phi = 0$, $\psi = 0$,

* 'Proceedings of the London Mathematical Society,' ser 2, vol. 6, pp. 468—478, and vol. 7, pp. 49—69.

$\chi = 0$, whatever value we give to a, b. Thus the coefficients of $a^r b^s$ for all values of r and s will all vanish for a common root, and we derive from $F = 0$, $2mn$ equations in x and y, the number of terms in each being also $2mn$.

From these equations we can at once write down the eliminant as a determinant of order $2mn$, whose elements are the coefficients of $a^r b^s x^t y^u$ in $F(x, y, a, b)$, r and s being the same for any row, and t and u the same for any column From this again we may derive, by multiplication, the result

$$\Delta = \{a, b\}\,\{\alpha, \beta\}\,E,$$

where E is the pure eliminant in the form just obtained, Δ is a determinant of order $2mn$ whose elements are $F(\alpha_r, \beta_r, \alpha_s, b_s)$, (a_1, b_1), (a_2, b_2) $\ldots$ (a_{2mn}, b_{2mn}), (α_1, β_1), (α_2, β_2) . . $(\alpha_{2mn}, \beta_{2mn})$ being two sets of $2mn$ pairs of arbitrary quantities, and $\{a, b\}$, $\{\alpha, \beta\}$ denote respectively the determinants whose rth rows are

$$| 1,\ a_r,\ b_r,\ a_r^2,\ a_r b_r,\ b_r^2,\ \ldots\ a_r^{n-1} b_r^{2m-1} |$$

and

$$| 1,\ \alpha_r,\ \beta_r,\ \alpha_r^2,\ \alpha_r \beta_r,\ \beta_r^2,\ \ldots\ \alpha_r^{2n-1} \beta_r^{m-1} |,$$

the terms that occur being those of

$$\Sigma a_r^s b_r^t\ (s < n, t < 2m) \quad \text{and of} \quad \Sigma \alpha_r^s \beta_r^t\ (s < 2n, t < m) \text{ respectively}$$

To obtain corresponding results for three quantics of the ordinary standard form $\Sigma A_{rs} x^r y^s\ (r + s \leqq n)$, we take $\frac{1}{2} n(n+1)$ [or $\frac{1}{2} n(n-1)$], equations derived from $F(x, y, a, b)$ by equating to zero the coefficients of $a^r b^s$ for $r + s < n$ [or $n-1$], and add to these $\frac{1}{2} n(n-1)$ [or $\frac{1}{2} n(n+1)$], equations got by multiplying $\phi = 0$, $\psi = 0$, $\chi = 0$ by $x^r y^s$ for $r + s < n-1$ [or n], thus obtaining a determinant of order $2n^2 - n$ [or $2n^2 + n$], and by suitable modifications it is possible to adapt this method to quantics of different degrees.

My present object is to obtain the eliminant of three quantics of the ordinary standard form of degree n, as a sum of products, n^2 at a time, of various values of $F(\alpha, \beta, a, b)$, and in order to make the process clearer, and also for the sake of the result obtained, I begin by finding the corresponding expression for the eliminant of two quantics in one variable.

If in $\phi(x)$ and $\psi(x)$ I change the variable from x to x^2, the result of eliminating between $\phi(x^2)$ and $\psi(x^2)$, considered as functions of x, will obviously be the square of the eliminant of $\phi(x)$ and $\psi(x)$ Thus the equation already obtained may be written for this case

$$\Delta = \mathrm{ARE}^2,$$

Δ being a determinant of order $2n$ whose elements (a_s, r_t) are

$$[\phi(a_s^2)\,\psi(r_t^2) - \phi(r_t^2)\,\psi(a_s^2)]/(a_s - r_t).$$

In this, as the a's and r's are arbitrary quantities, we may put $r_s = -a_s$

for all values of s, and we obviously get $(a_s, -a_t) = -(a_t, -a_s)$, so that Δ is now a skew symmetrical determinant, whilst A and R are identical.

But the expression for the square root of a skew symmetrical determinant is a known formula, so that, using this formula, and putting

$$[r, s] = (\phi(a_r^2)\psi(a_s^2) - \phi(a_s^2)\psi(a_r^2))/(a_r + a_s),$$

where $a_1, a_2, a_3, \ldots a_{2n}$ are $2n$ arbitrary quantities, I get

$$AE = \Sigma\pm[1,2][3,4][5,6] \quad [2n-1, 2n],$$

A being the product of the differences of the a's.

To obtain a similar result for the eliminant of three quantics, $\phi(x, y)$, $\psi(x, y)$, $\chi(x, y)$ of the ordinary standard form, of degree n, make the substitution $x = xy$, $y = x+y$. When this is done, the new functions $\phi(xy, x+y)$. are of the form $\Sigma A_{rs}x^r y^s$ $(r \leqq n, s \leqq n)$, and their eliminant is the square of the eliminant of the original quantics, since there is a $(1, 2)$ correspondence

Thus the formula already obtained may be written

$$\Delta = \{a, b\}\{\alpha, \beta\} E^2,$$

where the numbers of pairs of quantities (a, b) and (α, β) is $2n^2$, and Δ is the determinant of order $2n^2$, whose elements are $F(\alpha_r, \beta_r, a_s, b_s)$, given by

$$(\alpha_r - a_s)(\beta_r - b_s)F(\alpha_r, \beta_r, a_s, b_s) = (\phi(\alpha_r\beta_r, \alpha_r+\beta_r), \psi(\alpha_r b_s, \alpha_r + b_s), \chi(a_s b_s, a_s + b_s)).$$

If in this we put $a_s = \beta_s$, $b_s = \alpha_s$ for all values of s, the result is a skew symmetrical determinant, and, taking the square root, we have

$$\{\alpha, \beta\} E = \Sigma\pm[1,2][3,4][5,6] \ . \ [2n^2-1, 2n^2],$$

where $[r, s]$ is $F(\alpha_r, \beta_r, \beta_s, \alpha_s)$, as defined above, that is

$$[r,s] = \begin{vmatrix} \phi(\alpha_r\alpha_s, \alpha_r+\alpha_s), & \phi(\alpha_r\beta_r, \alpha_r+\beta_r), & \phi(\alpha_s\beta_s, \alpha_s+\beta_s) \\ \psi(\alpha_r\alpha_s, \alpha_r+\alpha_s), & \psi(\alpha_r\beta_r, \alpha_r+\beta_r), & \psi(\alpha_s\beta_s, \alpha_s+\beta_s) \\ \chi(\alpha_r\alpha_s, \alpha_r+\alpha_s), & \chi(\alpha_r\beta_r, \alpha_r+\beta_r), & \chi(\alpha_s\beta_s, \alpha_s+\beta_s) \end{vmatrix} / (\alpha_r - \beta_s)(\alpha_s - \beta_r),$$

and $\{\alpha, \beta\}$ is the determinant whose rth row is

$$| 1, \ \alpha_r, \ \beta_r, \ \alpha_r^2, \ \alpha_r\beta_r, \ \beta_r^2, \quad . \ \alpha_r^{2n-1}\beta_r^{n-1} | .$$

A curious corollary is the expansion of $\{\alpha, \beta\}$ itself in the form $\Sigma\pm(1,2)(3,4)(5,6) \ldots (2n^2-1, 2n^2)$, where

$$(r,s) = \begin{vmatrix} \alpha_r^n\alpha_s^n, & (\alpha_r+\alpha_s)^n, & 1 \\ \alpha_r^n\beta_r^n, & (\alpha_r+\beta_r)^n, & 1 \\ \alpha_s^n\beta_s^n, & (\alpha_s+\beta_s)^n, & 1 \end{vmatrix} / (\alpha_r - \beta_s)(\alpha_s - \beta_r),$$

which is at once obtained by taking ϕ, ψ, χ all of the form $Ax^n + By^n + C$.

Utilisation of Energy stored in Springs for the Production of Mechanical Work.

By A Mallock, F.R.S.

(Received April 27,—Read May 13, 1909)

Small pieces of mechanism which require little power are often driven by wound-up springs (generally of steel), but steel and all other known materials, with the exception of indiarubber, have such a small capacity for work that they are not of much use as sources of power if weight is a consideration

The capacity for work, or "dynamic worth," of a substance is the elastic work which can be stored in it divided by its mass. The dimensions of dynamic worth are therefore L^2/T^2 and the velocity of which it is the square represents the velocity which the work stored could impart to the mass in which it is contained Dynamic worth may also be defined in gravity measured by the height to which the stored work could raise the weight in which it is contained For steel this height is between 70 and 80 feet, and for all other known substances, except indiarubber, it is less. For good specimens of indiarubber, however, the dynamic worth is between 1000 and 1200 feet, and this singular property ought to render it valuable for small light engines whose duty is to give out a large power for a short time Thus 30 lbs of indiarubber would serve to produce something like a horse-power for one minute

I do not know that any heat engine has been yet made which will give out a horse-power for one minute for this weight, though internal combustion engines can now be made giving 100 horse-power on a weight of very much less than 3000 lbs. The object of this note is to show in what way the work stored in stretched indiarubber may be most fully utilised, and so utilised as to supply power at a constant rate.

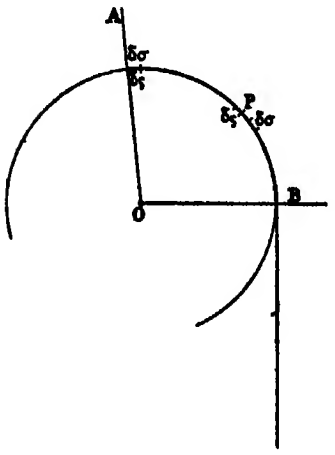
The most convenient form in which the indiarubber can be used is that of a long strip or band wound on a reel or drum under tension, and in order to convert the potential energy into mechanical work it will be shown that the conversion cannot be effected continuously, but must proceed in cycles, and that the condition of efficiency is that each portion of the elastic band whose contraction is to be utilised must be unwound from the drum without change of tension, and that the part so unwound must then be isolated by clamping or otherwise, and allowed to contract without contact with other bodies except at the clamped ends.

Consider the case of a long perfectly elastic band wound without tension on a reel A and let this reel be so geared to another reel B as to make B revolve at n times the rate of A. If the end of the band on A is attached to B, the band in passing from A to B will be stretched to n times its original length It is shown below that the work expended in winding from A to B is exactly twice the work required to stretch the band to n times its length, one half the work being taken up in overcoming the elastic reaction and the other half converted into heat by the friction of the band on itself or on the reel while its length is being changed from n to nl.

Conversely, if from the reel B the stretched cord is rewound on to A, no mechanical work can be gained, the whole of the potential energy due to the elasticity being taken up in overcoming the friction of the cord on itself or B while contracting from nl to l.

To avoid mechanical loss there must be no slip and therefore no change of tension while unwinding is in progress. If a suitable length of band is unwound without change of tension and then clamped where it leaves the reel, while its outer end is attached to an appropriate part of the engine and allowed to contract, the whole elastic work can be utilised in accelerating the motion of that part, and the process of unwinding and subsequent contraction repeated indefinitely as long as any stretched band remains on the reel. (A small model was shown on which this cycle is carried out automatically.)

With indiarubber there are thermal losses during contraction, independent of friction, which will prevent the efficiency of the engine being as high as if the cord was perfectly elastic, but the proposition as regards the frictional loss being exactly equal to the elastic work done seemed of sufficient interest to make it worth recording.



This proposition has an immediate application to the efficiency of belts, and shows that the more stretchable a belt is (*i.e.* the smaller is Young's modulus for the material) the lower is the efficiency of transmission.

To prove the proposition above stated, let AB (= S) be part of the circumference of a reel of radius r, A being the position where the tension of the band is t_1, and B the position where the band leaves the reel at tension t_2, and OA, OB being directions fixed in space. Then, as is known, the tension at any point P between A and B is $t_2 e^{-\mu s/r}$, where S is the arc BP and μ the

coefficient of friction. Let δs and $\delta\sigma$ be short lengths of the circumference of the reel and of the band respectively, which are equal and in contact at A. If the reel be now turned until δs arrives at P, $\delta\sigma$ will (in virtue of the extension of the band) be nearer to B than δs by the length $s-\sigma = \int_\sigma^s \frac{t}{E} ds$, E being the elastic modulus of the band, and the slip between the band and reel which occurs while the latter turns through ds is tds/E The normal pressure of the band at P is t/r, and the resistance to sliding is $\mu t/r$ The work therefore expended in overcoming the resistance to sliding per unit length of the band at P while the reel turns through the arc ds is

$$\frac{\mu}{rE} t^2 ds = \frac{\mu}{rE} t_2^2 ds e^{-2\mu s/r},$$

and the total work spent in friction between the band and reel between A and B for the arc of rotation ds is the integral of the last expression taken between the limits s and 0, or $\frac{1}{2E}(t_2^2-t_1^2)$ Thus if a length l_0 of band is unwound, the frictional work is equal to $\frac{l_0}{E} \frac{1}{2}(t_2^2-t_1^2)$, and independent of μ.

The elastic work expended in stretching a portion of natural length l_0 from a length l_1 and tension t_1, to a length l_2 and tension t_2, is $(l_2-l_1)\frac{1}{2}(t_2+t_1)$, or, since $l_2 = l_0\left(1+\frac{t_2}{E}\right)$ and $l_1 = l_0\left(1+\frac{t_1}{E}\right)$, it is $\frac{l_0}{E}\frac{1}{2}(t_2^2-t_1^2)$

Thus if the l in the previous formula is reckoned as the unstrained length of the unwound band, the elastic and frictional work are equal in amount

The frictional work must always represent a loss of available mechanical work, and hence to wind from a low tension to a higher demands the expenditure of twice the work required to stretch the band from one tension to the other, and if the band is unwound from a high tension to a low no mechanical power can be developed in the process as the work is taken up by friction

The Absolute Value of the Mechanical Equivalent of Heat, in terms of the International Electrical Units

By HOWARD T. BARNES, D Sc, Macdonald Professor of Physics, McGill University, Montreal.

(Communicated by Prof H. L Callendar, M A, LL.D, F R.S Received April 30,—Read May 27, 1909)

In 1902* there were published the results of an extended series of measurements of the capacity for heat of water at different temperatures between 0° and 100° C The continuous method of calorimetry was employed in the work, the details of which were developed by Prof H L. Callendar and myself. A steady electric current was used to heat a steady flow of water through a fine bore tube, bringing about steady temperature conditions in the calorimeter The energy given in joules per calorie was expressed in terms of the international electric units, and was consequently dependent on the values of the standards used in the measurements The very greatest possible care was exercised in selecting the standards employed and in making sure that they represented the values assumed for the international electrical units Previous to the commencement of the investigation of the capacity for heat of water, several years were devoted to a thorough study of the Clark cell as a standard of electromotive force† and to the determination of the temperature coefficient of the Clark cell From a large number of cells suitable examples were selected which were as perfect as could then be obtained. Additional cells were also made at different times and with different materials, in order that those actually used in the investigation could be expressed in terms of the mean of as large a number as possible In addition, a further investigation‡ of the Weston cell was made, and the mean of all the Clark cells was compared with the mean of all the Weston cells The value of this ratio was found to be

$$\frac{\text{Clark at } 15° \text{ C.}}{\text{Weston at } 20° \text{ C}} = 1\,40666,$$

which is practically coincident with the value of the ratio found by Jaeger

* 'Phil Trans,' A, vol 199, p 149 (1902)

† 'Roy Soc Proc,' vol 62, p 148 (1897), B. A Report, Toronto, 1897, 'Electrician,' vol 39, p 66 (1897), vol 40, p. 165 (1897)

‡ 'Journ. Phys. Chem.,' vol 4, p 339 (1900), vol 8, p. 196 (1904).

and Lindeck* for the cells prepared at that time at the Reichsanstalt They found

$$\frac{\text{Clark at } 15^\circ \text{ C}}{\text{Weston at } 20^\circ \text{ C}} = 1\,40665$$

While entire confidence was felt that we had cells which represented the true value of such combinations, the uncertainty still existed as to the absolute value of the cells In view of this fact, and until such time as future work might throw light on the question, we adopted the legal value in calculation. This is given as

1 4342 int. volts at 15° C

Recent investigations† have shown that this value of the Clark cell is too high The most probable absolute figure, so far as it is possible to fix it, is now given as

1 4330 int. volts at 15° C

for the cells set up according to the new specifications

There is, however, an important difference between the cells set up according to the old specifications, such as were used in our determinations of the mechanical equivalent of heat, and the cells set up according to the approved methods now adopted at the various standardising laboratories The fundamental difference lies in the treatment of the mercurous sulphate By the old specifications this was washed with water before being mixed with the zinc sulphate to form the paste According to the newer methods, no water is allowed to come in contact with the mercurous salt, which is specially made by various methods, all of which give results in very good agreement F A Wolff and Waters‡ have shown that the cells set up according to the old specification are 0 30 millivolt higher than the new cells

With the assistance of Dr H L Bronson we have recently equipped the laboratory with a set of modern standards, some of which were made by Dr Bronson in Washington, and directly compared with the cells at the Bureau of Standards. Other cells have been made here by him with the assistance of Mr A N Shaw, all of which have agreed very closely with the cells constructed in Washington Details of the comparisons of these cells will be published shortly. It has been possible for us to construct several cells in an exactly similar way to those previously used for the mechanical equivalent of heat measurements, and to compare them with

* 'Zeit. für Instk.,' vol. 21, p. 33 (1901), vol. 21, p 65 (1901).

† K. E Guthe, 'Bull Bureau Standards Washington,' vol. 2, p 33 (1906), T Mather and F E. Smith, 'Roy Soc. Proc.,' A, vol 80, p. 383 (1908).

‡ 'Bull. Bureau of Standards,' vol. 4, p. 64 (1907)

our new cells. The result of this test has shown that the mean of our cells, set up according to the old specifications, as our old cells were constructed, are 0 28 millivolt higher than the modern Clarks As a matter of fact, our new cells are exactly 18 microvolts (0·018 millivolt) higher than the reference standards at Washington, which brings the mean of our cells set up, according to the old specifications, to 0 30 millivolt We have also set up a number of inverted Weston cells, using the washed mercurous sulphate, and find them approximately the same amount higher than the modern Weston cells prepared according to the new specifications

The absolute value which we must therefore take for our cells in order to reduce the values of the mechanical equivalent must be 0 30 millivolt higher than the value assumed for the modern Clarks, or

$$1\,4330 + 0\,0003 = 1{\cdot}4333 \text{ int volts at } 15^\circ \text{ C}$$

It is possible now to express the original determinations on the basis of the new value for the Clark cell The value assumed for the resistance standards used in the investigation must be also assumed The actual resistance through which the heating current was passed was compared with 11 standards, all of which had certificates signed by Dr Glazebrook, and a twelfth standard from the Reichsanstalt All these standards were expressed in terms of the international ohm and agreed with each other to better than 1 part in 10,000, except one which was subsequently found to be in error and which was not included in the mean. It is exceedingly unlikely, therefore, that either the mean of the resistance standards or of the Clark cells were in error by as much as 1 part in 10,000 from the international units, whatever values may ultimately be assigned to these in C.G S units

In the calorimetry also great pains were taken to eliminate errors of thermometry and heat loss, and the agreement of the observations, taken at different times and under widely different conditions, was not far from 1 part in 10,000 The method of water jacketing rendered the calorimeter independent of external temperatures. This made the work at the extremes of the temperature range as accurate as those at ordinary temperatures. It is difficult to see where the measurements can be in error by more than 1 part in 10,000 at any part of the range

In the table following is given the values of the thermal capacity of water at different temperatures between 5° and 95° C in joules per calorie. Column 1 contains the original values expressed in terms of the Clark cell equal to 1·4342 int. volts at 15° C. Column 2 contains the same results reduced to the modern Clark equal to 1 4330 int. volts at 15° C., or what is the same value for the old Clarks, 1 4333 volts. In Column 3 is given the

values of Rowland by the direct mechanical method obtained between 5° and 35° C as corrected by Waidner and Mallory.

Table I.

Temperature	Joules per calorie Clark 1 4342	Joules per calorie Clark 1 4330	Rowland Mechanical
5	4 2105	4 2052	4 206
10	4 1979	4 1926	4 196
15	4 1895	4 1842	4 188
20	4 1838	4 1785	4 181
25	4 1801	4 1748	4 176
30	4 1780	4 1727	4 174
35	4 1778	4 1720	4 175
40	4 1773	4 1720	
45	4 1782	4 1729	
50	4 1798	4 1745	
55	4 1819	4 1766	
60	4 1845	4 1792	
65	4 1870	4 1817	
70	4 1898	4 1845	
75	4 1925	4 1872	
80	4 1954	4 1901	
85	4 1982	4 1929	
90	4 2010	4 1957	
95	4 2038	4 1985	
Means	4 1888	4·1835	

The means of the two columns are seen to be, for the interval between 5° and 95° C.—

For Clark..... 1·4342 4 1888 joules per calorie
Clark . . 1 4330 4·1835 „

It is a matter of interest to compare these values with the direct mechanical measurements of Rowland, and of Reynolds and Moorby. On account of the limited range of Rowland's experiments comparison can only be made between 5° and 35° C The mean value over this range from Rowland's curve is found to be—

Rowland's mean value.. ..... 4·185 joules

From the second column of results, the mean value over the same range is found to be—

Electrical mean (Clark 1 4330) . .. . 4 1826 joules

Here a divergence of 1 part in 2000 is shown between the two values, but when we enquire into the limits of accuracy which Rowland assigns for the values he gives, we find that this is about 1 part in 1000. Hence we find an

agreement of 1 part in 2000, which is better than Rowland gives as the probable accuracy of his measurements.

When we come to the absolute measurements of Reynolds and Moorby, we find probably greater accuracy on account of the method they adopted. They used the continuous method with steady temperatures and heated the water from the freezing to the boiling point The value they obtained was the mean over the entire range, and was found to be—

Reynolds' and Moorby's mean . 4·1832 joules

Prof A. W Smith* has recently pointed out that their results are really for an interval of temperature between 1° and 100° C To express this mean value accurately between 0° and 100° C, it becomes necessary to increase their value slightly, since the value of "J" increases rapidly between 1° and 0° C Assuming the variation of the specific heat of water from our determinations, this value is increased to

4·1836 joules

for the mean calorie

Over the same interval our mean value must be increased slightly, making allowance for the increase at both ends of the temperature range This brings the value

4·1835 between 5° and 95° C

up to 4·1849 between 0° and 100° C,

which is in accord with their value to 1 part in 4000 The value is therefore seen to be between that of Rowland and that of Reynolds and Moorby The excellent agreement shown here indicates that we must, at the present time, have a knowledge of the electrical units not far from the truth, both as regards the Clark cell and resistance The agreement with Rowland to within the limits of accuracy of his measurements serves to link his absolute mechanical measurements with those of Reynolds and Moorby in a very satisfactory way, and to show that they obtained almost identical values.

The value of the 15° C calorie is a matter of considerable importance—

Rowland gives this as .. .. . 4·188 joules
Our value comes ... . 4·184 „

Assuming the variation of the specific heat, Reynolds' and Moorby's value comes—

4·183.

* 'Monthly Weather Review,' vol. 35, p. 458 (1907)

A paper has recently appeared* in which the 15° C calorie has been discussed Our value of this quantity is given as—

4 187 joules

This value is given as expressed in terms of a Clark cell equal to 1 4334 int. volts at 15° C The measurements for the Clark cell here quoted were carried out by an absolute method in the Macdonald Physics Building by Mr R O King and described by Prof Callendar,† but were only preliminary and no record has been kept of the particular cell used Although the value is remarkably close to the correct figure for our old cells, it can hardly be taken as representing the mean Clark cell which was adopted in the calculations for the mechanical equivalent Mr King eventually intended to obtain more accurate readings for our Clark cell mean, but was unfortunately called off on other work. The work is being continued in this laboratory after a lapse of several years, and it is hoped that further results may be communicated later

[*Note by Prof. H L Callendar*—I designed the apparatus used by Mr R. O King to read to an order of accuracy of 1 in 100,000, and personally assisted him in all the fundamental measurements. I had no doubt at the time that the result which I worked out from Mr King's observations, though not so perfect as might have been obtained in a more extended investigation, correctly represented to 1 in 10,000 the absolute value of the Clark cells at that time employed in our investigations in terms of the international ohm It is very satisfactory to find that Dr Barnes' recent comparisons agree so well with the old absolute measurement, but I think the absolute value of the mechanical equivalent deduced from the electrical standards may possibly be in error by at least 1 part in 4000]

* 'Verh. der Deut Phys Ges.,' vol 6, p 589 (1908).
† 'Phil Trans,' A, vol 199, p 55 (1902)

An Approximate Determination of the Boiling Points of Metals.

By H. C. Greenwood, M.Sc., Beyer Fellow of the University of Manchester.

(Communicated by Prof E. Rutherford, F.R.S.—Received April 30,—Read May 27, 1909)

Despite the facility with which high temperatures can be reached and maintained constant by means of electric heating, no general investigation of the boiling points of the metals has yet been carried out, and such information as is available has in many cases been obtained by considerable extrapolation. Moreover, the published data are remarkably discordant, as will be seen from the individual results quoted below.

In the course of an extended experimental investigation, H. Moissan* has made observations on the vaporisation of metals at high temperatures by observing the loss of weight of a considerable mass of metal heated for definite periods of time in his arc furnace. O. P. Watts† has attempted to deduce from these experiments approximate values for the boiling points of the metals. In addition to the uncertainty due to the fact that many metals possess a high vapour tension at temperatures much below their actual boiling points, considerable errors are caused by the fact that Moissan does not appear to have measured the expenditure of energy in the furnace, which varies widely according to the conductivity of the vapours surrounding the arc. Also, in many of his experiments the temperature of ebullition must have been altogether modified by carburisation.

Since it is now possible to perform effectively the heating and to make relatively accurate temperature measurements in the region concerned, the remaining difficulty is largely due to our ignorance of any material capable of remaining gas-tight at sufficiently high temperatures.

For approximate measurements it has, however, been found possible to circumvent this difficulty and yet obtain a sufficiently definite proof that actual boiling is taking place.

In the present investigation the following metals have been studied: aluminium, antimony, bismuth, chromium, copper, iron, lead, magnesium, manganese, silver, tin.

Experimental Methods.

In some unpublished work carried out by Dr. L. Bradshaw in this laboratory, measurements were made of the loss in weight of a crucible,

* H. Moissan, 'Comptes Rendus,' vol. 142, p. 425 (1906).

† O. P. Watts, 'Trans. Amer. Electrochem. Soc.,' 1907, vol. 12, p. 141.

containing the metal under investigation, maintained for a definite time at constant temperature, it being hoped that, by repeating the measurements at fixed points over a wide range of temperature, the loss by volatilisation would enable a fairly close approximation of the boiling point to be deduced. It was found, however, that the volatilisation occurs over such a large temperature interval that in starting the present investigation other methods were resorted to

In the first place thin-walled crucibles, containing a considerable amount of metal, were heated, either in a Moissan arc furnace* or in a carbon tube furnace, the energy consumption being kept constant and chosen so as to be capable of ensuring in the enclosure a temperature considerably higher than the boiling point of the metal under investigation

During the heating the temperature of the outside wall of the crucible was measured at regular intervals by means of an optical pyrometer; it being anticipated that when the relatively large mass of metal entered into ebullition some definite indication would be obtained that the crucible walls ceased to show an increase in temperature The measurements, however, were distinctly disappointing, the crucible walls rising considerably above the boiling point of the metal

The method eventually adopted was to employ a vertical carbon tube resistance furnace,† in which was suspended a long graphite crucible which contained the metal under investigation (see fig. 1). The depth of metal employed was usually about 30 mm‡ Temperature readings of the outer walls of the crucible were taken by means of a Wanner optical pyrometer, a side tube being provided exactly opposite the lower end of the crucible, and being so arranged that only the radiation from the crucible walls could fall on the pyrometer, the side tube was kept clear of vapours by a current of hydrogen

The temperature of the heating tube, which could be readily raised to 2700°, was under delicate control by adjustment of the current passing through it.

The measurements of the boiling points were carried out by slowly raising the temperature of the crucible and observing the surface of the metal from above through an absorbing glass. At first the surface of the molten metal remains perfectly still, but as the boiling point is approached a slight agitation

* This method was employed by Féry ('Annales de Chimie et de Phys.,' sér 7, vol. 28, p. 425). His values are 1040° C for zinc and 2100° C for copper.

† *Cf* Hutton and Patterson, 'Trans. Faraday Soc.,' vol. 1, No 2, p 187

‡ The tall crucible employed acts as a reflux condenser, so that the quantity of metal does not rapidly decrease. Moreover, comparative experiments proved that wide variations in the height of the metal had no influence on the boiling point indicated.

of the surface is observed which soon becomes vigorous. In the case of most of the metals studied, the difference between the temperature indicated when a gentle agitation is first apparent and that at which the ebullition has

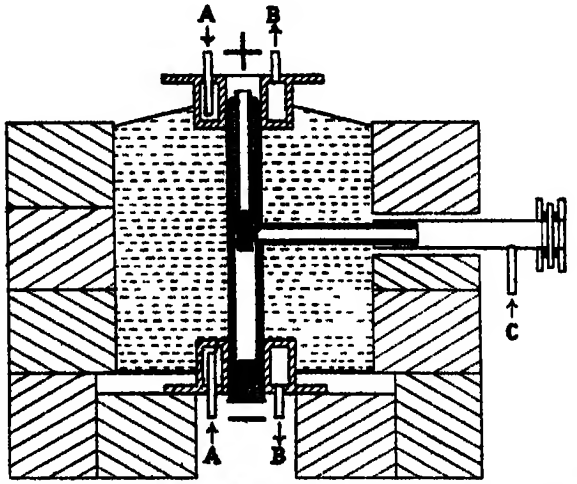


FIG 1.—Resistance furnace, consisting of vertical carbon tube electro-coppered at the ends and soldered into brass castings, provided with water circulation at A and B. Temperature readings taken down the side tube of carbon, fixed into a brass tube with a window at the end, a current of hydrogen being admitted at C. The whole furnace was packed in crushed wood charcoal, while a thin walled graphite crucible contained the metal to be studied.

Scale 1 8

become so violent that globules of metal are being ejected from the top of the tall crucible does not exceed 100°. By taking the boiling point as that temperature at which ebullition becomes decided, quite concordant results were obtained in different experiments, as will be seen from the detailed measurements in three experiments with silver, given below.

From the study of a large number of metals in this manner, it appears probable that the temperature at which the vaporisation becomes sufficiently rapid to cause a decided projection of drops from the surface may be taken with fair approximation as the boiling point, in the three experiments quoted this temperature is 1955°.

The question as to whether the temperatures measured on the outer surface of the crucible really indicate sufficiently accurately the actual temperatures of the metal is certainly an important one. Measurements made up to 1500° in comparison with a thermo-element indicate that the difference is not very great, but in order to obtain further confirmation and attempt to make use of a somewhat different method of measurement, an apparatus as shown in

I				II				III			
Time from commencement	Temperature.	Current amperes		Time from commencement.	Temperature	Current amperes		Time from commencement	Temperature	Current amperes	
mins	°			mins	°			mins	°		
12	1800	320	Clear and still	16	1770	260	Perfectly clear and still	6	1440	330	Clear and still
13	1820	320	" "	18	1815	260	" " "	9	1560	330	" "
14	1860	320	Clear, gentle agitation at edges, centre still.	20	1860	260	" " "	12	1670	360	" "
				21	1875	290	Slight trace of movement at edges	13	1730	360	" "
15	1905	330	All surface in gentle agitation, no drops thrown up					16	1790	360	" "
				22	1880	290	Surface quiet.	18	1840	370	" "
16	1915	330	" " "	23	1915	290	Gentle agitation, no drops thrown up	20	1860	370	Slight agitation at edges
17	1935	330	Drops thrown up very gently					21	1880	380	" " "
				24	1925	290	" " "	22	1900	380	Clear, gentle agitation all over surface
18	1935	330	" " "	25	1955	290	Decided agitation, drops thrown up gently				
19	1955	330	Drops quite decided.					24	1920	380	Agitation decided, no drops thrown up
20	1955	330	Drops thrown half way up crucible	26	1955	300	" " "				
				27	1975	300	Drops half way up crucible	25	1935	390	Agitation decided, occasional drops
21	1965	330	Drops sent out of crucible	28	1975	300	Vigorous, drops projected out of crucible	27	1955	390	Drops sent up decidedly
22	1975	330	" " "					29	1980	390	Drops vigorous, half way up crucible
				29	1980	300	" " "				
								30	2000	390	Drops projected out of crucible.

fig 2 was employed Here the heating is effected from the inside by a rod of carbon, an annular crucible containing the metal, and the temperature being read on the outer surface of the crucible as before, thus reversing the effect of errors in temperature measurement due to the thermal conductivity of the

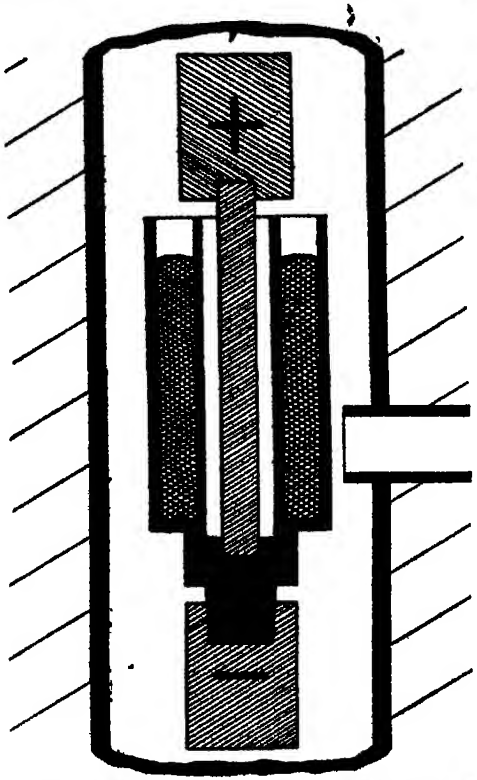


Fig 2.—Annular crucible turned out of graphite, heating being effected electrically by means of the central carbon rod, which is supplied with current from the two thick graphite rods. The crucible is fixed inside a long wide carbon tube surrounded by Kieselguhr, and temperature readings taken through a side tube, just as with the apparatus shown in fig 1.

Scale 2 5.

crucible walls, and altogether removing any possibility of reflexion of the radiation from the hotter surface of the heating tube.

Experiments made with lead proved a close agreement between the results obtained by the two methods, but all attempts to obtain limitation of the temperature of the outer wall whilst the metal was boiling, and when the heating rod was maintained at an excessively high temperature, proved abortive.

The current of hydrogen which was passed through the side tube of the

furnace (shown in fig. 1) was found to have quite a marked influence on the ebullition of the metal. If the current of hydrogen is stopped when the metal is gently boiling ebullition ceases Moreover, when nitrogen was employed instead of hydrogen, the temperature readings were concordant in similar experiments, but were always considerably higher (50° to 100°) than those obtained in a hydrogen atmosphere

This curious and unexpected effect appears to be due to the ease with which hydrogen permeates the crucible walls, removing and diluting the column of heavy vapour If, after interrupting the admission of hydrogen for a brief period, the stream be restarted, it can be distinctly seen, on looking down the crucible, that the vapours above the metal surface are immediately dislodged by gas passing through the crucible walls Using a slow current of nitrogen, on account of its much higher density, the values obtained are practically the same as if no gas at all were admitted.

A large number of measurements were made in a nitrogen atmosphere and the results are indicated below, but it seems probable that the values obtained with hydrogen approximate more closely to the boiling points of the metals at atmospheric pressure.

The use of a graphite crucible is naturally limited to such metals as do not appreciably combine with, or dissolve, carbon under the conditions of the experiment As will be seen, modifications have to be introduced in the case of other metals.

Temperature Measurements

Throughout this investigation the temperatures have been measured by a Wanner pyrometer sighted on the outer wall of the crucible containing the metal This was made possible by the provision of a side tube as shown in fig 1, the tube being kept free from vapours by a current of hydrogen

In order to ensure the accuracy of the temperature readings, the current passing through the standard lamp of the pyrometer was carefully adjusted, an ammeter being included in the lamp circuit. Comparison with a thermo-element up to 1500° C (using the correction for the difference between the optical and thermo-electric scales of temperature)* showed that the temperature scale provided with the instrument closely concorded with the measurements of the thermo-element.

In order, however, to render the results capable of subsequent correction, the pyrometer was standardised by measuring the "black body" temperatures of the melting points of platinum, rhodium, and iridium These metals, which were specially prepared of a high degree of purity by

* *Cf* Burgess, 'Trans. Amer. Electrochem. Soc.,' vol 11, p. 247.

Messrs. Johnson, Matthey, and Co, were used in the form of strips 4 mm. wide by 8 cm long, and were heated electrically, the pyrometer being sighted upon them

The individual values, as measured with this pyrometer under identical conditions to the measurements for the boiling points, were as follows :—

Platinum	Rhodium	Iridium
1545°	1670°	1995°
1560	1655	1985
1555	1680	2025
1545	1680	1990
—	1680	2035
Mean 1551	1673	2006

Holborn and Henning* give the following "black body" values of the melting points :—platinum, 1545°, rhodium, 1650°, iridium, 2000°

In view of the fact that such fixed points can readily be determined, and consequently the relative accuracy of the measurements given in the present work may be conveniently checked, it has been thought best to publish the results as they were obtained, without correction, it being clearly understood that the temperatures are all given on the optical scale

The only correction introduced is a deduction of 20° from the values in the summary for temperatures below 1500°, as these were determined without any absorption glass before the pyrometer and standardisation under these conditions indicated this difference of 20°

The main object of the investigation was to obtain approximate values for the boiling points measured under closely similar conditions, and thus clear away the uncertainty of the published data, which in several cases amounts to some hundreds of degrees. In order to obtain more accurate results, it remains to introduce refinements in the method which it was impossible to adopt in an attempt to extend the work over a large number of metals

Copper.—Practically the only determination previously recorded of the boiling point of copper is that of Féry,† his value being 2100° C Wartenberg,‡ from measurements of the vapour densities of the metals, and by observing at what temperatures they begin to assume the gaseous state rapidly, deduced approximate values for the boiling points of a few metals, he classes copper with tin and aluminium as probably above 2200° C.

* Sitzungsber K. Akad. Wiss. Berlin,' 1905, vol. 12, p. 311

† 'Annales de Chimie et de Physique,' sér 7, vol. 26, p. 428 (1903).

‡ 'Z. für anorg Chemie,' vol 56, p 320 (1908).

When measurements were made by the general method described above the different stages of the vaporisation were sharply indicated, a very clear surface of the metal being visible throughout

The following results were obtained for electrolytic copper at intervals of several months, the atmosphere being hydrogen, which was admitted through the sighting tube for the pyrometer

First gentle agitation of surface	Decided ebullition	Violent ejection of material from the crucible
2250°	2300°	2350°
2300	2320	
2280	2320	2400
2230	2275	
2275	2320	2375

Boiling point Mean approx 2310°

In nitrogen, as explained previously, the readings are considerably higher, the first agitation of the surface occurring at 2430°, decided ebullition at 2450°, and violent projection of material at 2475°

An examination of the copper subsequent to the experiments indicated that a small amount of carbon had been dissolved, and, upon cooling, given up in the form of graphite

Tin.—Carnelley and Williams* attempted to measure the boiling points of some metals by suspending above the surface of the highly heated metal small iron or fireclay tubes containing metals of known melting point, they record the boiling point of tin as between 1435° and 1505° C.,† and state that it boils very well and is more easily volatile than lead

Wartenberg,‡ on the other hand, gives a probable value above 2200° C

Moissan's experiments indicated that tin is less volatile than copper

In the direct measurements made by the method described, the surface of the molten metal remained very clearly visible and the different stages were sharply indicated The following values for tin were obtained in a hydrogen atmosphere

In nitrogen the first general agitation of the surface occurs at 2220°, ebullition at 2350°, and violent projection of material at 2400°.

On subsequent examination, the metal did not appear to have been appreciably affected by the carbon.

* Carnelley and Williams, 'Journ. Chem. Soc.,' 1879, p. 563.

† M.P of nickel and iron.

‡ *Loc. cit.*

First gentle agitation of the surface	Decided ebullition	Violent ejection of material from the crucible
2150°	2275°	2320°
2100	2250	2300
2150	2250	2300
2210	2295	2350
2200	2270	2340
2150	2270	2320
2170	2250	2300

Boiling point Mean approx 2270°

Silver.—The value deduced by Wartenberg* from his vapour density determinations is 2070° C With silver assaying 99 9 per cent. the following results were obtained in hydrogen, the readings being very concordant

First gentle agitation of the surface	Decided ebullition	Violent ejection of material from the crucible
1900°	1955°	1975°
1915	1955	1975
1915	1960	2000
1900	1955	2000

Boiling point Mean approx 1955°

Silver is apparently very little affected by the carbon with which it was in contact at these high temperatures

In nitrogen the first gentle agitation occurs at 2020°, ebullition at 2050° and projection of material at 2075°

Lead.—Carnelley and Williams† give the boiling point as between 1435° and 1505° C, whilst Wartenberg estimated it at 1580° C

With Merck's "extra pure" lead, the following direct determinations were made in a hydrogen atmosphere —

First gentle agitation of the surface	Decided ebullition.	Violent ejection of material from the crucible
1475°	1530°	1585°
1475	1505	1540
1470	1540	1580
1475	1525	1600
1475	1525	1570

Boiling point Mean approx 1525°.

* *Loc. cit.* † *Loc. cit.*

In nitrogen the first gentle agitation of the surface commences at 1525°, boiling is distinctly visible at 1570°, and projection of material at 1600°.

Bismuth.—Carnelley and Williams found the boiling point to be between* 1084° and 1435° C, Barus,† by extrapolation from the values obtained at low pressure, gives as the approximate boiling point at atmospheric pressure 1550° C

Using Merck's "extra pure" bismuth, the following readings were obtained in hydrogen, the surface of the metal being somewhat obscured above 1425° C by the condensing metallic vapours After the experiment the metal seems to be quite unchanged

First gentle agitation of the surface	Decided ebullition	Violent ejection of material from the crucible
1415°	1435°	1460°
1400	1425	1480
1420	1450	1500
—	1450	1500

Boiling point Mean approx 1440°

In nitrogen the values deduced are 1450° for agitation of the surface 1500° for boiling point, and 1530° for violent boiling

Antimony.—Several measurements have been recorded Carnelley and Williams state that it lies between 1084° and 1435° C, Mensching and Meyer‡ above 1437° C, and Biltz and Meyer§ 1500° to 1700°

Although the observation of the surface of the metal is in this case rendered somewhat difficult by the clouds of metallic vapour, the readings were fairly consistent

The metal employed was Merck's "extra pure" antimony, and was apparently little affected by carbon after prolonged boiling

First gentle agitation of the surface	Decided ebullition.	Violent ejection of material from the crucible.
1420°	1455°	1500°
1400	1450	1470
1425	1470	1510
1425	1460	1505

Boiling point. Mean approx. 1460°.

In nitrogen the corresponding values were 1480°, 1530°, and 1570°.

* M.P of copper and nickel.

† Barus, "Bull U S Geolog Survey, No 103," 'Amer. Journal. Sci.,' (3), vol. 46, p. 332

‡ Mensching and Meyer, 'Annalen,' vol 240 (1887), p. 317

§ Biltz and Meyer, 'Ber.,' 22, No. 1 (1889), p. 725.

Boiling Point Determinations of Metals which readily Carburise

In these cases very great difficulties were encountered which for long made it impossible to obtain even roughly concordant results Numerous attempts were made to apply the magnesia tubes of the Berlin porcelain factory as crucible materials, but even with very gradual heating they almost invariably cracked before the boiling point of the metal had been reached Eventually, after further fruitless efforts with magnesia and thoria, a method was devised to "brasque" carbon crucibles with highly shrunk pure magnesia,* although still many of the experiments proved failures owing to the break-down of the lining Only the results of these experiments in which the lining remained free from cracks, and in which none of the liquid metal had come in contact with the carbon, were considered suitable for the purpose in view The graphite crucibles were 15 cm long and 2 5 cm internal diameter, a thick paste of the finely powdered magnesia, mixed with saturated magnesium chloride solution, was placed in the tube, and by means of a wooden former a uniform lining 2 mm thick was obtained After drying slowly at 200° C, the crucible was placed in the furnace, and the temperature gradually raised to about 1700° C, hydrogen chloride being given off copiously during the process It was found that the minimum risk of the brasque cracking was secured by adding the charge of metal, after allowing the crucible only to cool to about 1300° C., the experiment being performed immediately afterwards The magnesia brasques prepared in this manner remained perfectly hard and coherent after heating to 1800° C, and even when subjected to 2500° C Some trouble is caused by the fact that at about 1700° C the magnesia begins to react with carbon, giving a dark grey sublimate. This action is, however, not sufficiently vigorous to seriously interfere with the observation of the metal, except, perhaps, at the high ebullition temperature of iron

Aluminium.—Deville stated that aluminium was not volatile at a white heat, and recently Wartenberg has estimated that the boiling point lies above 2200° C. In consideration of the facility with which this metal vaporises *in vacuo*, and also from observations on its behaviour in the electric furnace, it was suspected that this value was considerably too high

In addition to its great affinity for carbon, another difficulty encountered with aluminium is the tenacious surface film of oxide which always covers the molten metal.

On gradually raising the temperature of the brasqued crucible with its

* Prepared by heating pure calcined magnesia, packed around a carbon rod maintained at a high temperature by passing through the rod an electric current.

charge of aluminium (about 3 cm. deep), there is no visible motion of the surface of the metal, and no marked sublimation until 1700° is reached. At 1790°, as read on the outside of the crucible, a sudden very vigorous agitation of the surface is observed: in some cases the metal frothed right up the crucible, and globules of metal were ejected very freely from the top of the crucible, a marked noise being also noticed.

To estimate the error due to the low thermal conductivity of the lining, the boiling points of silver and copper were determined under similar conditions in brasqued crucibles. The values obtained, however, were practically the same as those indicated above, so that no correction appears to be necessary. We may therefore fix the boiling point of aluminium as approximately 1800°. The metallic residue in the crucible, in all experiments which gave a definite indication of ebullition, was found to be only very slightly carburised.

How greatly carburisation affects the ebullition can be judged from the fact that aluminium heated in an unbrasqued graphite crucible to 2100° shows not the slightest sign of boiling.

Manganese.—The difficulties in this case were even greater than with aluminium, on account both of the higher temperature of vaporisation and of the marked corrosive action of the metal upon the magnesia brasque. Consequently only a few of the numerous experiments proved successful.

The metal employed was Merck's "extra pure" fused manganese, which was found to be free from aluminium. The different stages of the boiling were quite clearly defined owing to the absence of any surface film. A slight agitation of the surface was apparent at 1850°, ebullition at 1900°, and at 1950° the metal vapour was burning at the top of the crucible with a large yellowish flame.

The boiling point may be given approximately as 1900°.

In an unbrasqued crucible no marked vaporisation of the metal was obtained at 2200°, thus showing the great effect of carburisation.

First gentle agitation of surface	Decided ebullition	Very vigorous ebullition.
°	°	°
1850	1875	—
1850	1900	—
1860	1890	1925

Chromium.—In this case also the brasque is to some extent acted upon by the metal, but fairly definite observations could be made, showing the agitation of the surface to commence at about 2175° and boiling at about

2250°. In an unbrasqued crucible no marked volatilisation is observed at 2500°.

Magnesium.—Until recently the measurement of Ditte,* who found the boiling point to be 1100°, has not been seriously thrown in doubt, but Wartenberg's† published results group magnesium with copper, tin, and aluminium, with boiling point above 2200° It is difficult to see to what this uncertainty is due In a graphite crucible, steady ebullition is clearly visible at 1100°, and measurements made with a protected thermo-element immersed in the molten metal indicate a well defined constancy of temperature for some minutes about 1120°, the metal distilling off There seems little reason, therefore, for classing magnesium with metals of high boiling point.

Iron.—With iron of 99·9 per cent. purity, ebullition was found to set in at about 2450° Observations of the surface were rendered difficult by the products of the reaction between the magnesia and carbon, which at this high temperature is very vigorous The carbon content of the metal after the experiments was under 0 1 per cent.

Conclusion

Subject to the corrections which may be necessary when the temperature scale has been more accurately fixed, the following approximate measurements may be given of the boiling points as determined in the present investigation :—

	°C		°C
Aluminium	1800	Lead	1525
Antimony	1440	Magnesium	1120
Bismuth	1420	Manganese	1900
Chromium	2200	Silver	1955
Copper	2310	Tin	2270
Iron	2450		

Some of the incidental expenses of this research have been borne out of the funds provided by the Government Grant Committee of the Royal Society I wish to express my indebtedness to Dr. R S Hutton for his continual interest and advice during the progress of the work.

* 'Comptes Rendus,' vol. 73 (1871), p 108

† *Loc. cit.*

On Electrostatic Induction through Solid Insulators.

By Prof. HAROLD A WILSON, F.R.S., King's College, London.

(Received May 18,—Read June 17, 1909.)

The following paper contains an account of a series of experiments on the variation of the capacity of condensers having solid dielectrics with the potential difference and the time. The influence of temperature on the results has also been investigated. It is not proposed to give any detailed account of previous work on this subject here. A very full list of papers is given in a paper by Schweidler entitled "Studien uber die Anomalien im Verhalten der Dielektrika," 'Ann. der Physik,' No. 14, 1907, p. 711.

Let A and B be two guard ring parallel plate condensers, the dielectric in A being air and in B any insulator. Let the two guard rings be permanently connected to earth, and the two small plates permanently connected together. Suppose initially the large plates and small plates are connected to earth, the small plates then disconnected, and then the large plate of the condenser B raised to a potential $+V$ and that of A to a potential $-V$. After a time t, measured from the instant at which the plates were charged, the total charge q on the small plates will be given by the equation

$$(V+e)\,C-(V-e)\,C'+eC''=q,$$

where C is the capacity of the condenser A, C' that of B, C'' the capacity of the backs of the small plates and their connecting wire, and e the potential of the small plates. The charge q will be very small if the dielectric in B is a good insulator, and we may put it equal to $kVC't$, where k is a constant approximately. Hence

$$V(C-C')+e(C+C'+C'')=kVC't. \qquad (1)$$

Suppose, now, another experiment is done in which both large plates are charged to the same potential v, and let the resulting potential of the small plates be e' just after charging the large plates, then

$$-v(C+C')+e'(C+C'+C'')=0 \qquad (2)$$

C' is a function of t, but it does not vary very much, so that if C'' is large compared with $C+C'$ we may take $C+C'+C''$ to be constant without sensible error, in which case (1) and (2) give

$$C'/C=\left(1+\frac{ev}{e'V}\right)\Big/\left(1-\frac{ev}{e'V}+kt\right) \qquad (3)$$

When t is large C' will vary very slowly with the time, so that then k can

be found from the variation of e with the time. Then knowing k, C'/C can be found as a function of the time.

If during an experiment C is altered to $C+\alpha$, then we have

$$V(C-C')+e(C+C'+C'') = V(C-C')+V\alpha+f(C+\alpha+C'+C''),$$

where f is the potential of the small plates after the change in C Hence

$$\frac{e}{V} = \frac{f}{V}+\frac{\alpha}{C+C'+C''}\left(1+\frac{f}{V}\right)$$

Thus so long as f is small compared with V, the difference between e/V and f/V will be independent of f This enables a change in C to be allowed for very easily

For a particular value of t equation (3) shows that C'/C depends on e/V, so that if C' is independent of V then e/V should be independent of V

If the two condensers are of nearly equal capacities, e will be small compared with V, so that we may make V large, say 500 volts, and measure e with a quadrant electrometer to, say, 0·01 volt. This will enable variations in C' of 1 part in 10,000 or more to be measured The experiment described provides therefore a sensitive method of determining small variations in the capacity of B

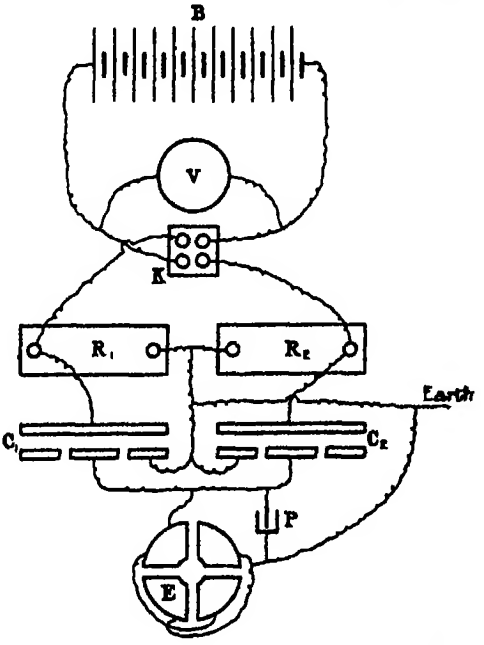
The apparatus used is shown diagrammatically in the figure A large battery B of small secondary cells was connected through a reversing commutator K to two megohm resistances R_1 and R_2 in series The guard rings of the two condensers were connected to the junction between R_1 and R_2 The large plates of the condensers were connected as shown respectively to the battery ends of the resistances The small plates were connected together and to one pair of quadrants of a quadrant electrometer E The other pair of quadrants was connected to the guard rings and to the case of the instrument The two pairs of quadrants could be connected together by means of a wire dipping in a mercury cup P The point of junction between R_1 and R_2 was connected to earth through the gas mains, while the battery and the other parts of the apparatus were well insulated. The potential difference due to the battery was indicated by a Kelvin's multicellular electrostatic voltmeter V.

The condensers were contained in a water-jacketed copper oven, provided with a temperature regulator, which maintained the temperature constant within one-tenth of a degree The air condenser was specially made to fit into the oven. Its guard ring was a square brass plate, which fitted into the oven like a shelf. The large disc was mounted on the guard ring, and could be moved up and down parallel to itself by means of a screw and nut. The nut could be turned slowly by means of a worm worked by a rod passing

through a hole in the oven door. The other condenser was placed a few centimetres below the guard ring, so that the two small plates were opposite each other, and the wire leading to the electrometer was passed through an ebonite plug fitted into the door of the oven.

The electrometer was put up close to the oven door, and the wire leading to it and the mercury cup key were surrounded by metallic screens

The method of making an experiment was as follows —The two pairs of quadrants were connected together, and the battery disconnected so that



everything was at zero potential The apparatus was left in this condition until all residual charge from previous experiments had disappeared. When this was the case, on insulating the small plates and their pair of quadrants the electrometer needle remained steady. The battery was then connected at a time determined with an accurate chronometer, and the electrometer reading was noted at subsequent times The air condenser was adjusted so that the average electrometer deflection during the time in which the observations were made was as small as possible. When once the air condenser was adjusted, a complete series of observations with different potentials was made without altering the air condenser The oven was kept at a constant temperature day and night The sensibility of the electrometer for potential difference was determined with a standard cadmium cell. It was usually about 150 scale divisions per volt.

Experiments on Ebonite.

A plate of ebonite was first used for the condenser B It was 15 cm square and 1 3 cm thick Both sides were coated with thin silver foil, and an annular strip of this, 1 mm wide and 5 cm in diameter, was cut off in the middle of one side The strip of bare ebonite thus exposed was covered with paraffin wax, which was found to improve the insulation considerably Contact with the silver foil was made by means of platinum-tipped springs pressing tightly against it

The following table contains the results of a series of experiments at 30° 2 C. Column 1 gives the times to the nearest hour at which each experiment was begun. Column 2 gives the temperature, which remained very constant Column 3 gives the potential difference maintained between the ends of the total resistance Columns 4 to 12 give the values of $-e/V$ at a series of times in minutes. Thus each horizontal row contains the results of a series of measurements of the potential of the small plates at different times after charging up the large plates The values of $-e/V$ are given in volts per thousand volts —

Table I

Time	Tempera-ture	2V	1′	2′	4′	7′	10′	15′	20′	25′	30′
hours	° C	volts									
3	30·15	+120	8 28	6 96	5 44	4 12	3 32	2·00	1 28	0·72	−0·12
4	30·17	−120	8 28	6 88	5 28	3 64	2 52	1 00	0·20	−0·60	−1 16
5	30·15	+230	8 64	7·08	5 52	4·04	3 00	1 80	0·96	0 28	−0 40
22	30 17	−230	9 32	7 88	6 44	5 12	4 12	3 04	2 16	1 44	0 20
23	30 13	+462	—	—	6 06	4 68	3 72	2·64	1·64	0·96	0 44
24	30·15	−462	—	—	5 84	4 48	3 48	2 28	1 40	0·60	0·12
71	30 14	+936	—	—	—	—	—	2 92	2 24	1 56	0 92
72	30·18	−936	—	—	—	—	—	3·00	2·12	1 40	0·76
74	30 20	+120	9 36	8 10	6·82	5 24	4 62	3 26	2 10	1 68	1 04
75	30·19	−120	9 46	8·10	6 82	5 78	4 20	3 40	2·96	1·78	1 14
77	30 17	+230	9·96	7 98	6 60	5 40	4 50	3 44	2 64	2·04	1 54
95	30·17	−230	10·16	8 66	7 16	5 78	4·96	3 92	3·06	2 46	1 96
96	30·20	+460	9 84	8 42	6 92	5·62	4 70	3 60	2 76	2·08	1 60
97	30·20	−460	9 74	8 38	6 78	5 46	4 52	3 36	2 52	1 80	1 20
98	30·15	+932	—	—	7 06	5 74	4 66	3·62	3·04	2 36	1 84
100	30·18	−932	—	—	7 02	5 68	4 74	3·64	2 82	2 10	1 52
Mean			9·304	7 844	6 412	5 056	4·092	2·932	2 12	1·416	0·782

Between 72 and 74 hours the capacity of the air condenser was altered in order to get the electrometer deflections when using large potentials on the scale for a longer time. The alteration diminished $-e/V$ by −3 46 volts per 1000 volts, so I have added this quantity to all the values of $-e/V$ after

72 hours. The results in the table are therefore for a constant capacity of the air condenser. The values of $-e/V$ are given instead of $+e/V$ because nearly all the values of $+e/V$ are negative

An inspection of this table shows that $-e/V$ is nearly independent of V, but it varies with the time in two ways It diminishes with t, the time in minutes, since the condensers were charged in a particular experiment and also it increases with the time (given in hours), since the series of experiments was begun The second variation is very slow, so that it is scarcely appreciable in under about 10 hours Each vertical column can be divided into four groups containing results got at times only differing by a few hours, so that we may take the means of the results in each group The following table contains these means —

Table II

Time	1′	2′	4′	7′	10′	15′	20′	25′	30′	Mean.
hours										
4	8 40	6 98	5 42	3 94	2·94	1 60	0·82	0 14	−0 56	3 30
23	9 32	7 88	6 12	4 76	3 78	2 66	1 74	1·00	0 26	4 168
74	9 60	8 06	6 74	5 48	4 44	3 20	2 42	1 70	1·08	4 746
97	9 92	8 48	6 98	5 66	4 76	3 62	2 84	2 16	1·60	5 114

For each value of t, $-e/V$ increases nearly equally with the time The mean increase per hour during the first 19 hours is 0 046, during the next 51 hours it is 0 011, and during the next 23 hours it is 0 016 Since this slow increase is nearly the same for the different values of t we can separate the two effects and take the means of the numbers in each vertical column in Table I to give the variation of e/V with t and the means of the numbers in each horizontal row in Table II to give the slow variation of e/V with the time

When e/V is negative the air condenser has the greater capacity, so that it appears that the capacity of the ebonite condenser was slowly diminishing relatively to the capacity of the air condenser

The value of v/e' was found from the following results·—

v in volts	Electrometer deflection	Deflection per volt.
8·3	130	15·65
12 5	194	15 53
20·4	320	15·67
	Mean . .	15·62

The electrometer gave 342 divisions with 2·04 volts. Hence

$$v/e' = \frac{342}{2\cdot04\times15\cdot69} = 10\cdot74.$$

For the purpose of calculating the slow variation of the capacity we may take $C'/C = 1+2ev/e'V$. Substituting the mean values from Table II in this formula gives the following numbers —

Time in hours	4.	23	74	97
C'/C	0 929	0 910	0 896	0 890

Thus the ratio of the capacities of the condensers diminished by about 4·2 per cent. in 93 hours This variation was probably connected with the fact that the temperature had recently been raised from about 15° C to 30° C The variation is more rapid during the first 19 hours than afterwards. After the temperature had been kept at 50° C for several days the variation of the ratio at 30° C. was again examined The following values of e/V were obtained Each number is the mean of several observations with different potentials —

Time in hours	0	20	47
$+e/V$ volts per 1000	1 48	1 58	1 71

In this case e/V was positive, so that the ebonite condenser had the greater capacity, and the increase of e/V with the time shows that its capacity was increasing relatively to that of the air condenser It appears, therefore, that after the temperature has been raised the capacity of the ebonite condenser falls slowly at constant temperature for a long time, while after a fall of temperature its capacity slowly rises This effect is probably analogous to the slow variations which take place in the volume of a thermometer bulb after a large temperature change.

This variation of the ratio of the two capacities might be due to a variation in the capacity of the air condenser The distance between the plates of this condenser was 2 9 mm. during these experiments on ebonite. If this altered by 0 1 mm this would account for the 4 2 per cent. change in the ratio of the capacities. The condenser was rigidly constructed, so that it does not seem at all likely that this can be the explanation of the variation To test the working of the apparatus some experiments were tried with two air condensers. The second one consisted of a shallow brass box, the top of

which was closed with a thick ebonite plate This plate carried a brass disc inside the box and about 3 mm from the bottom The bottom of the box and the disc formed the condenser plates. A wire from the disc passed through the ebonite plate and was surrounded by an earthed guard ring to prevent leakage from the box to the disc across the ebonite This condenser was made specially rigid It was put in the oven at 30° C instead of the ebonite condenser, and the adjustable air condenser was adjusted until no deflection was obtained with −950 volts There was then a deflection of about three scale divisions in one minute with +950 volts The electrometer gave 170 divisions for one volt With ±472 volts the deflection was about two scale divisions in one minute After 24 hours −950 volts still gave no deflection This experiment shows that the capacity of the adjustable air condenser remained constant to within about 2 parts in 10,000 during 24 hours, although just before the experiment it was cooled down The variation was at least 50 times less in this experiment than with the ebonite condenser, which seems to show that the variation was really due to the ebonite and not to the air condenser I have since repeated this experiment with the two air condensers, and found that the ratio of their capacities did not vary by 1 part in 100,000 in 90 hours at constant temperature

The variation of the capacity with the time of charging will now be considered

The mean values of $-e/V$ given in Table I were plotted on square paper and a curve drawn through them Tangents were very carefully drawn to this curve, and from the inclinations of the tangents the following values of $\frac{d}{dt}\left(-\frac{e}{V}\right)$ were obtained

Time in minutes	1	2	3	4	6	8	10	15.	20	25	30
$10^3 \frac{d}{dt}\left(-\frac{e}{V}\right)$	2 38	1 00	0·686	0 58	0 408	0 33	0 278	0 190	0 154	0 134	0 116
	2 34	1·92	1·94	2 16	2 21	2 32	2 38	2 25	2 28	2 35	2 28

The last row contains numbers got by subtracting 0·04 from the values of $10^3 \frac{d}{dt}\left(-\frac{e}{V}\right)$ in the second row, and multiplying by the times These numbers for the larger times are nearly constant, so that it appears that $\frac{d}{dt}\left(-\frac{e}{V}\right) - 0{\cdot}00004$ is inversely proportional to the time since the condenser was charged. After a long time the rate of variation would therefore be

0·00004 per minute, so that this may be taken to be the effect due to the conduction through the ebonite. Adding 0 00004 t to the values of $-e/V$, we get what should be the values of $-e/V$, which would have been obtained if the ebonite had insulated perfectly The following table contains these corrected values of $-e/V$ in volts per 1000, and the values of C'/C deduced from them by means of the equation

$$\frac{C'}{C} = \frac{1+10\,74\,e/V}{1-10\,74\,e/V},$$

which is got from (3) by putting in the value found for v/e' and $k = 0$

The fourth column contains values of C'/C calculated from the formula

$$C'/C = 0\,7874+0\,115 \log (1+t)$$

Time in minutes	$-e/V$ (Corrected)	C'/C (Found)	C'/C (Calculated)
1	9 344	0 8175	0 8220
2	7 924	0 8429	0 8423
4	6 572	0 8678	0 8678
7	5 336	0 8915	0 8913
10	4 492	0 9076	0 9072
15	3 532	0 9270	0 9259
20	2 920	0 9390	0 9394
25	2 416	0 9491	0 9502
30	1 982	0 9581	0 9589

If the capacity of the ebonite condenser is put equal to 1 when $t = 0$ its capacity at any subsequent time is given by the equation

$$C = 1+0\,1461 \log_{10}(1+t),$$

which agrees with the values found to about 1 part in 1000 from 2 to 30 minutes

The variation of the capacity with the time cannot be represented at all exactly by a formula like $A+B(1-e^{-\alpha t})$, for this makes the rate of variation proportional to $e^{-\alpha t}$ instead of to $(1+t)^{-1}$.

Schweidler* found that the rate of variation of the capacity of some of his condensers was proportional to t^{-n} where n was a constant. His values of n were less than unity

Trouton and Russ† found that the current due to the recovery of the residual charge of condensers could be represented by the formula $C = a/(t+b)$, where a and b are constants. This gives for the total charge recovered in a time t, $Q = a \log (1+t/b)$

* *Loc cit*

† 'Phil Mag,' May, 1907.

A series of experiments was done on the ebonite condenser at 40° C After raising the temperature it was necessary to slightly alter the capacity of the air condenser The following table contains the mean results obtained. The conductivity of the ebonite was not appreciable

Time (t) in minutes	$-e/V$ Volts per 1000	C′/C (Found)	C′/C (Calculated)
1 5	16 24	0 ·7028	0 ·6959
2 5	13 90	0 ·7399	0 ·7381
5 0	10 24	0 8015	0 8057
7 5	7 68	0 8476	0 8492
12 5	4 66	0 9046	0 ·9072
17 5	2 62	0 9452	0 ·9467
22 5	1 ·06	0 ·9774	0 9767
27 5	− 0 12	1 002	1 ·001

The last column contains numbers given by the equation

$$C'/C = 0\,5811 + 0\,2885 \log (1+t)$$

Putting $C' = 1$ when $t = 0$ gives

$$C' = 1 + 0\,4965 \log (1+t)$$

It appears, therefore, that the capacity varies with the time 3·4 times as rapidly at 40° C as at 30° C

Another set of measurements was made at 50° C, but at this temperature the insulation between the small plate and the guard ring was not perfect, so that reliable results could only be obtained for times of charging up to about five minutes The following table contains the mean results obtained —

Time (t) in minutes	$-e/V$	C′/C	C′/C (Calculated)
1	16 2	0 ·703	0 ·703
2	11 4	0 781	0 775
4	7 ·0	0 859	0 ·864
7	2 6	0 ·946	0 ·946

Thus at 50° C the capacity varied as much in 7 minutes as in 15 minutes at 40° C

The last column contains numbers given by the equation

$$C'/C = 0\,582 + 0{\cdot}404 \log (1+t),$$

which gives, on putting $C' = 1$ when $t = 0$,

$$C' = 1 + 0\,693 \log (1+t).$$

Paraffin Wax

A paraffin wax condenser was substituted for the ebonite one It had a capacity about three times larger It was found that with the wax the deflection of the electrometer was simply a linear function of the time after charging This was taken to mean that the capacity did not change appreciably, and that the increase in the deflection was due to a small conduction current The increase amounted to about 0 006 volt per minute with 500 volts, but was nearly independent of the potential difference This makes the conductivity of the paraffin with 500 volts about 10^{-19}

The electrometer deflection for $t = 0$ was found by producing the straight line drawn through the points representing the observed deflections The following are some of the results obtained in this way at 30° C —

Potential in volts	e in volts	e/V	C'/C
60	0 056	0 000933	1 0093
118	0 125	0·00106	1·0106
247	0 242	0 000982	1·0098
493	0 470	0·000954	1 0095
Mean		0 000982	1 0098

In this case the capacity was independent of the potential to about 1 part in 2000

At 40° C similar results were obtained The leak amounted to about ·0 036 volt per minute, and again was nearly independent of the potential difference The conductivity therefore increased six times for a rise of 10° C

At 50° C the leak was 0 2 volt per minute with 60 volts, 0 23 with 120, 0 43 with 240, and 0 6 with 480. The current was therefore nearly proportional to the square root of the potential difference

Quartz

An attempt was made to work with a condenser having a plate of crystalline quartz cut perpendicular to the optic axis for its dielectric. The quartz, however, did not insulate sufficiently well, although it was very carefully cleaned and embedded in highly insulating paraffin wax The leak was through the quartz, not over its surface.

Sulphur

A series of measurements was made with a sulphur condenser at 30° C. The following are the mean results —

Time (t) in minutes	$-e/V$ (Volts per 1000)	C′/C (Found)	C′/C (Calculated)
10	16 12	0 879	0 880
15	9 82	0 925	0 928
20	4 12	0 968	0 972
25	−0 84	1 007	1 007
30	−4 8	1 040	1 040

The last column contains the numbers given by the formula—

$$C'/C = 0\,743 + 0\,65 \log (1 + t/16)$$

Making $C' = 1$ when $t = 0$, this gives

$$C' = 1 + 0\,875 \log (1 + t/16)$$

The capacity of the sulphur condenser increased by 18 per cent. from 10 to 30 minutes The ebonite condenser at 30° C increased in capacity 5 5 per cent in the same time The conductivity of the sulphur was not appreciable

Conclusion

The experiments described above show that for a definite time of charging the capacity of a condenser is nearly independent of the potential difference The variation is not more than 1 part in 1000, and is probably really much less than this Small variations appear to occur, but they are evidently due to slight temperature changes or to the after effects of the previous chargings There is consequently no reason to suppose that anything of the nature of true hysteresis exists The capacity is a function of the time of charging, and in the case of ebonite and sulphur varies from 5 per cent to 20 per cent in the course of half an hour This large variation of the specific inductive capacity of sulphur and ebonite makes these substances very unsuitable for use as insulators in experiments in which exact electrostatic measurements are to be made, unless care is taken that the total capacity is large compared with the capacity due to the insulation.

The capacity (C) at constant potential difference (V) is given very approximately by the formula—

$$C = C_0 \{1 + B \log (1 + pt)\},$$

where C_0 is the initial value of the capacity, t the time of charging, and B

and p are constants. The following table contains the values found for B and p.—

Dielectric	Temperature	B	p
	°C		
Ebonite	30	0 1461	1
"	40	0 4956	1
"	50	0 693	1
Paraffin wax	--	0	—
Sulphur	30	0 875	0·0625

The constant p appears, therefore, to be independent of the temperature

The charging current or $V\,dC/dt$ is equal to $VC_0B/(t+1/p)$ Schweidler* determined the charging current with a galvanometer, and considered that it was proportional to t^{-n}, where n was rather less than unity. I find that his results for glass can be represented fairly well by a formula of the type $a/(t+b)$ The following are his results for a glass condenser :—

Time (t) in seconds	Current (y)	$(t+10)y$
10	16	320
20	10	300
30	6 6	264
50	5 1	306
80	3 1	279
100	3 0	330
150	2 0	320

The currents in the above table are those due to a constant potential difference of 300 volts After 900 seconds the condenser was disconnected from the battery and connected directly to the galvanometer, when the following currents were observed —

t	(y)	$(t+10)y$
10	18	360
20	9 8	294
30	7 3	292
50	5·0	300
80	3·9	351
100	3 1	341
150	2·0	320

Here t is reckoned from the instant at which the condenser was connected to the galvanometer If a condenser is charged at constant potential for a

* *Loc cit*

long time and then discharged through a galvanometer, then the discharge current should, according to Pellat's theory as generalised by Schweidler,* be equal to the charging current. Schweidler's results show that this is approximately true If we assume this, then according to my results we should expect the discharge current to be given by the expression $VC_0B/(t+1/p)$, which is of the same form as that found by Trouton and Russ† to represent the results of their experiments on the discharge of condensers They did not determine the absolute values of the constants in their formulæ

The apparatus used in these experiments was purchased with a grant from the Government Grant Committee of the Royal Society, to whom, therefore, I wish to express my indebtedness A great many preliminary experiments with different forms of apparatus were tried, and many long series of observations were taken and rejected before the results described above were obtained My thanks are due to my assistant, Mr H G Martyn, B Sc, for the great pains which he took with these long and tedious experiments

* *Loc. cit.* † *Loc cit*

The Measurement of Dielectric Constants by the Oscillations of Ellipsoids and Cylinders in a Field of Force.

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(Communicated by Sir Oliver Lodge, F R.S Received February 22,—Read March 11, 1909)

(Received in revised form April 28, 1909)

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1 This paper deals with the measurement of dielectric constants by a method which, though simple, does not appear to have been previously employed in the same form Ellipsoids and cylinders accurately cut from crystals and other insulating material are suspended between flat plates connected to an alternating transformer supplying a sinuous electrification of 1500 to 2000 volts amplitude, and the period of small swings of the suspended rod or ellipsoid, with and without the field, is observed From these measurements the dielectric constant can be found

The method is capable of a high order of accuracy, and the results obtained agree closely with the best of previous determinations, especially

those made by Hopkinson for different varieties of glass It also lends itself to work with liquids, and some examination can be made of the polarisation of dielectrics in a unidirectional field.

The several methods which have been used in the measurement of the dielectric constants of insulators are fully described by Gray* and by Winkelmann† A complete bibliography of the subject is given by the latter and by Landolt and Bornstein‡ In most of those methods recorded, observations have been made of the change in the electrified state of charged plates when the dielectric substance is introduced between them Boltzmann determined the constants of solids by measuring the translational force on a small sphere suspended in a field of force of known intensity, and Quincke those of liquids by the tension and pressure caused in them by electrification In the foregoing the dielectric was in every case at rest, and with the exception of Boltzmann's experiments, the effects observed have been external to the cell containing the dielectric The method now to be described differs from these in that the dielectric is ellipsoidal or cylindrical in form, and makes small oscillations about a position of symmetry in line with the field

The apparatus is really a condenser, with a portion of its dielectric free to move The electrostatic field was maintained in sine wave alternation at a frequency of about 80 a second, the oscillations of the rods never exceeded one a second By the use of an alternating field, the influence of "elastic afterworking" and of conduction currents are avoided, and the conditions are more nearly those for which the constants are usually required

After some preliminary experiments, a short reference to the method was made in an inaugural address§ It was found later that Beaulard was at the time of the early experiments investigating the dielectric constant of water by enclosing it in a hollow glass ellipsoid suspended in a field maintained by the usual spark device in a state of oscillation at two million alternations a second His value is very interesting, and is probably of the right order, though the experiment did not aim at a high degree of accuracy||

The differences between the recorded values of the constant K are not on the whole to be explained by differences of composition In view of the importance of its accurate determination under alternating fields of low frequency or infinite wave-length, the following measurements were

* 'Absolute Measurements in Electricity and Magnetism,' vol 1, chap VII, § 2

† 'Handbuch der Physik' (2te Auflage), vol. 4, (1), pp. 92—150

‡ Landolt and Börnstein, 'Phys. Tabellen,' Tab 240.

§ 'Journal Inst. Elec. Engrs.,' 1905, Part 176, vol. 36, p. 186

|| 'Science Abstracts,' No 2097, November, 1905, 'Revue Electrique,' August 30, 1905

made, in which the solid ellipsoids and cylinders used were worked into shape with all possible care Great pains were taken to obtain a steady voltage, and to constantly compare it with a standard unidirectional voltage

The points in the paper which perhaps call for special note are the small mass of substance used, in the solids not exceeding a third of a gramme and in liquids a tenth of a gramme, the close agreement between theory and experiment in § 5, and the practical method adopted of dealing with cylindrical specimens.

2 The restoring couple upon a dielectric ellipsoid with its axis inclined to an electrostatic field of force is a quadratic function of its dielectric constant * When the ellipsoid is freely suspended by an elastic fibre, the couple may be measured by the rotation of a torsion head required to bring the body to any fixed angle with the direction of the field, or by the period of small oscillations about the position of rest. The motion in the latter case is that of a "quadrantal pendulum," the restoring moment of which reaches a maximum at an angular displacement of 45 degrees The general theory of this motion has been given by Thomson and Tait,† and a solution for large amplitudes by Greenhill,‡ but in the above case there is, in addition to the polarisation couple, that of the elastic control

From measurements of the couple, the dielectric constant can be found in terms of the strength of the field, the dimensions of the body, the constant of the surrounding medium, and the time of oscillation or angle of twist of the suspension fibre

When the medium is air or vacuum, the method is absolute in the sense that the constant is determined from one set of measurements

The equation of motion of a quadrantal pendulum under torsional control and with negligible damping is

$$I\ddot{\theta} + a\theta + b \sin\theta \cos\theta = 0, \tag{1}$$

where I is the moment of inertia about the axis of suspension, a the restoring moment of the suspension per unit twist, and $b \sin\theta \cos\theta$ the polarisation couple, θ being the angle made by the longer axis with the undisturbed field.

In the case of an isotropic dielectric whose constant is K in a medium of constant K_0,

$$b = F^2V \frac{(K-K_0)^2(L-N)}{\{4\pi + (K-K_0)L\}\{4\pi + (K-K_0)N\}}, \tag{2}$$

where V is the volume of the body, F the intensity of the field in electro-

* Maxwell, 'Electricity and Magnetism,' vol. 2, §§ 437 *et seq.*

† 'Natural Philosophy,' vol. 1, § 322.

‡ 'Applications of the Elliptic Functions,' A. G. Greenhill, p. 103.

static units, L and N coefficients depending only upon the shape of the body.

In the case of a prolate ellipsoid of eccentricity e,*

$$L = 2\pi\left(\frac{1}{e^2}-\frac{1-e^2}{e^3}\log\frac{1+e}{1-e}\right), \text{ the transverse coefficient,}$$

$$N = 4\pi\left(\frac{1}{e^2}-1\right)\left(\frac{1}{2e}\log\frac{1+e}{1-e}-1\right), \text{ the longitudinal coefficient.} \quad (3)$$

When the oscillations are so small that the square of the amplitude may be neglected,

$$I\theta+(a+b)\theta = 0 \quad (4)$$

The period is therefore

$$T = 2\pi\sqrt{\left(\frac{I}{a+b}\right)}, \quad (5)$$

and $$(a+b) = 4\pi^2n^2I$$

The free period $T_0 = 2\pi\sqrt{(I/a)}$, so that

$$a = 4\pi^2n_0^2I \quad \text{and} \quad b = 4\pi^2(n^2-n_0^2)I \quad (6)$$

Hence from equation (2)

$$\frac{(K-K_0)^2F^2V(L-N)}{\{4\pi+L(K-K_0)\}\{4\pi+N(K-K_0)\}} = 4\pi^2(n^2-n_0^2)I, \quad (7)$$

which gives

$$(K-K_0)^2\left\{\frac{F^2}{n^2-n_0^2}\frac{V}{I}\frac{(L-N)}{4\pi^2}-LN\right\}-(K-K_0)4\pi(L+N)-16\pi^2 = 0, \quad (8)$$

the equation by which K is determined when K_0 is known In this $F^2/(n^2-n_0^2)$ is the term derived from the movement of the ellipsoid, the other terms are found from its dimensions and mass

When the amplitude is so large that the period is sensibly increased, this must be reduced to the case of infinitely small swings.

Writing $$\theta+\frac{a}{I}\theta+\frac{b}{2I}\sin 2\theta = 0, \quad (9)$$

and with an amplitude α, we have

$$t = \sqrt{I}\int\frac{d\theta}{\sqrt{\{(a\alpha^2-\frac{1}{2}b\cos 2\alpha)-a\theta^2+\frac{1}{2}b\cos 2\theta\}}}$$

Expanding $\cos 2\theta$, and rejecting terms from θ^6 upwards, we have for the period

$$T = 4\sqrt{\left(\frac{3I}{b}\right)}\int_0^\alpha\frac{d\theta}{\sqrt{(\gamma-\beta\theta^2+\theta^4)}}, \quad (10)$$

where $$\beta = 3\left(\frac{a}{b}+1\right), \qquad \gamma = 3\left(\frac{a}{b}\alpha^2+\sin^2\alpha\right) \quad (11)$$

* *Vide* Maxwell, *loc. cit*

Writing $\frac{1}{p^2}=\frac{\beta}{2}+\sqrt{\left(\frac{\beta^2}{4}-\gamma\right)}$, $\frac{1}{q^2}=\frac{\beta}{2}-\sqrt{\left(\frac{\beta^2}{4}-\gamma\right)}$.

$$T=4p\sqrt{\left(\frac{3I}{b}\right)}\int_0^{\pi/2}\frac{d\phi}{\sqrt{(1-k^2\sin^2\phi)}}, \tag{12}$$

where $\sin\phi=p\theta$, and $k=p/q$

For indefinitely small amplitudes γ, and therefore k, are zero, and in this case

$$p=\frac{1}{\sqrt{\beta}},\quad T=4\sqrt{\left(\frac{3I}{b\beta}\right)}\cdot\frac{\pi}{2} \tag{13}$$

For finite amplitudes

$$T_1=\frac{4}{\sqrt{\{\frac{1}{2}\beta+\sqrt{(\frac{1}{4}\beta^2-\gamma)\}}}}\sqrt{\left(\frac{3I}{b}\right)}\ F(\eta,\tfrac{1}{2}\pi), \tag{14}$$

in which $\sin\eta=k$, and the square of the frequency of oscillation reduced to indefinitely small amplitudes is therefore

$$n_1^2=n^2\frac{1}{\frac{1}{2}+\sqrt{(\frac{1}{4}-\gamma\beta^{-2})}}\left\{1+\tfrac{1}{4}\frac{\frac{1}{2}-\sqrt{(\frac{1}{4}-\gamma\beta^{-2})}}{\frac{1}{2}+\sqrt{(\frac{1}{4}-\gamma\beta^{-2})}}\right\}^2, \tag{15}$$

which for most practical purposes may be written

$$n_1^2=n^2(1+\tfrac{3}{4}\gamma\beta^{-2}) \tag{16}$$

The value of β upon which k depends, itself requires a knowledge of b If it is impossible to work with small amplitudes this is a real difficulty, but if, as in the following experiments, the swing rarely exceeded 5 degrees, the reduction to zero amplitude may be omitted in a first approximation, and b calculated from equation (6) This value may then be used to find β, and the reduction made as above If a still closer approximation is required the process may be repeated

The logarithmic decrement of the motion was in most cases 0·06 The correction of the square of the frequency for this increases it in the ratio $(1+\lambda^2/\pi^2)$, that is 1 00036

The experiment then consists in observing n, n_0, and F, correcting n for amplitude and damping, and inserting the value of $F^2/(n^2-n_0^2)$ in equation (8), to calculate the value of K

Arrangement of Experiment

3. The ellipsoids were suspended by a quartz fibre, about 0·001 cm. diameter and 30 cm long, between square brass plates b, b held apart by glass distance pieces g, g at their corners (see fig 1) and enclosed in a glass vessel having a thick glass top carrying a suspension tube, torsion head, and circular scale. To reduce damping and to accelerate the drying of the specimens the vessel was in most of the determinations exhausted to

a vacuum of 700 mm and a dish of phosphoric anhydride placed at the bottom The size of the plates and their distance apart were chosen after trial with the stream line method* to determine the best proportions of the field under the distorting influence of the ellipsoids With dielectric ellipsoids 2 cm long a convenient distance is 6 cm. apart, with plates 10 cm square. At this distance the feeble induced charges on the suspended body were, as shown by the results obtained for flint glass and quartz, insufficient

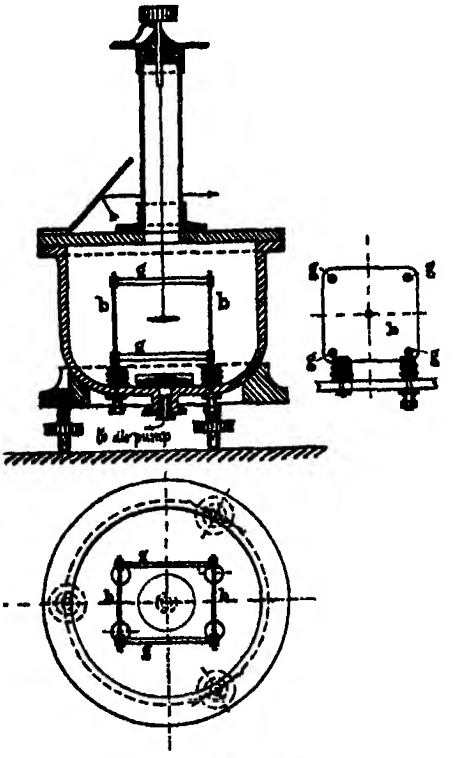

Fig 1 —Testing Cell

to change the distribution of electricity on the plates by any appreciable amount. At 4 cm apart the period is increased by a few hundredths of a second. When, however, a metallic ellipsoid of the same size is placed in the field it is necessary to space the plates 10 cm apart to reduce its reaction on them to less than a measurable amount The motion of the specimen was observed by reflexion from a mirror into a telescope having an eye-piece with five cross wires radiating from the centre at angles of

* Hele Shaw and Hay, 'Phil. Trans.,' vol 195, ser A, pp 307—327

10 degrees apart. They were set with regard to the field by focussing the central one upon the ellipsoid brought to rest in a strong field of force.

The potential difference between the plates could be raised to 10,000 volts obtained from a small oil-cooled transformer In order to get very steady voltages the steam end of a 24-kilowatt Parsons turbogenerator was disconnected from the generator so that the latter could be run independently as a motor Slip rings were fitted over part of the commutator, and the machine driven from large storage cells as an inverted converter (fig 2).

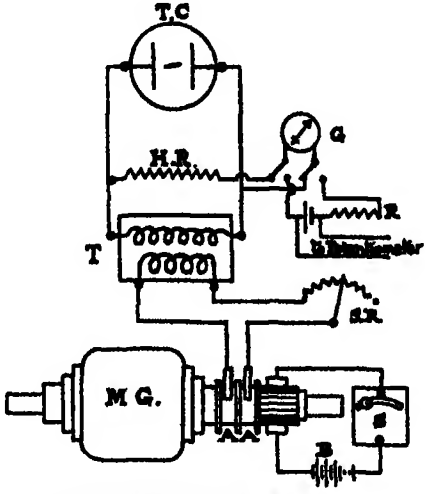

Fig 2.—Arrangement of Apparatus.

B, storage battery.
S, motor starter
M G, motor generator
A A, alternating current slip rings
S R., switch of current regulator
T, step-up transformer
H R, 99,000 ω resistance
G, hot wire galvanometer
R., standard resistance, 100 ω.
T C, testing chamber.

All slight flickering of the alternating voltage which had previously given trouble was in this way avoided, the maximum variation of pressure during several minutes—the time of observation of the period of the ellipsoids—being, as a rule, one part in 800.

Difficulty was for some time experienced in obtaining readings of the voltage consistent to 0 1 per cent Several forms of voltmeter were tried without success. The means eventually adopted was to connect a high resistance across the terminals in series with a Duddell twisted strip ammeter which was enclosed in a metal case surrounded by flannel. It was found necessary, on account of the influence of slight changes of temperature upon the readings, to calibrate with direct current before and after each set

of observations with alternating current. When this was done very consistent readings were obtained

Working by the method of oscillations the period was increased by air resistance to the extent of about 4 parts in 1000 The damping in every case was so small that a correction could be made by observation of the logarithmic decrement as indicated in § 2 Taking this into account no measurable effect was found from the change of the ionised state of the surrounding air when the pressure was reduced to 755 mm of mercury, below which it was found not advisable to go on account of the occurrence of luminous discharge between the plates

In working, a torsional swing was given to the ellipsoid, from the period of which and the known moment of inertia the elastic constants of the suspension were found The voltage was then applied and the new period found The ratio of the square of the intensity of the field to the difference of the squares of the frequency of oscillation with and without the field was the object of the experiment.

The glass and quartz ellipsoids and cylinders were suspended by forming a small drop of fused canada balsam on the quartz fibre, slightly warming the surface of the solid and quickly melting the drop into contact with it by the approach of a hot rod If the ellipsoid were then out of balance, as tested on a levelled mirror, the attachment was broken and the process repeated, usually a dozen or more times before a perfect suspension was obtained The other substances were hung in a cocoon silk saddle

4 The substances selected for trial of the method were optical flint glass of densities 4 67, 4·1, and 3 3, and quartz cut parallel and perpendicular to the optical axis The former were chosen because their constants have been very fully determined, especially by Hopkinson,* and the latter on account of the calculations of their dielectric constants which have been made, from very accurate optical observations, upon the electromagnetic theory of dispersion Ellipsoids of revolution of each of these substances were made by Messrs Hilger, and, tested by optical enlargement, were found to agree so closely with the true elliptic section that no difference could be measured between a photographic enlargement, by projection, of the ellipsoids, and a calculated ellipse, having the same major axis, 10 inches.

The ellipsoids were carefully measured, their volumes and densities determined, and their moments of inertia compared with a standard cylinder of heavy glass also made by Hilger, which together with a shorter cylinder was used in the determination of the influence of the ends of cylinders upon the polarisation torque The measurements are given in Table I. It can be

* 'Original Papers,' vol 2.

shown* that, on account of the low inductive constants of dielectrics, the influence of slight deviations from the true form does not rise in magnitude in the result

Table I.—Measurements of Glass and Quartz Ellipsoids

Substance	Mass	Volume	Density	Length	Diameter	L	N	I
	gramme	c c		cm	cm			gm cm^2
Quartz ∥	0 1420	0 0534	2 659	2 013	0 2285	6 1336	0 3013	0 ·02918
,, ⊥	0 1491	0 ·0560	2 663	2 014	0 2311	6 1255	0 3132	0 ·0308
Flint glass	0 2527	0 0543	4 654	2 ·022	0 2243	6 1276	0 3055	0 ·0519
,,	0 2363	0 0573	4 12	2 ·006	0 2352	6 117	0 3292	0 ·04747
,,	0 2091	0 ·0633	3 34	2 ·015	0 2452	5 848	0 3432	0 ·04360

Quartz

5 When a uniaxal crystalline ellipsoid is suspended in a field of force, the restoring couple upon it depends upon the inclinations of the optic axis both to the direction of the field and to the axis of suspension

The general expression for the couple upon an æolotropic ellipsoid is, with the previous notation,†

$$F^2V\left[\frac{4\pi(K_1-K_2)+(K_1-K_0)(K_2-K_0)(L-N)}{\{4\pi+(K_1-K_0)L\}\{4\pi+(K_2-K_0)N\}}\right]\sin\theta\cos\theta \qquad (17)$$

For an ellipsoid cut perpendicular to the optic axis this will be greatest when the optic axis is parallel to the suspension In this case K_1 and K_2 are each equal to K⊥, the coefficient of dielectric polarisation at right angles to the optic axis When the ellipsoid is cut parallel to the axis, the coefficient of longitudinal polarisation is K∥, the transverse coefficient K⊥ We proceed therefore to find first the dielectric constant for the ellipsoid cut perpendicular to the optic axis

Quartz, perpendicular to the optic axis—

Ellipsoid.—Length 2 0145 cm, diameter 0 2311 cm., mass 0 149 gramme, volume 0 0560 c c, density 2·66 L = 6 1255, N = 0 3132, I = 0·0308 gm.-cm.^{2}

A series of observations was made with the optic axis inclined to and parallel to the axis of suspension, giving the following values, each of which is the mean of from 20 to 30 observations θ is the angle between the two above axes in degrees

* *Vide* Chrystal, 'Encyc Britannica,' ed 9, Art "Magnetism," p. 245

† 'Encyc Britannica,' *loc cit*, *eq* (82).

θ	$\frac{F^2}{n^2-n_0^2}$
0	139 25
−45	148 20
+45	141 90
90	141 10

A curve of these is drawn in fig 3, the dip in which points to a marked change in the apparent electrical behaviour of the crystal when the polarisation is everywhere exactly at right angles to the optic axis, *i e*, the condition under which the ordinary ray is transmitted The couple on the ellipsoid is a maximum at this point, but the approach to it is so steep that a very slight angular displacement from the axis of suspension causes a marked difference in the period. A calculation of the variation of the couple on the ellipsoid with the angle θ shows a dip of this nature but not quite so steep as in fig 3

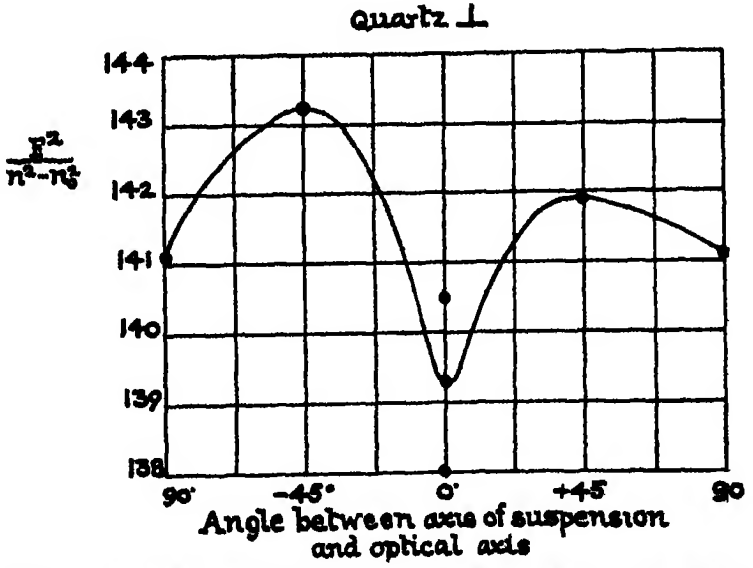

Fig 3.—Change of horizontal couple with vertical inclination of optical axis.

The value 139 25 is the mean of two series taken with different suspensions, the greatest care being taken to have the axes in line The average value of the first series was 138 and of the other 140 5 It was impossible to obtain perfect alignment, a difference of 0 01 mm on the circumference corresponding to an angular displacement of 4 degrees

The determining equation (8) for this ellipsoid is

$$(139\,25\times 0\,2676-1\,9185)(K-K_0)^2-80{\cdot}91\,(K-K_0)-157\,9=0,$$

from which $K-K_0=3\,5490.$

Taking K_0 for a vacuum of 700 mm $=0{\cdot}9995$,

$$\mathbf{K1=4{\cdot}5485.}$$

The following observations were made on the ellipsoid of quartz cut parallel to the optic axis over an interval of six months with the same suspension in each case Each row is the mean of ten readings, but the pressure was not at this time very steady —

Table II

D	i	P D	F^2	T	n^2	$\frac{F^2}{n^2-n_0^2}$
38 31	0 ·0201	1990	1 24	9 495	0 ·01109	135 ·06
30 ·65	0 ·01787	1769	0 ·9801	10 425	0 009201	134 44
38 ·07	0 ·01983	1963	1 2069	9 57	0 ·01092	133 84
38 87	0 ·01872	1853	1 ·0750	10 ·03	0 00994	133 86
40 20	0 ·02045	2023	1 ·2816	9 35	0 01144	134 48
31 ·30	0 ·01801	1783	0 ·9955	10 36	0 ·00931	134 52
41 ·76	0 ·02087	2066	1 3372	9 187	0 ·01185	134 52
37 ·98	0 ·01988	1969	1 2129	9 553	0 ·01096	134 ·02
					Mean	134 34

Here D is the observed hot-wire galvanometer deflection, i the corresponding current, P.D the potential difference across the plates, F the electrostatic intensity of the field calculated from the voltage, assuming "v" $=3\times10^{10}$, n the frequency of swing

In order to reduce n^2 to the frequency for zero amplitude, the reading nearest to the mean is taken, that is the second row in the table, we have then from equation (6)—

$$a=4\pi^2\times 0\,02912\times 0\,00191=0{\cdot}00219,$$

$$b=4\pi^2\times 0\,02912\,(0\,009201-0\,001910)=0\,0083605,$$

and from (11),

$$\beta=3\left(\tfrac{0\,00219}{0\,0083605}+1\right)=3\,7857,$$

$$\gamma=3\,(0{\cdot}2619\times 0\,004873+0\,0048658)=0{\cdot}01846$$

Thus $\gamma/\beta^2=0\,018426/14\,3314=0\,001285,$

and by (16),

$$n^2=0\,009201\,\{0\,5+\sqrt{(0\,25-0\,001285)}\}^{-1}\left(1+\tfrac{1}{4}\frac{0\,5-\sqrt{(0\,25-0{\cdot}001285)}}{0\,5+\sqrt{(0\,25-0{\cdot}001285)}}\right)^2,$$

that is $n^2 = 0{\cdot}009215$ This is increased to allow for damping in the ratio 1·00036, giving finally $n^2 = 0\,009218$ The corresponding value of $F^2/(n^2-n_0^2)$ is 134 11 The influence of the correction has been to reduce the above quantity in the ratio 134 11/134 44, that is 0·9974 Applying, now, this reduction factor to the mean value 134 34, we obtain finally

$$F^2/(n^2-n_0^2) = 134\,0$$

Measurements of the ellipsoid gave $I = 0\,02913$, $V = 0\,0534$ cc, $L = 6{\cdot}1336$, $N = 0{\cdot}3013$ Thus, from equation (8),

$$(134\,0 \times 0\,2716 - 1\,848)(K-K_0)^2 - 80\,86\,(K-K_0) - 157\,9 = 0,$$

from which $(K_1-K_0) = 3\,6073$, giving finally, with $K_0 = 0\,9995$ and $K\perp = 4\,5485$,

$$\mathbf{K\parallel = 4{\cdot}600.}$$

It is interesting to compare these results with those obtained from optical data by the Ketteler and Sellmeier dispersion formula—

$$K_\infty = \mu_\lambda^2 - \frac{M_1}{\lambda-\lambda_1^2} + \frac{M_2}{\lambda_2^2-\lambda^2} + \frac{M_3}{\lambda_3^2-\lambda^2},$$

where K_∞ is the dielectric constant for exceedingly long waves, μ the index of refraction at wave-length λ, M_1, M_2, M_3, constants derived at the wave-lengths λ_1, λ_2, λ_3, at which selective absorption occurs These constants have been found for quartz by Rubens and Nichols,* using a bolometer as a detector of waves coming from an Auer burner, reflected several times from the surfaces of quartz blocks, and separated by a grating From their results the following values were obtained—

$$\lambda_1 = 0\,1031, \qquad \lambda_2 = 8\,85, \qquad \lambda_3 = 20\,75$$
$$M_1 = 0\,01065, \qquad M_2 = 44\,224, \qquad M_3 = 713\,55,$$

the wave-lengths being in 10^{-4} cm

If μ is taken for the D line, and for the ordinary ray the formula gives K (ord.) = 4 578

The above values of the constants M_1, M_2, M_3 are usually quoted as being appropriate to the ordinary ray The constants for a similar dispersion formula for the extraordinary ray have not been calculated, but if μ is taken for the D line and for the extraordinary ray we get as an estimate K (ext) = 4 6065.

The above formula has recently been slightly modified by Maclaurin,† who has worked out the constants for rocksalt and sylvin but not for quartz It

* 'Wied. Ann.,' vol 60, p 418, 1897

† 'Roy Soc Proc.,' A, vol. 81, p. 367, 1908

is probable that the value 4·6065 would be reduced in conformity with his treatment by as much possibly as 0·2 per cent.

In comparing results obtained from optical data with those derived from direct electrical experiment, the electric polarisation must be in the same direction in both cases. Since the electrical polarisation in a light wave is transverse, values of K calculated by the above formula, using the index of refraction for the ordinary ray, obtained by the transmission of light along the optic axis, correspond to those found directly by exposing the quartz to an electric field at right angles to the axis. Thus K(ord.) as calculated should agree with K⊥, and K(ext.) with K∥. We have

Observed	Calculated
K⊥ 4·548	K ord. 4·578
K∥ 4·600	K ext. 4·606

It may be remarked that the mean of the observed values of K⊥ and K∥, which is 4·572, agrees better with the value of the corresponding constant in the dispersion formula than K⊥.

There are two further points bearing upon the agreement of the observed and calculated values which must be considered. The expression "at right angles to the axis" does not specify the precise direction within the crystal in which the cutting was made. The value K⊥ is therefore subject to a possible variation of a small fraction of 1 per cent. from the mean. Again, the composition of quartz is known to vary, but fine clear crystals are pure silica. Both Nichols and Rubens, and also Hilger, remark that the crystals used by them were very fine, and the agreement indicates that the molecular state was very nearly the same in both cases. There is no recorded investigation of the influence of change of composition on the dielectric constant of quartz, but judging by the small difference between the constants K∥ and K⊥, it would be small.

At the end of the paper, values of K obtained by Fellinger, Ferry, Curie, and Romich and Nowak are given, the mean of which for K∥ is 4·66 and for K⊥ 4·54.

Fused Quartz.

The density of fused quartz is about 0·86 that of crystalline. In order to see how the dielectric constant depends upon density as distinct from structure, an ellipsoid was made of quartz, carefully fused and perfectly clear. It

had the following dimensions. diameter, 0 2291 cm , length, 2 003 cm., mass, 0 1265 gramme, volume, 0 055 c c , L = 6 125, N = 0 311, density, 2 30

The mean value for $F^2/(n^2-n_0^2)$ was 162 25, from which

$$(162\,25 \times 0\,316 - 1{\cdot}904)(K-K_0)^2 - 80\,82(K-K_0) - 157\,9 = 0,$$

giving $$K-K_0 = 2\,78 \quad \text{and} \quad \mathbf{K = 3\,78}$$

The ratio of this to the mean of K|| and K⊥ is 0·826 The change in the dielectric constant is from this about 4 per cent less than the corresponding change in density in keeping with the fact that the influence of the crystalline structure in quartz upon the dielectric constant is comparatively small

Flint Glass

6. The dielectric constant of flint glass has been very fully investigated by Hopkinson,* using a guard-ring condenser

It is common in modern optical work to use a heavy flint glass of density about 4 67 Its departure from Maxwell's law $K = \mu^2$ is great, but its large dielectric constant renders it a suitable material for a standard dielectric ellipsoid to be used in alternating fields of low frequency

In order, however, to make a comparison over the whole range of Hopkinson's research three ellipsoids were made of densities 4 65, 4 1, and 3 3, the measurements of which are given in Table I

For the first, diameter 0 2243 cm , length, 2 022 cm , mass, 0 2527 , volume, 0 0543 , L = 6 1276, N = 0 3055, I = 0 0519, the mean of many observations of $F^2/(n^2-n_0^2)$ corrected for amplitude and damping gave the value 83 57

We have then

$$(83\,57 \times 0\,1543 - 1\,872)(K-K_0)^2 - 80\,84(K-K_0) - 157\,9 = 0,$$

from which $$K-K_0 = 9\,645$$

In this experiment the vessel was not exhausted, so that

$$K_0 = 1, \quad \text{and} \quad \mathbf{K = 10\,645} \quad (\Delta = 4\,65)$$

In order to fix the position of the curve between the densities 3 66 and 4 5, the ellipsoid of density 4 12 was made, the measurements of which gave—

Volume = 0 05736 c c, I = 0 04747, L = 6 117, N = 0 3292

The mean value of $F^2/(n^2-n_0^2)$, corrected as before, was 87·6 at a frequency of 87 5 Thus

$$(87\,6 \times 0\,1776 - 2\,013)(K-K_0)^2 - 81\,0(K-K_0) - 157{\cdot}9 = 0,$$

giving $$\mathbf{K = 8{\cdot}52} \quad (\Delta = 4{\cdot}12).$$

* 'Phil. Trans.,' Part I, 1878, pp. 17—23 , Part II, 1881, pp 355—373.

The ellipsoid of light flint glass, of density 3 34, melting point 674° C, had

Volume = 0 0633 c c, I = 0 04360, L = 5 848, N = 0 3432

The mean value of $F^2/(n^2-n_0^2)$ was 94 65 at a frequency of 86

Thus for this glass

$$(94\,65 \times 0\,2054 - 2\,007)(K-K_0)^2 - 77\,8\,(K-K_0) - 157\,9 = 0,$$

from which $\mathbf{K = 6{\cdot}98}$ ($\Delta = 3\,30$)

Comparing these values with Hopkinson's, there is seen to be very close agreement in the dense glasses, but not with the light glass, the difference in that case being 2 3 per cent In working with light flint glass the composition and melting point must be known in order to make a fair comparison, because of the possibility of obtaining the same density by a variation of the proportions The composition of the heavier glasses is more closely fixed by the proportion of lead required to give the greater density

The symmetry of the curve of fig 4 shows that the variation of density

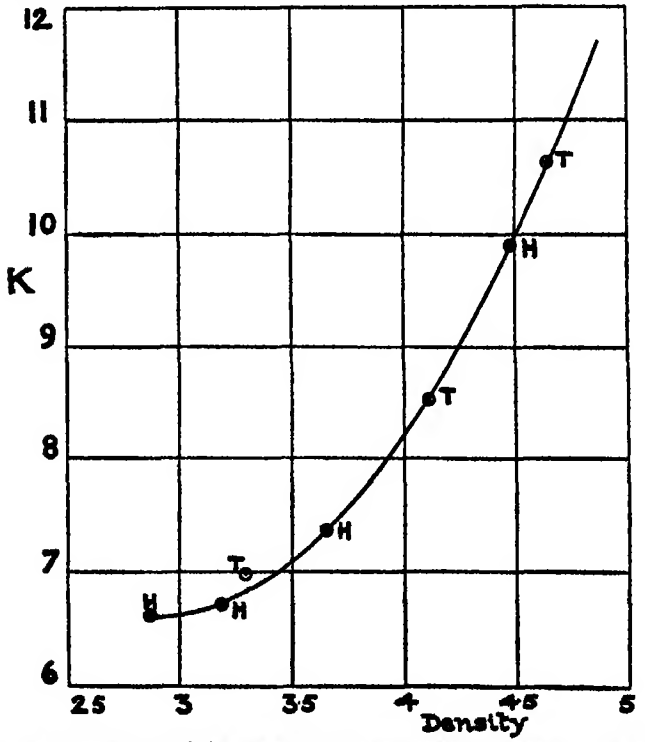

Fig 4.—Variation of the dielectric constant of flint glass with density

was obtained by a systematic change of the proportion of lead,* and while the curve is valuable in itself, especially near the origin, there must always

* *Vide* Hopkinson, *loc. cit*

be a possible variation of several per cent. in the value of the dielectric constant of light flint glass from different makers at any given density

7 The curve in fig 4 is parabolic and agrees very well with

$$K - K_1 = b(d - d_1)^2, \tag{18}$$

where d is the density, K the dielectric constant, and $K_1 = 6\,61$, $d_1 = 2{\cdot}89$, $b = 1\,2965$

The density, 2 89, is that of nearly the lightest flint glass made From this to the value 4 65 the change of the dielectric constant is proportional to the square of the change of density

The relation between K and μ which is shown by fig. 5 is also a quadratic function expressed by

$$K - K_1 = c(\mu_F - \mu_1)^2 \tag{19}$$

in which $K_1 = 6\,61$, $\mu_1 = 1\,549$, $c = 95\,6$

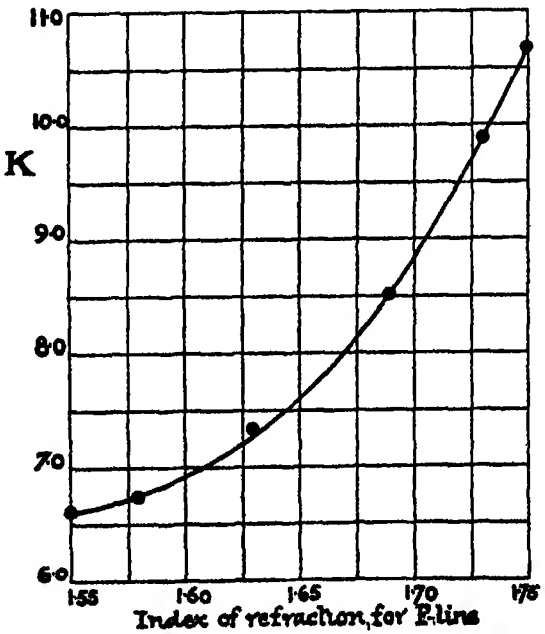

Fig 5.—Dielectric constant and index of refraction Flint glass

The following table gives corresponding values of d, K, and μ, and the derived values of c —

Table III

d	K	μ_F	c
2 8(7)	6 61	1 549	—
3 2	6 72	1 5838	95
3 66	7 3(7)	1 6347	95
4 12	8 52	(1 690)	96
4 50	9 89	1 735	95
4 65	10 645	(1 758)	96 9

From (18) and (19),

$$\mu_F - \mu_1 = g(d - d_1), \tag{20}$$

in which $\mu_1 = 1\,549$, $d_1 = 2\,89$, $g = 0\,1155$

The value 2 89 has been taken rather than 2 87, for the zero of the curve of density is nearer the former value, and if Hopkinson's experimental determinations of a in the empirical relation*

$$\mu - 1 = a + bf(\lambda)$$

are drawn with respect to the density, the density corresponding to $a = 0\,549$ upon a straight mean line is 2 89

The value 0 1155 of the coefficient g was calculated from Hopkinson's experiments, being the slope of the a, d line, but from 18, 19, and 20, $g = \sqrt{(b/c)}$

If for c the last figure in the foregoing table is taken,

$$g = \sqrt{\tfrac{1\,203}{96\,9}} = 0\,1156$$

Moulded Ellipsoids

8 Many substances for which the dielectric constant is required in practice cannot be conveniently worked when hard, but may be moulded Paraffin wax, shellac, and gutta-percha are examples To deal with these a mould was made by preparing two blocks of lead $1 \times 2 \times 4$ cm Between these a steel ellipsoid, 3 cm long, was placed, and the blocks crushed together in a powerful press A semi-ellipsoidal cavity was in this way formed in each. Castings could not be made by pouring into the mould on account of the smallness of the cavity and the difficulty in removing air bubbles The substances moulded were all of a kind which becomes plastic on heating They were therefore warmed until, when placed between the plates and pressed, they took the form of the mould, any excess being squeezed out between the blocks. These were then separated and the excess

* 'Roy Soc Proc.,' No. 182, pp 1—8, 1877

removed, the blocks warmed, and the process repeated until no excess was found On being removed they were then very fair ellipsoids, but in every case required trimming by scraping away of some excess along the joint Substances which, like sulphur or resin, have a relatively high melting point, solidify quickly, and adhere to the mould even when smeared with oil or glycerine, could be more conveniently prepared by forming into cylinders, as described in §§ 15 and 16

In every case great care had to be taken to thoroughly dry the specimen and the containing vessel The latter was kept warm, and when it was necessary to admit air in order to remove the specimen after a test, this was done through tubes of phosphorus pentoxide When moisture was suspected the specimens were always allowed to stand for several hours in the alternating field and *in vacuo*, in order by the internal heating caused by dielectric hysteresis to help to dry the surface

Paraffin Wax

9 Moulded ellipsoid of pure clear wax of density 0 914—Melting point 55° 8 C, diameter 0 2895 cm, length 2 93 cm, volume 0 141 c c, moment of inertia by comparison with standard glass cylinder 0 0607 gm cm^2, mass 0 1290 gramme, eccentricity 0 995, L = 5 98, N = 0 2715, temperature 17° C, vacuum 700, frequency 86 The free period was 52 5 seconds Period 19 1 seconds in a field F = 1 025 units. $F^2/(n^2-n_0^2) = 442\ 5$

The determining equation is

$$\left(\frac{442\ 5\times 0\ 141\times 5\ 708}{39\ 4\times 0{\cdot}0607}-1\ 623\right)(K-K_0)^2-12\ 566\times 6\ 251\,(K-K_0)-157\ 9=0,$$

and $$\mathbf{K = 2{\cdot}32_6}$$

For freshly-moulded wax the highest value found was 2 4. Increasing the frequency from 60 to 84 decreased K by 0 7 per cent

Bees-wax

10 Ellipsoid—Diameter 0 3194 cm, length 3 005 cm, mass 0 1521 gramme, volume 0 160 c.c, density 0 951, L = 5 9, N = 0 30, temperature 18° C Free period 73 seconds Period 10 21 seconds in a field F = 1 261 units

Here $$F^2/(n^2-n_0^2) = 169,$$

and $$\left(\frac{169\times 5{\cdot}6\times 1\ 41}{39\ 4}-1\ 773\right)(K-K_0)^2-78\,(K-K_0)-157{\cdot}9=0,$$

so that $$\mathbf{K = 4\ 75.}$$

This wax, from a lemon-yellow cake, one of the Royal Agricultural Society's prize winners, was worked in warmed moulds, and dried *in vacuo* over phosphorus pentoxide for a week

Shellac

11 Ellipsoid —Diameter 0 323 cm , length 3 1 cm , volume 0 169 c c , mass 0 196 gramme, density 1 15, moment of inertia 0 0965, L = 5 92, N = 0 3019, temperature 17° C. Free period 40 4 seconds Period 6 92 seconds in field F = 1 16 units $F^2/(n^2-n_0^2) = 66\ 22$

Thus $147\ 8\,(K-K_0)^2-78\ 05\,(K-K_0)-157\ 9 = 0,$

from which $\mathbf{K = 2{\cdot}49.}$

The shellac was dark orange flake before melting, and was carefully heated until only the most minute bubbles of air were found in it when solid Like paraffin, shellac polarises quickly, and the higher values which have been obtained by previous experimenters may have been in some cases from too long contact

Sealing Wax

12 I Ellipsoid —Diameter 0 317 cm , length 3 05 cm , volume 0 1604 c c , moment of inertia 0 1263 gm cm^2, mass 0 274 gramme, L = 5 915, N = 0 299 Free period 38 0 seconds Period 11 73 seconds in F = 1 155 units $F^2/(n^2-n_0^2) = 203\ 1$

The determining equation is

$$\left(\frac{203\ 1\times 0\ 1604\times 5{\cdot}915}{39\ 4\times 0\ 1263}-1\ 769\right)(K-K_0)^2-12\ 566\times 6\ 214\,(K-K_0)-157\ 9 = 0,$$

from which $\mathbf{K = 4{\cdot}56,}$ at a density 1 63

In making this ellipsoid, a stick of the best sealing-wax was melted once, and from the plastic mass sufficient taken to form the ellipsoid between hot moulds Another ellipsoid was formed from the same wax, but the heating was inadvertently continued for half an hour The density of the new ellipsoid was 1 68, and, measured in the same way as above, the dielectric constant had risen to 5 2 The wax contained vermilion, button lac, resin, and "mineral," about half being compounds of mercury and lead and half vegetable resins

Gutta-percha

13 This ellipsoid was made from new thin clear gutta-percha tissue (Silvertown best quality), dried, melted, and moulded In melting the tissue to a homogeneous mass, the heat was continued for about half an hour, and was such that bubbles formed in the plastic substance There is reason to believe that this heating was excessive, but the case is given as an example It is possible that something of the nature of carbonisation may have taken place

The ellipsoid was dried for several months over phosphoric anhydride, and

then exposed to an alternating field of about 300 volts per centimetre for four hours in a vacuum of 700 mm before taking a reading. These precautions were taken, on account of the high values of K obtained, to reduce the risk of the presence of moisture

The measurements were—

Diameter 0·317 cm., length 2·952 cm., volume 0·147 c.c., mass 1·35 grammes, density 0·92, L = 5·9, N = 0·30 Free period 50·6 seconds. Period in a field F = 1·046, 7·49 seconds. $F^2/(n^2-n_0^2) = 62·8$

Hence $\left(\frac{62·8\times2·46\times5·6}{39·4}-1·77\right)(K-K_0)^2-78(K-K_0)-157·9 = 0,$

giving $$\mathbf{K = 6·3} \text{ (see § 16)}$$

Chatterton Compound

14 The sample was of ordinary commercial insulating compound Its composition was unknown, though it apparently consisted largely of gutta-percha and pitch

Ellipsoid.—Diameter 0·315 cm., length 2·952 cm., volume 0·160 c.c., moment of inertia 0·083 gm. cm.^{2}, mass 0·1821 gramme, temperature 18° C., density 1·13, L = 5·91, N = 0·299 Free period 30·85 seconds Period 10·3 seconds in F = 1·187 units $F^2/(n^2-n_0^2) = 151·24$

Thus $\left(\frac{151·24\times0·160\times5·61}{39·4\times0·083}-1·769\right)(K-K_0)^2-78(K-K_0)-157·9 = 0,$

so that $$\mathbf{K = 3·98.}$$

Dielectric Cylinders

15 Many dielectrics are difficult to shape into ellipsoids even approximately true in form, which can be cast or turned to very accurate cylinders The electrical intensity within a cylinder in a field of force is, however, not uniform, and the reaction coefficients L and N have, strictly speaking, no definite meaning for cylinders. If not more than three-figure accuracy is required, as in commercial testing, it may be assumed that the polarisation couple upon cylinders can be expressed by $b \sin\theta\cos\theta$, where b is of the form given in equation (2) If the cylinder is long compared with its diameter, we may take the transverse coefficient $L = 2\pi$, the value which it has for infinitely long cylinders The value of the longitudinal coefficient N, derived from equation (7), is

$$N = \frac{\frac{(K-K_0)}{4\pi^2}\frac{F^2}{(n^2-n_0^2)}\frac{V}{I}L-\frac{16\pi^2}{(K-K_0)}-4\pi L}{\frac{(K-K_0)}{4\pi^2}\frac{F^2}{(n^2-n_0^2)}\frac{V}{I}+4\pi+(K-K_0)L},$$

and is therefore not independent of the dielectric constant By measure-

ment of the polarisation couple upon cylinders of known dielectric constant, a series of empirical values of N can be obtained, covering the range of K required in practice, for all convenient ratios m of length to diameter

The following measurements were made for the purpose of finding N in this way, and figs 6 and 7 are drawn from them

Metallic Cylinders (K infinite)

(a) Length 2 433 cm, diameter 0 1604 cm, $m = 15\,07$, $\Delta = 8\,72$, $L = 6\,283$ Free period more than 5 minutes Period in a field $F = 1\,147$, 4 44 seconds, from which $F^2/(n^2-n_0^2) = 25\,92$ and $N = 0\,149$

(b) Length 1 571 cm, diameter 0 1604 cm, $m = 9\,8$, $\Delta = 8\,72$ Free period negligibly great Period in $F = 1\,15$, 3 98 seconds. $F^2/(n^2-n_0^2) = 21\,0$ and $N = 0\,228$

(c) Length 0 96 cm, diameter 0 1604 cm, $m = 5\,98$, $\Delta = 8\,72$ Free period over a minute Period in $F = 1\,15$, 3 51 seconds $F^2/(n^2-n_0^2) = 16\,27$ and $N = 0\,376$

Dense Flint Glass

(a) Length 3 014 cm, diameter 0 220 cm, $m = 13\,68$, $L = 6\,283$, $K = 10\,75$, $\Delta = 4\,68$ Free period 90 6 seconds Period in a field $F = 1\,174$, 13 61 seconds Thus $F^2/(n^2-n_0^2) = 271\,5$, from which $N = 0\,258$.

(b) Length 2 004 cm, diameter 0 2258 cm, $m = 8\,88$, $K = 10\,65$, $\Delta = 4\,65$. Free period 17 51 seconds Period in $F = 1\,219$, 8 41 seconds $F^2/(n^2-n_0^2) = 136\,34$ and $N = 0\,422$

(c) Length 1 504 cm, diameter 0 2189 cm, $m = 6\,88$, $\Delta = 4\,68$, $K = 10\,75$. Free period (on silk suspension) several minutes. Period in $F = 1\,147$, 8 02 seconds $F^2/(n^2-n_0^2) = 84\,7$ and $N = 0\,532$

Light Flint Glass.

(a) Length 3 009 cm, diameter 0 2295 cm, $m = 13\,1$, $\Delta = 3\,23$, $K = 6\,92$ Free period several minutes Period in $F = 0\,805$, 22 78 seconds. Thus $F^2/(n^2-n_0^2) = 336$ and $N = 0\,34$

(b) Length 1 87 cm, diameter 0 2295 cm, $m = 8{\cdot}14$, $\Delta = 3\,23$, $K = 6\,92$. Free period 69 seconds Period in $F = 0\,853$, 14 26 seconds $F^2/(n^2-n_0^2) = 148$ and $N = 0\,56$.

(c) Length 1 03 cm, diameter 0 2295 cm, $m = 4\,48$, $\Delta = 3\,23$, $K = 6\,92$. Free period 27 6 seconds Period in $F = 0\,883$, 8·1 seconds $F^2/(n^2-n_0^2) = 54\,7$ and $N = 0\,88$

Paraffin Wax.

The value previously found for paraffin wax is that for annealed wax free from structure. It was found to be practically impossible to obtain cylinders free from initial strain The method adopted was to melt the wax and draw it into a short piece of barometer tube slightly warmed to prevent the wax solidifying at the circumference and shearing With a little practice

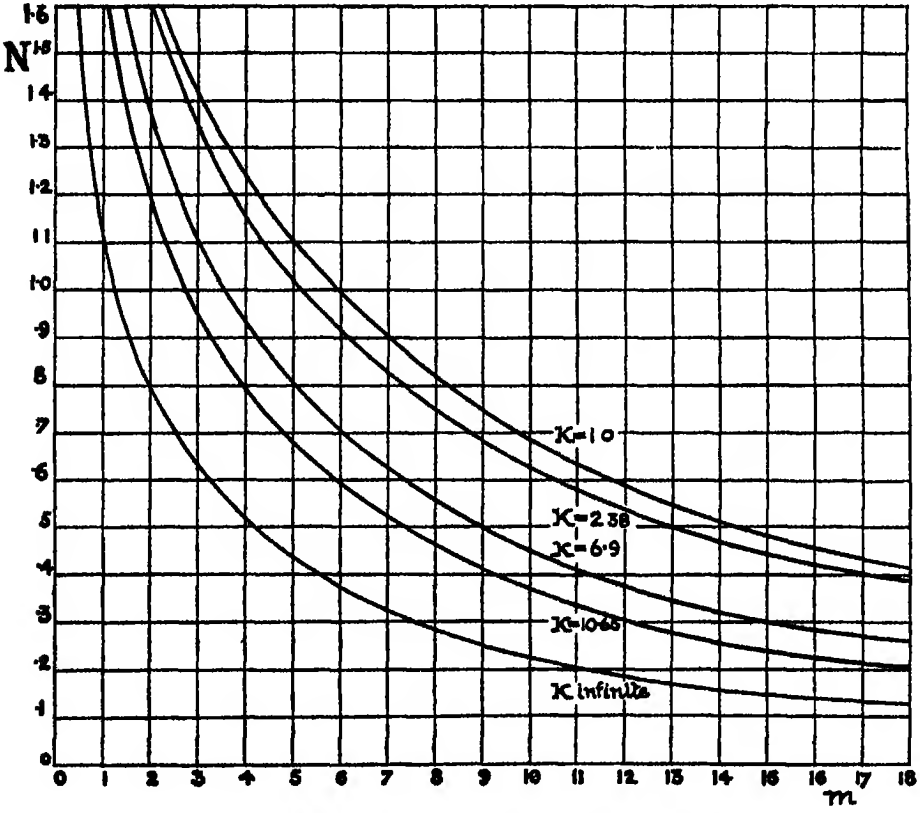

Fig 6.—Variation of reaction coefficient N with m and K.
m = length/diameter of cylinder

cylinders were made, the diameter of which did not differ at any point by 0 001 inch from the mean

(*a*) Length 3 56 cm, diameter 0 217 cm, $m = 16\,36$ Free period 50 4 seconds. Period in field $F = 1{\cdot}132$, 23·03 seconds. $F^2/(n^2-n_0^2) = 871$, which with $K = 2\,36$ gives $N = 0\,41$.

(*b*) Length 2·64 cm, diameter 0 216 cm, $m = 12{\cdot}28$ Free period 30·6 seconds Period in field $F = 1\,15$, 17 seconds $F^2/(n^2-n_0^2) = 550$ and with $K = 2\,38$, $N = 0\,55$.

(c) Length 1·24 cm, diameter 0 217 cm, $m = 5{\cdot}71$. Free period 10 seconds. Period in F = 1 232, 6·82 seconds. $F^2/(n^2 - n_0^2) = 135\,5$, giving with K = 2 38, N = 0·95

Fig 6 is a record of these experimental values of N, but in finding the appropriate constant to use in any particular case, fig 7 will be more useful. It is not necessary, however, to know any of these curves with great accuracy, for in working out the following tests the procedure was to assume any likely value for N, having regard to the dimensions of the specimen

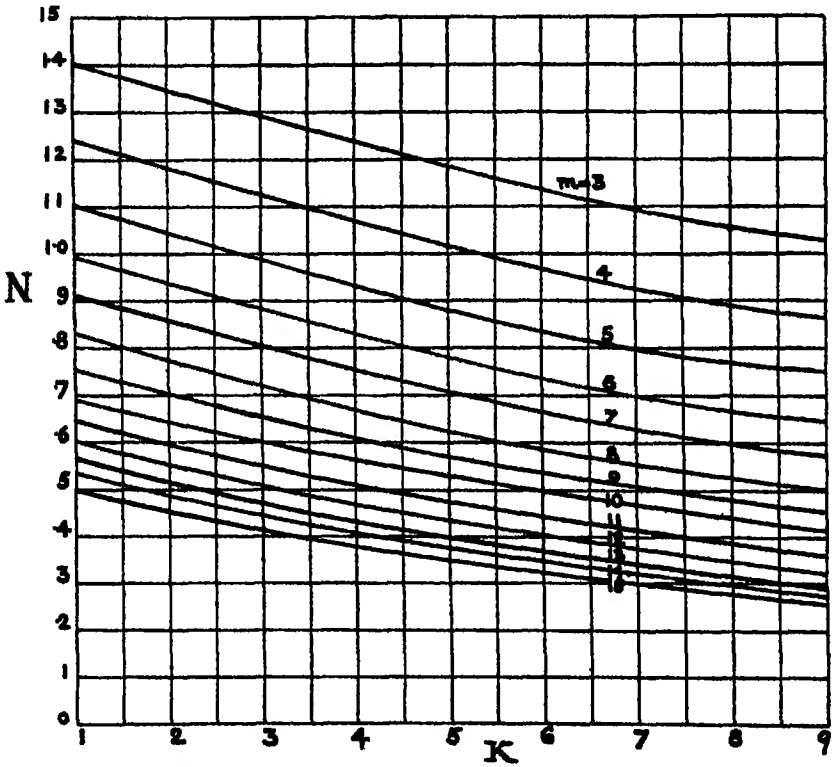

Fig 7.—Change of N with K for different ratios of length to diameter of cylinder

and the nature of the substance, and to calculate the corresponding dielectric constant, then from the curves to find N for the new value of K, and in this way by successive approximation to reach a point where the changes became less than experimental error The use of the curves facilitates the finding of K by this method of trial and error The value of N taken as determining that of K was usually found at the third approximation. When the point K, N for the given ratio m agreed with the curves of figs 6 or 7 it was taken as final

16 The cylinders were shaped by turning and moulding That of *ebonite*, $\Delta = 1\,22$, was turned from a sheet 1 cm thick of the best Silvertown material, and gave a value 2 79 A very clear sample of *amber*, $\Delta = 1\,06$, was turned by holding fine sand-paper pressed between the fingers upon the specimen, and gave 2 80 A cylinder turned from a piece of old *ivory*, $\Delta = 2{\cdot}0$, gave $K = 6\,9$ For *canada balsam*, $\Delta = 1\,025$, a pale clear yellow sample melted and cast, cooling slowly, $K = 2\,72$ For *resin*, $\Delta = 1\,089$, prepared in the same way, a clear light brown sample, $K = 3{\cdot}09$ Sublimed *sulphur*, $\Delta = 1\,99$, heated to melting point and cast in a glass tube, cooling slowly with radial structure not reaching quite to the centre, gave $K = 4\,03$ A *gutta-percha* cylinder, $\Delta = 0\,815$, was prepared by rolling gently a small piece cut from a rod of Silvertown "super" quality warmed over an incandescent lamp, giving $K = 4\,43$ A cylinder of pure soft black *indiarubber*, obtained from the makers and tested in fields of different intensities to eliminate electrostriction, gave finally $K = 3\,08$

Olive Oil

17 The sample had a density 0 915 at 20° It was dried by shaking with fused calcium chloride

In order to eliminate as far as possible the influence of the necessary envelope, a tube was made by rolling a piece of cigarette paper 2 cm. square around an oiled steel cylinder having 0 238 cm diameter The oil caused the three layers used to adhere by capillarity, and the tube was dipped in the oil and wound up tightly on the cylinder Made in this way a tube has sufficient stiffness to retain its shape perfectly when suspended filled with liquid

The measurements were as follows —

Length of tube 2 00 cm, diameter 0 238 cm, thickness 0 0023 cm, mass of tube 0 0055 gramme, mass of oil contained 0 0814 gramme, ratio of length to bore 8 3 Free period of paper tube alone 7 seconds Period in field 6 seconds Free period full of oil 31 6 seconds Period in a field $F = 1{\cdot}065$, 11·06 seconds. The moment of inertia of the cylinder alone $I_1 = 0\,001926$ and of the oil alone 0 0272 gm. cm^2

The polarisation couple is therefore

$$39\,4(0\,0272 \times 0\,00718 - 0\,0000132) = 0{\cdot}00717 \text{ dyne-cm per radian}$$

The volume of the oil was 0 089 c.c

Assuming by trial a depolarising factor $N = 0\,7$, we have

$$(K-K_0)^2\left(\frac{1\,135 \times 0\,089 \times 5\,58}{0{\cdot}00717} - 4\,39\right) - 12{\cdot}56 \times 6\,28\,(K-K_0) - 157\,9 = 0,$$

from which $\mathbf{K = 3{\cdot}16.}$

The average value of all the determinations quoted by Winkelmann is 3·106. Hopkinson found 3·16 The commonest adulterant of olive oil is cottonseed oil, which has a dielectric constant of 3·88.

Heavy "White" Paraffin Oil.

18 Used for electric condensers, having an exceedingly high resistivity. Density, 0·885 at 27°·5 C.

Paper tube 1·99 cm long, 0·238 cm diameter, mass of liquid used 0·0787 gramme, ratio of length to diameter 8·37 Free period of tube filled with oil 25·7 seconds Period in a field F = 1·098, 12·15 seconds Polarisation couple 0·004735 dyne-cm /radian, and the equation with N = 0·71 .

$$121·84\,(K-K_0)^2-87·9\,(K-K_0)-157·9 = 0,$$

giving $$\mathbf{K = 2·55.}$$

For this oil $\mu_D = 1·484$, $\mu_D^2 = 2·2$

Water

19 Proceeding as before, distilled water was enclosed in a paper tube 2·02 cm. long, 0·238 cm diameter, or, allowing for the wetting of the paper, 0·24 cm

Mass of water in tube 0·0884 gramme Free period, full, 27·6 seconds. Period in a field F = 1·075, 1·76 seconds. Polarisation couple 0·364 dyne-cm /radian

From this the cylinder had apparently an infinite dielectric constant, which means that at the frequency of 80 a second electrical conduction has time to come into full operation.

Collected Results

20 For convenience of reference the results obtained in the paper are collected below, together with some previous determinations at frequencies not widely different from those of the paper There is on the whole close agreement between the new values and those quoted The differences between the best previous determinations are from 2 to 5 per cent By the method of the paper an accuracy of one-tenth per cent is obtained without difficulty except that of moisture or other conducting agency The work has been extended to the slow polarisation of dielectrics in unidirectional fields.

I wish to thank my late students, Mr O J. Williams, B Sc, and Mr B. J M Lane, B.Sc, for much help, especially in the development of the method, and my friend, Dr T. H Havelock, for several valuable suggestions.

Table IV ($\sim$ = 80 to 85)

Substance	K	Some previous determinations
Quartz ‖	4·600	5 06, Fellinger, 4 46, Ferry, 4 55, Curie; 4 6, Romich and Nowak Mean of these, 4 66
Quartz ⊥	4 548$_5$	4 69, Fellinger; 4 38 Ferry; 4 49, Curie, 4 6 Romich and Nowak Mean of these, 4 54
Fused quartz	3 78	
Flint glass—		
Δ = 4·65	10 04	10 65 }
Δ = 4 12	8 52	8 52 } Hopkinson, from fig 5
Δ = 3 30	6 98	6 82 }
Paraffin wax	2 326	2 32, Boltzmann; 2 32, Northrup, 2 20, Hopkinson, 1 94, v Pirani
Bees wax	4·75	3 67 Cavendish
Shellac	2 40	3 04, Winkelmann, 2·95 to 2·73, Wullner
Sealing wax	{ 4 56 / 5 2 }	4 3 (Lupton)
Gutta percha	4 43	2 5 to 4 2 (Lupton)
Chatterton compound	3 98	
Ebonite	2 79	2 76, Schiller, 2 72, Winkelmann, 2 86, Elsas, 2 55, Ferry
Amber	2 80	
Ivory	6 90	
Canada balsam	2 72	
Resin	3 09	3 0, Lombardi
Soft indiarubber	3·08	2 34, Schiller (? hard raw rubber)
Sulphur	4 03	4 05, Fellinger 4 00, Curie, 3 90, Boltzmann, 4 2, v Pirani Mean of these, 4 03
Olive oil	3 16	3 16, Hopkinson Mean of many determinations, 3 106 (in Winkelmann)
Heavy paraffin oil, Δ = 0 885	2 55	
Water	∞	75 to 80

Notes concerning Tidal Oscillations upon a Rotating Globe.

By Lord Rayleigh, O M , F R S

(Received May 6,—Read May 27, 1909)

Speculations on tidal questions are much hampered by our ignorance of the peculiar influence of the earth's rotation in any but the simplest cases. The importance of this element was first appreciated by Laplace, and he succeeded in obtaining solutions of various problems relating to a globe completely covered with water to a depth either uniform throughout, or at any rate variable only with latitude His work has been extended by Kelvin, G Darwin, and Hough For an excellent summary, reference may be made to Lamb's ' Hydrodynamics,' which includes also important original additions to the theory

But it must not be overlooked that a theory which supposes the globe to be completely covered with water has very little relation to our actual tides Indeed, in practice, tidal prediction borrows nothing from Laplace's theory, unless it be to look for tidal periods corresponding with those of the generating forces And this correspondence, although perhaps first brought into prominence in connection with Laplace's theory, is a general mechanical principle, not limited to hydrodynamics If the theory of terrestrial tides is to advance, it can only be by discarding the imaginary globe completely covered with water and considering examples more nearly related to the facts, as was done in some degree by Young and Airy in their treatment of tides in canals It is true that we are unlikely to obtain in this way more than very rough indications, but even such are at present lacking I am told that opinions differ on so fundamental a question as whether the Atlantic tides are generated in the Atlantic or are derived from the Southern Ocean Probably both sources contribute, but a better judgment, based on some sort of discussion on *a priori* principles, does not appear hopeless. In this connection, it is interesting to observe that a comparison of spring and neap tides shows that the moon is more effective relatively to the sun than would be expected from the ratio of the generating forces. This indicates some approach to synchronism with a natural free oscillation That the approach is closest in the case of the moon indicates that the free period is longer than those of the actual lunar and solar tides

Were it not for the complication due to the earth's rotation rendering all tidal problems *vortex* problems, as Kelvin put it, questions such as this could be treated without great difficulty, and perhaps illustrated by models.

There is nothing improbable in an oscillation backwards and forwards across the Atlantic having a period somewhat exceeding 12 lunar hours, but a treatment at all precise demands the inclusion of the rotation This suggests the problem of the oscillations of a rotating ocean bounded by two meridians

The present paper does not profess to make more than a modest contribution to the subject It commences by developing further the theory of the free vibrations of a plane rectangular sheet of liquid, initiated in a former paper,* but only under the restriction that the angular velocity is relatively small.† Subsequently, the corresponding problem for an ocean on a rotating globe, bounded by two meridians, is attempted, but with limited success Probably a better command of modern mathematical resources would lead to further results

Plane Rectangular Sheet

If ζ be the elevation, u, v the component velocities at any point, the equations of free vibration, when these quantities are proportional to $e^{i\sigma t}$, are‡

$$\left.\begin{aligned} i\sigma u - 2\omega v &= -g\, d\zeta/dx, \\ i\sigma v + 2\omega u &= -g\, d\zeta/dy, \end{aligned}\right\} \tag{1}$$

and

$$\frac{d^2\zeta}{dx^2} + \frac{d^2\zeta}{dy^2} + \frac{\sigma^2 - 4\omega^2}{gh}\zeta = 0, \tag{2}$$

in which ω denotes the angular velocity of rotation, h the uniform depth of the water, and g the acceleration of gravity The boundary walls will be supposed to be situated at $x = \pm \frac{1}{2}\pi$, $y = \pm y_1$

When ω is evanescent, one of the principal vibrations is represented by

$$u_0 = \cos x, \qquad v_0 = 0, \tag{3}$$

and ζ_0 is proportional to $\sin x$, so that

$$\sigma_0^2 = gh \tag{4}$$

This determines the frequency when $\omega = 0$ And since by symmetry a positive and a negative ω must influence the frequency alike, we conclude that (4) still holds in general so long as ω^2 can be neglected. The equation for ζ is at the same time reduced to

$$d^2\zeta/dx^2 + d^2\zeta/dy^2 + \zeta = 0. \tag{5}$$

Taking u_0 and v_0 as given in (3) and the corresponding ζ_0 as the first

* 'Phil. Mag.,' vol 5, p 297, 1903

† The condition is satisfied in the case of terrestrial lakes of moderate dimensions, especially if they are situated near the equator

‡ Kelvin, 'Phil. Mag.,' August, 1880; Lamb, 'Hydrodynamics' (3rd ed.), § 206.

approximation, we add terms u_1, v_1, ζ_1, proportional to ω, whose forms are to be determined from the equations

$$\iota\sigma_0 u_1 = -g\, d\zeta_1/dx, \tag{6}$$

$$\iota\sigma_0 v_1 = -g\, d\zeta_1/dy - 2\omega\cos\alpha, \tag{7}$$

$$(d^2/dx^2 + d^2/dy^2 + 1)(\zeta_1, u_1, v_1) = 0, \tag{8}$$

v_1, as well as ζ_1 and u_1, satisfying (8), since $\cos x$ does so They represent in fact a motion that would be possible in the absence of rotation under forces* parallel to v and proportional to $\cos x$ This consideration shows that u_1 is an odd function of both x and y, and v_1 an even function.

The former investigation proceeded from the assumption for u_1 of the form

$$u_1 = A_2 \sin 2x + A_4 \sin 4x + \quad , \tag{9}$$

which provides for the boundary condition to be satisfied at $x = \pm\frac{1}{2}\pi$, whatever functions of y the coefficients A_2, etc, may be. The value of v_1 thus obtained was

$$v_1 = \frac{2\omega\iota}{\sigma}\left\{\cos x - \frac{2}{\pi}\frac{\cos y}{\cos y_1} - \frac{4}{3\pi}\frac{\cosh(\sqrt{3}\ y)}{\cosh(\sqrt{3}\ y_1)}\cos 2x \right.$$
$$\left. + \frac{4(-1)^m}{(4m^2-1)\pi}\frac{\cosh\{y\sqrt{(4m^2-1)}\}}{\cosh\{y_1\sqrt{(4m^2-1)}\}}\cos 2mx + \quad \right\}, \tag{10}$$

where $m = 1, 2, 3$, etc

This value of v_1 may be employed to obtain a correction to σ_0 If we introduce terms u_2, v_2, ζ_2, proportional to ω^2, our equations (1) become, with retention of ω^2,

$$\iota\sigma(u_0+u_1+u_2) - 2\omega v_1 = -g\frac{d}{dx}(\zeta_0+\zeta_1+\zeta_2),$$

$$\iota\sigma(v_1+v_2) + 2\omega(u_0+u_1) = -g\frac{d}{dy}(\zeta_1+\zeta_2),$$

or with regard to the equations satisfied by the terms with zero and unit suffixes,

$$\iota\sigma u_2 + \iota(\sigma-\sigma_0)u_0 - 2\omega v_1 = -g\, d\zeta_2/dx,$$
$$\iota\sigma v_2 + 2\omega u_1 = -g\, d\zeta_2/dy$$

These are the equations that would apply to a rotationless sheet under the action of forces parallel to x and y proportional to $\iota(\sigma-\sigma_0)u_0 - 2\omega v_1$ and $2\omega u_1$ respectively, and of speed σ In order that the motion thus determined should be, as has been supposed, of the second order in ω, it is necessary that these forces should include no component capable of stimulating the principal motion For this purpose the force parallel to y

* It will be observed that these forces are not derivable from a potential

may be omitted from consideration as operating only upon v_1. Accordingly the condition to be satisfied is

$$\iint \{\iota(\sigma-\sigma_0)\, u_0 - 2\omega v_1\}\, u_0\, dx\, dy = 0, \tag{11}$$

an equation which may be regarded as giving a correction to σ_0.

In the present case the integration, between the limits $\pm\frac{1}{2}\pi$ for x and $\pm y_1$ for y, is straightforward and we get

$$\sigma-\sigma_0 = \frac{64\omega^2}{\pi^3\sigma_0}\left\{\frac{\pi^2}{16} - \frac{\tan y_1}{2y_1} - \frac{1}{1^3\ 3^2}\frac{\tanh\{v_1\sqrt{(1\ .\ 3)}\}}{y_1\sqrt{(1\ \ 3)}}\right.$$
$$\left. - \frac{1}{3^3\ 5^3}\frac{\tanh\{y_1\sqrt{(3\ \ 5)}\}}{y_1\sqrt{(3\ \ 5)}} - \ \right\} \tag{12}$$

The limiting values of x have been supposed for the sake of brevity to be $\pm\frac{1}{2}\pi$. If we denote them by $\pm x_1$, we are to replace x, y, y_1 in (10), (12) by $\frac{1}{2}\pi x/x_1$, $\frac{1}{2}\pi y/x_1$, $\frac{1}{2}\pi y_1/x_1$ respectively. At the same time (4) becomes

$$\sigma_0^2 = \frac{\pi^2 g h}{4x_1^2} \tag{13}$$

The method fails if y_1 is equal to an odd multiple of x_1. It would then become necessary to modify the initial assumption, as formerly explained in treating the case of the square, and $\sigma-\sigma_0$ would rise in magnitude so as to be of the first order in ω.

Equation (12) is not convenient in its application to the case where y_1 is very small. If we expand the tangent and hyperbolic tangents in powers of y_1, we obtain convergent series whose sums are zero for the terms independent of y_1 and proportional to y_1^2, but for higher powers of y_1 the series are divergent and no satisfactory conclusion can be drawn.

I have applied (12) to calculate the value of $\sigma-\sigma_0$ for the case where $y_1 = \frac{1}{2}\sqrt{3}$. For the various terms of the series within braces involving hyperbolic tangents, we get (with reversed signs) 0·102692, 0·003200, 0·000448, 0·000108, 0·000035, 0·000014, etc., giving in all about 0·106510. Also $\tan 2y_1/2y_1 = 0{\cdot}514368$, $\pi^2/16 = 0{\cdot}616850$. Hence

$$\sigma-\sigma_0 = -\frac{64\omega^2}{\pi^3\sigma_0} \times 0{\cdot}00403.$$

The inadequacy of (12) to deal satisfactorily with the case where y_1 is small led me to seek another solution. Here we assume in the first instance a form for v_1 which satisfies the conditions at $y = \pm y_1$, viz.,

$$v_1 = A_1 \cos\frac{\pi y}{2y_1} + \ldots + A_{2m+1}\cos\frac{(2m+1)\pi y}{2y_1}, \tag{14}$$

where $m = 0, 1, 2$, etc, making $v_1 = 0$ when $y = \pm y_1$ Hence, by (7),

$$-\frac{g}{i\sigma}\zeta_1 = \frac{2\omega y}{i\sigma}\cos x + \frac{2y_1 A_1}{\pi}\sin\frac{\pi y}{2y_1} + \ldots + \frac{2y_1 A_{2m+1}}{(2m+1)\pi}\sin\frac{(2m+1)\pi y}{2y_1},$$

no arbitrary function of x being added, since ζ_1 is odd in y

Further, by (6),

$$u_1 = \frac{2\omega i}{\sigma} y\sin x + \frac{2y_1}{\pi}\sin\frac{\pi y}{2y_1}\frac{dA_1}{dx} + \ldots + \frac{2y_1}{(2m+1)\pi}\sin\frac{(2m+1)\pi y}{2y_1}\frac{dA_{2m+1}}{dx}, \tag{15}$$

which is to be made to vanish when $x = \pm\frac{1}{2}\pi$ for all values of y between $\pm y_1$. Now, between these limits,

$$y = \frac{8y_1}{\pi^2}\left\{\sin\frac{\pi y}{2y_1} - \frac{1}{3^2}\sin\frac{3\pi y}{2y_1} + \ldots + \frac{(-1)^m}{(2m+1)^2}\sin\frac{(2m+1)\pi y}{2y_1}\right\} \tag{16}$$

Hence, when $x = \frac{1}{2}\pi$,

$$\frac{dA_1}{dx} = \frac{2\omega}{i\sigma}\frac{4}{\pi}, \qquad \frac{dA_{2m+1}}{dx} = \frac{2\omega}{i\sigma}\frac{4(-1)^m}{(2m+1)\pi} \tag{17}$$

Now, since v_1 satisfies (8),

$$\frac{d^2A_{2m+1}}{dx^2} + \left(1 - \frac{\pi^2(2m+1)^2}{4y_1^2}\right)A_{2m+1} = 0,* \tag{18}$$

whence, v_1 being an even function of x, if $y_1 < \frac{1}{2}\pi$,

$$A_{2m+1} = B_{2m+1}\cosh px, \tag{19}$$

where
$$p_1^2 = \frac{\pi^2}{4y_1^2} - 1, \qquad p_{2m+1}^2 = \frac{(2m+1)^2\pi^2}{4y_1^2} - 1 \tag{20}$$

If $y > \frac{1}{2}\pi$, (19) changes its form for one or more of the values of m. In (19) B_{2m+1} is a constant whose value is to be found from (17) We get, when $x = \frac{1}{2}\pi$,

$$\frac{dA_{2m+1}}{dx} = pB_{2m+1}\sinh(\tfrac{1}{2}p\pi) = \frac{2\omega}{i\sigma}\frac{4(-1)^m}{(2m+1)\pi};$$

so that finally

$$v_1 = \frac{2\omega}{i\sigma}\frac{4}{\pi}\frac{\cosh p_1 x\ \cos(\pi y/2y_1)}{p_1\sinh(\frac{1}{2}p_1\pi)} + \\ + \frac{2\omega}{i\sigma}\frac{4(-1)^m}{(2m+1)\pi}\frac{\cosh px\ \cos\{(2m+1)\pi y/2y_1\}}{p\sinh(\frac{1}{2}p\pi)} \tag{21}$$

Also from (15)

$$u_1 = \frac{2\omega i}{\sigma}\left[y\sin x - \frac{8y_1}{\pi^2}\frac{\sinh p_1 x}{\sinh(\frac{1}{2}p_1\pi)}\sin\frac{\pi y}{2y_1}\right. \\ \left. + \ldots + \frac{(-1)^m 8y_1}{(2m+1)^2\pi^2}\frac{\sinh px}{\sinh(\frac{1}{2}p\pi)}\sin\frac{(2m+1)\pi y}{2y_1}\right] \tag{22}$$

* The circumstances are such as to justify the differentiation under the sign of summation. (Stokes' 'Collected Papers,' vol. 1, p 281)

The introduction of (21) into (11) gives the correction to σ_0 in another form We find

$$\sigma - \sigma_0 = -\frac{512\omega^2 y_1^2}{\sigma_0 \pi^5} \Sigma \frac{\coth(\frac{1}{2}\pi p_{2m+1})}{(2m+1)^4 p_{2m+1}}, \tag{23}$$

where $m = 0, 1, 2, 3$, etc Whatever be the value of y_1, p_{2m+1} becomes larger as m increases, and ultimately $\coth(\frac{1}{2}\pi p_{2m+1}) = 1$ If y_1 be small enough, this occurs even for $m = 0$, and we may then omit the coth in (23) If, further, 1 can be neglected in comparison with p_1^2, we may take

$$p_{2m+1} = (2m+1)\pi/2y_1,$$

and (23) becomes

$$\sigma - \sigma_0 = -\frac{1024\omega^2 y_1^3}{\sigma_0 \pi^6} \Sigma \frac{1}{(2m+1)^6} = -\frac{1024\omega^2 y_1^3}{\sigma_0 \pi^6} \times 1\,00452, \tag{24}$$

so that the correction is of the *third* order in y_1, or in y_1/x_1, if we replace y_1 by its general value, viz, $\frac{1}{2}\pi y_1/c_1$

Comparing (12) and (23), we see that

$$\frac{\tan y}{2y_1} - \frac{1}{1^2\ 3^2} \frac{\tanh\{y_1\sqrt{(1\ \ 3)}\}}{y_1\sqrt{(1\ \ 3)}} - \quad -\frac{\pi^2}{16} = \frac{8y_1^2}{\pi^3} \Sigma \frac{\coth(\frac{1}{2}\pi p_{2m+1})}{(2m+1)^4 p_{2m+1}} \tag{25}$$

If we take $y_1 = \frac{1}{2}/\sqrt{3}$, we find from (20) $p_1 = 5\,3487$, so that all the coths on the right of (25) are nearly equal to unity The first term ($m = 0$) gives 0 0040199 and the two following are 0 0000165 and 0 0000013, so that the right-hand member of (25) is 0 00404, in sufficient agreement with the number previously calculated from the series on the left

So far we have supposed that the type of vibration is founded upon $u_0 = \cos(\frac{1}{2}\pi x/x_1)$ There is no difficulty in generalising the solution so far as to apply to the type

$$u_0 = \cos\frac{(2l+1)\pi x}{2x_1}, \tag{26}$$

where l is an integer We find

$$v_1 = \frac{2\omega}{i\sigma} \Sigma \frac{4(2l+1)(-1)^{l+m}}{2x_1(2m+1)} \frac{\cosh px \cos\{(2m+1)\pi y/2y_1\}}{p \sinh px_1}, \tag{27}$$

where the summation relates to m, taking in succession the values 0, 1, 2, etc, and

$$p^2 = \frac{(2m+1)^2\pi^2}{4y_1^2} - \frac{(2l+1)^2\pi^2}{4x_1^2}. \tag{28}$$

It is assumed that

$$y_1 < \frac{x_1}{2l+1}, \tag{29}$$

otherwise one or more terms corresponding to the lower values of m will change their form.

A process similar to that already employed when $l = 0$ gives for the corrected value of σ,

$$\sigma - \sigma_0 = \frac{64\omega^2}{\sigma_0}\frac{(2l+1)^2 y_1^2}{\pi^2 x_1^3}\Sigma\frac{\coth px_1}{(2m+1)^4 p}, \tag{30}$$

which agrees with (23) when we put $l = 0$, $x_1 = \frac{1}{2}\pi$ It is to be observed that the general value of σ_0 is now given by

$$\sigma_0^2 = \frac{(2l+1)^2\pi^2 gh}{4x_1^2}, \tag{31}$$

so that (30) may also be written

$$\frac{\sigma - \sigma_0}{\sigma_0} = -\frac{256\,\omega^2 y_1^2}{\pi^4 gh}\Sigma\frac{\coth px_1}{(2m+1)^4 px_1}, \tag{32}$$

in which l does not appear directly

There is also another class of primary vibrations in which the motion is parallel to x and is expressed by

$$u_0 = \sin(l\pi x/x_1), \qquad v_0 = 0, \tag{33}$$

l being an integer For this case we find, in the same way,

$$u_1 = \frac{2\omega}{i\sigma_0}\frac{l\pi}{x_1}\frac{y}{x_1}\cos\frac{l\pi x}{x_1} - \frac{2\omega}{i\sigma_0}\frac{8ly_1}{\pi x_1}\Sigma\frac{(-1)^{l+m}\cosh px}{(2m+1)^2\cosh px_1}\sin\frac{(2m+1)\pi y}{2y_1}, \tag{34}$$

$$v_1 = \frac{2\omega i}{\sigma_0}\Sigma\frac{4l(-1)^{l+m}}{(2m+1)x_1}\frac{\sinh px}{p\cosh px_1}\cos\frac{(2m+1)\pi y}{2y_1}, \tag{35}$$

where m takes the values 0, 1, 2, etc., and

$$p^2 = \frac{(2m+1)^2\pi^2}{4y_1^2} - \frac{l^2\pi^2}{x_1^2} \tag{36}$$

It is here assumed that $y < 2l/x_1$, so that p^2 is positive even when $m = 0$ In this case u_1 is an even function of x and an odd function of y, while v_1 is odd in x and even in y

The value of σ_0^2 is now given by

$$\sigma_0^2 = \frac{ghl^2\pi^2}{x_1^2}, \tag{37}$$

and for the correction to σ_0 we have

$$\frac{\sigma - \sigma_0}{\sigma_0} = -\frac{256\omega^2 y_1^2}{\pi^4 gh}\Sigma\frac{\tanh px_1}{(2m+1)^4 px_1} \tag{38}$$

It will not be forgotten that in this formula, as well as in (32), ω^4 is neglected.

In the examples hitherto given the primary motion ($\omega = 0$) is parallel to one of the sides of the rectangle I will now take an example from the

square, where the primary motion is symmetrical with respect to x and y and defined by

$$\zeta_0 \propto \cos x \cos y, \tag{39}$$

the sides of the square being the lines $x = \pm\pi$, $y = \pm\pi$ In harmony with (39) we get

$$u_0 = \sin x \cos y, \qquad v_0 = \cos x \sin y, \tag{40}$$

and since ζ_0 satisfies

$$\frac{d^2\zeta_0}{dx^2} + \frac{d^2\zeta_0}{dy^2} + \frac{\sigma_0^2}{gh}\zeta_0 = 0,$$

we see that

$$\sigma_0^2 = 2gh \tag{41}$$

Also

$$(d^2/dx^2 + d^2/dy^2 + 2)(\zeta_1, u_1, v_1) = 0 \tag{42}$$

The equations of the next approximation, analogous to (6), (7), are

$$\iota\sigma_0 u_1 - 2\omega \cos x \sin y = -g\, d\zeta_1/dx, \tag{43}$$

$$\iota\sigma_0 v_1 + 2\omega \sin x \cos y = -g\, d\zeta_1/dy, \tag{44}$$

and they are the same as if impressed forces $2\omega \cos x \sin y$, $-2\omega \sin x \cos y$ acted parallel to u and v respectively and there were no rotation From this we may infer that u_1 is even in x and odd in y, while v_1 is odd in x and even in y

The procedure is much the same as before We assume

$$v_1 = \Sigma V_{2m+1} \cos \tfrac{1}{2}(2m+1)y, \tag{45}$$

where $m = 0, 1, 2$, etc, making $v_1 = 0$ when $y = \pm\pi$ From (43), (44) we deduce

$$u_1 = \frac{4\omega}{\iota\sigma} \cos x \sin y + \Sigma' \frac{2}{2m+1} \frac{dV_{2m+1}}{d\iota} \sin \frac{2m+1}{2} y, \tag{46}$$

so that

$$0 = \Sigma' \frac{2}{2m+1} \frac{dV_{2m+1}}{dx_\pi} \sin \frac{2m+1}{2} \eta - \frac{4\omega}{\iota\sigma} \sin y, \tag{47}$$

since $u_1 = 0$ when $x = \pm\pi$

Now, between the limits $\pm\pi$ for y

$$\sin y = \frac{8}{\pi}\left\{\frac{1}{3}\sin\frac{y}{2} + \frac{1}{1\ 5}\sin\frac{3y}{2} - \frac{(-1)^m}{(2m-1)(2m+3)}\sin\frac{(2m+1)y}{2}\right\} \tag{48}$$

And thus

$$\frac{2}{2m+1}\frac{dV_{2m+1}}{dx_\pi} + \frac{4\omega}{\iota\sigma}\frac{8(-1)^m}{\pi(2m-1)(2m+3)} = 0. \tag{49}$$

But V_{2m+1} satisfies

$$\frac{d^2V_{2m+1}}{dx^2} - \frac{(2m+1)^2}{4}V_{2m+1} + 2V_{2m+1} = 0, \tag{50}$$

and hence, being odd in x, takes the form

$$V_{2m+1} = B_{2m+1} \sinh px, \tag{51}$$

where

$$p^2 = \tfrac{1}{4}(2m+1)^2 - 2 \tag{52}$$

This form obtains when $m > 0$ When $m = 0$, $p = \frac{1}{2}\iota\sqrt{7}$, and

$$V_1 = B_1 \sin(\tfrac{1}{2}\sqrt{7}\ x) \tag{53}$$

Using (49), (51) to determine B, we get

$$v_1 = \frac{4\omega}{\iota\sigma}\left\{\frac{4\sin(\frac{1}{2}\sqrt{7}\ x)\ \cos\frac{1}{2}y}{\pi . 3\ \frac{1}{2}\sqrt{7}\ .\cos(\frac{1}{2}\sqrt{7}\ \pi)} + \right.$$

$$\left. -\frac{4(-1)^m(2m+1)\sinh px}{\pi(2m+3)(2m-1)p\cosh p\pi}\cos\frac{(2m+1)y}{2}\right\}, \tag{54}$$

in which the first term may be deduced from the general term by putting $m = 0$ if we remember that

$$\sinh(\iota x) = \iota\sin x, \qquad \cosh(\iota x) = \cos x$$

Also from (46)

$$u_1 = \frac{4\omega}{\iota\sigma}\left\{\cos x\sin y - \frac{8(-1)^m\cosh px}{\pi(2m-1)(2m+3)\cosh p\pi}\sin\frac{(2m+1)y}{2}\right\} \tag{55}$$

It is evident, however, that there must be another expression for u_1 analogous to that given for v_1, and such as would be obtained by starting from

$$u_1 = \Sigma U_{2m+1}\cos\tfrac{1}{2}(2m+1)x \tag{56}$$

instead of from (45) We may, in fact, interchange x and y if we reverse the sign of ω. Thus

$$u_1 = \frac{4\omega}{\iota\sigma}\left\{ + \frac{4(-1)^m(2m+1)\sinh py}{\pi(2m+3)(2m-1)p\cosh p\pi}\cos\frac{(2m+1)x}{2} + \right\} \tag{57}$$

In applying these results to find a correction to σ_0, we have, much as before,

$$\iota\sigma_0 u_2 + \iota(\sigma-\sigma_0)u_0 - 2\omega\iota v_1 = -g\,d\zeta_2/dx,$$

$$\iota\sigma_0 v_2 + \iota(\sigma-\sigma_0)v_0 + 2\omega\iota u_1 = -g\,d\zeta_2/dy,$$

and thus

$$\iint\{(\sigma-\sigma_0)u_0 + 2\iota\omega v_1\}u_0\,dx\,dy + \iint\{(\sigma-\sigma_0)v_0 - 2\iota\omega u_1\}v_0\,dx\,dy = 0 \tag{58}$$

In accordance with what has been said, if one of these integrals vanishes, so does the other, and we may confine our attention to the former In the first place

$$\iint u_0^2\,dx\,dy = \pi^2 \tag{59}$$

In integrating $v_1 u_0$ we have

$$\int_{-\pi}^{+\pi}\cos\tfrac{1}{2}(2m+1)y\ \cos y\,dy = -\frac{4(2m+1)(-1)^m}{(2m-1)(2m+3)} \tag{60}$$

and

$$\int_{-\pi}^{+\pi}\sinh px\sin x\,dx = \frac{2\sinh p\pi}{p^2+1} \tag{61}$$

Thus
$$\sigma-\sigma_0 = \frac{256\omega^2}{\pi^2\sigma_0}\Sigma\frac{(2m+1)^2\tanh p\pi}{(2m-1)^2(2m+3)^2 p\pi\,(p^2+1)}, \tag{62}$$

p being given by (52).

For calculation of the first term under the sign of summation ($m = 0$) the form must be modified We find

$$\frac{-8\tan(\frac{1}{2}\pi\sqrt{7})}{27\pi\sqrt{7}} = -0\,057309 \tag{63}$$

The most important term is the next for which $m = 1, p = \frac{1}{2}$ Under the sign of summation we have

$$\frac{8}{5^3}\frac{3^2\tanh(\frac{1}{2}\pi)}{\pi} = 0{\cdot}16816 \tag{64}$$

The following terms are 0 00166, 0 00021, and 0 00005, so that altogether we may take as the sum of the terms under the sign of summation +0 1128 Accordingly

$$\sigma-\sigma_0 = -\frac{\omega^2}{\sigma_0}\times 2\,925\,, \tag{65}$$

and this result, being already of the right dimensions, applies whatever may be the size of the square It may be remarked that the sign of the correction is the opposite of that applicable to the *circle*, for which approximately*

$$\sigma-\sigma_0 = +2\omega^2/\sigma_0 \tag{66}$$

These results are, of course, applicable only under the restriction that ω is small compared with σ, the latter quantity depending on the size and depth of the sheet of liquid In the case of lakes and seas upon the rotating earth, we have also to remember that ω depends upon the latitude At the equator ω vanishes.

Spherical Sheet of Liquid

An attempt will now be made to apply similar methods to the free vibrations of an ocean on a rotating globe, the water being of uniform depth h, and bounded by vertical walls coincident with two meridians, $\phi = 0$ and $\phi = 2\phi_1$. Using a similar notation, we have as the general equations†

$$i\sigma u - 2\omega v\cos\theta = -\frac{g}{a}\frac{d\zeta}{d\theta}, \tag{67}$$

$$i\sigma v + 2\omega u\cos\theta = -\frac{g}{a\sin\theta}\frac{d\zeta}{d\phi} \tag{68}$$

with the equation of continuity

$$i\sigma\zeta = -\frac{h}{a\sin\theta}\left\{\frac{d(u\sin\theta)}{d\theta} + \frac{dv}{d\phi}\right\} \tag{69}$$

* Lamb, *loc. cit.*, p 306

† Lamb, *loc. cit*, p. 314

Here θ denotes the colatitude, a is the radius of the globe; ω its angular velocity of rotation, u, v the velocities along and perpendicular to the meridian As is usual, we shall write μ for $\cos\theta$ when convenient

By (67), (68), u and v may be expressed in terms of ζ, and substitution in (69) will then give an equation in ζ only When $\omega = 0$, this equation is the well-known one,

$$\frac{d}{d\mu}(1-\mu^2)\frac{d\zeta}{d\mu} + \frac{1}{1-\mu^2}\frac{d^2\zeta}{d\phi^2} + \frac{\sigma^2 a^2}{gh}\zeta = 0 \qquad (70)$$

We will suppose that the primary motion—that which would obtain if $\omega = 0$—is represented by ζ_0, u_0, v_0, and that $v_0 = 0$, so that the primary motion is wholly in latitude And we will begin with the further supposition that

$$\zeta_0 \propto \mu, \qquad u_0 = \sin\theta. \qquad (71)$$

Substitution in (70) shows that

$$\sigma_0^2 a^2 = 2gh \qquad (72)$$

The motion is that which might obtain equally over the complete sphere, the liquid heaping itself alternately at the two poles.

It is to be observed that, under the circumstances here contemplated, (70) holds good so long as ω^2 can be neglected, since all that is required in its formation is the omission of ωv and of $\omega\, du/d\phi$ We write it in the form

$$\frac{d}{d\mu}(1-\mu^2)\frac{d\zeta_1}{d\mu} + \frac{1}{1-\mu^2}\frac{d^2\zeta_1}{d\phi^2} + 2\zeta_1 = 0 \qquad (73)$$

and we observe that (73) is satisfied also by u_1 and by

$$\iota\sigma v_1 \sin\theta + 2\omega \sin^2\theta\cos\theta \qquad (73A)$$

if ζ_1, u_1, v_1 are the correctional terms proportional to ω If we substitute $\sin^2\theta\cos\theta$, or $\mu(1-\mu^2)$, in the left-hand member of (73), we get $-6\mu + 10\mu^3$, so that

$$\left[\frac{d}{d\mu}(1-\mu^2)\frac{d}{d\mu} + \frac{1}{1-\mu^2}\frac{d^2}{d\phi^2} + 2\right](v_1 \sin\theta) = \frac{2\omega}{\iota\sigma}(6\mu - 10\mu^3) \qquad (74)$$

Here $v_1 \sin\theta$ vanishes at the limits of ϕ If we assume

$$v_1 \sin\theta = \Sigma V_m \sin(m\pi\phi/2\phi_1), \qquad (75)$$

we may deduce for the left-hand member of (74)

$$\Sigma\left[\frac{d}{d\mu}(1-\mu^2)\frac{dV_m}{d\mu} - \frac{1}{1-\mu^2}\frac{m^2\pi^2}{4{\phi_1}^2}V_m + 2V_m\right]\sin\frac{m\pi\phi}{2\phi_1}. \qquad (76)$$

For the expansion of the right-hand member we have, between 0 and $2\phi_1$,

$$1 = \frac{4}{\pi}\left\{\sin\frac{\pi\phi}{2\phi_1} + \frac{1}{3}\sin\frac{3\pi\phi}{2\phi_1} + \frac{1}{5}\sin\frac{5\pi\phi}{2\phi_1} + \ldots\right\}, \qquad (77)$$

so that for the general term

$$\frac{d}{d\mu}(1-\mu^2)\frac{dV_m}{d\mu}-\frac{1}{1-\mu^2}\frac{m^2\pi^2}{4\phi_1^2}V_m+2V_m=\frac{2\omega}{i\sigma}\frac{8(3\mu-5\mu^3)}{m\pi}, \qquad (78)$$

m being an odd integer For even values of m, V_m vanishes.

The complete integral of (78) comprises, as complementary function, two functions of μ, one odd and one even, each multiplied by an arbitrary constant In the present case we have to do only with the odd function and its coefficient is to be determined by the consideration that V_m remains finite at the poles ($\mu = \pm 1$) A complete treatment presents considerable difficulties. Reference may be made to *Theory of Sound*, § 338 In the present case the n of spherical harmonics is unity, but s ($= m\pi/2\phi_1$) is not necessarily integral, still less an integer not exceeding 1

When ϕ_1 is small, the calculation simplifies, for then the second term on the left of (78) predominates, and ultimately we have

$$V_m = -\frac{2\omega}{i\sigma}\frac{32\phi_1^2}{m^3\pi^3}(3\mu-8\mu^3+5\mu^5), \qquad (79)$$

which with (75) determines $v_1 \sin\theta$ for the extreme case We may pursue the approximation with respect to ϕ_1 by substituting from (79) in the first and third terms of (78). For this purpose we may use

$$\left[(1-\mu^2)\frac{d}{d\mu}(1-\mu^2)\frac{d}{d\mu}+2(1-\mu^2)\right]\mu^n$$
$$= n(n-1)\mu^{n-2}-2(n^2-1)\mu^n+(n^2+n-2)\mu^{n+2} \qquad (80)$$

As was to be expected, the term in μ ($n = 1$) contributes nothing

The result of substituting $3\mu-8\mu^3+5\mu^5$ is thus

$$-48\mu+228\mu^3-320\mu^5+140\mu^7,$$

and accordingly the second approximation to V_m is

$$V_m = (79)-\frac{2\omega}{i\sigma}\frac{512\phi_1^4}{m^5\pi^5}(-12\mu+57\mu^3-80\mu^5+35\mu^7) \qquad (81)$$

When $v_1 \sin\theta$ is known, the corresponding terms in ζ_1 and u_1 may be found from (67), (68)

We will now apply (79) to calculate a correction to σ_0 in the manner already employed for plane sheets We have to make

$$\iint\{i(\sigma-\sigma_0)u_0-2\omega v_1\cos\theta\}u_0\sin\theta\,d\theta\,d\phi = 0, \qquad (82)$$

where $u_0 = \sin\theta$, and the integrations extend from -1 to $+1$ for μ, and from 0 to $2\phi_1$ for ϕ. The calculation is straightforward, and we find

$$\sigma-\sigma_0 = \frac{16\omega^2\phi_1^2}{35\sigma_0}, \qquad (83)$$

where Σm^{-4} (m odd) has been replaced by its equivalent, $\pi^4/96$. A continuance of the approximation from (81) gives another term in (83) involving (as well as ω^2) ϕ_1^4 and Σm^{-6}. It appears that

$$\sigma - \sigma_0 = \frac{4\omega^2}{\sigma_0}\frac{3.128\,\phi_1^2}{35\pi^4}\left\{\Sigma m^{-4} - \frac{16.11}{9}\frac{\phi_1^2}{\pi^2}\Sigma m^{-6}\right\}$$

$$= \frac{16\omega^2\phi_1^2}{35\sigma_0}\left\{1 - \frac{88\phi_1^2}{45}\right\}, \qquad (84)$$

when we substitute for Σm^{-4}, Σm^{-6}, their values, viz., $\pi^4/96$ and $\pi^6/960$. If it were important, the approximation with respect to ϕ_1 could be carried further without much difficulty.

We will now pass on to consider the most important primary mode in which ζ_0 is an *even* function of μ, proportional to the zonal harmonic of order 2, or to $\mu^2 - \frac{1}{3}$. In agreement with this we take

$$u_0 = \cos\theta \sin\theta, \qquad v_0 = 0 \qquad (85)$$

Substituting the value of ζ_0 in (70), we see that

$$\sigma_0^2 a^2 = 6gh, \qquad (86)$$

and ζ_1 satisfies

$$\frac{d}{d\mu}(1-\mu^2)\frac{d\zeta_1}{d\mu} + \frac{1}{1-\mu^2}\frac{d^2\zeta_1}{d\phi_2} + 6\zeta_1 = 0, \qquad (87)$$

an equation satisfied also by u_1 and by

$$\iota\sigma v_1 \sin\theta + 2\omega\cos^2\theta\sin^2\theta \qquad (88)$$

Substituting (88) in (87), we get

$$\left[\frac{d}{d\mu}(1-\mu^2)\frac{d}{d\mu} + \frac{1}{1-\mu^2}\frac{d^2}{d\phi^2} + 6\right](v_1\sin\theta) = -\frac{4\omega}{\iota\sigma_0}(1 - 6\mu^2 + 7\mu^4), \qquad (89)$$

the analogue of (74). Retaining (75) and understanding, as before, that m is an odd integer, we get, with use of (77) for the general term,

$$(1-\mu^2)\frac{d}{d\mu}(1-\mu^2)\frac{dV_m}{d\mu} - s^2 V_m + 6(1-\mu^2)V_m$$

$$= -\frac{2\omega}{\iota\sigma}\frac{8}{m\pi}(1 - 7\mu^2 + 13\mu^4 - 7\mu^6), \qquad (90)$$

where $s = m\pi/2\phi_1$.

This equation may be treated in the same way as was (78), but it may be well to introduce a modification which would be convenient in pursuing the approximation further. We will divide (90) by the factor which multiplies the parenthesis on the right, taking

$$V_m' = V_m \div -\frac{2\omega}{\iota\sigma}\frac{8}{m\pi},$$

and assume*

$$V_m' = K_0 + K_2\mu^2 + \ldots + K_n\mu^n, \qquad (91)$$

* Thomson and Tait's 'Natural Philosophy,' 2nd ed., Part I, p. 210.

where n is even Substituting in the right-hand member of (90), we find as the coefficient of μ^n

$$(n+1)(n+2)\,K_{n+2}-(2n^2-6+s^2)\,K_n+\{(n-1)(n-2)-6\}\,K_{n-2} \qquad (92)$$

Hence

$$\begin{aligned}
n=0,\quad 1 \quad & 2\,K_2-(s^2-6)\,K_0 && = 1,\\
n=2,\quad 3 \quad & 4\,K_4-(s^2+2)\,K_2-6K_0 && = -7,\\
n=4,\quad 5 \quad & 6\,K_6-(s^2+20)\,K_4 && = 13,\\
n=6,\quad 7 \quad & 8\,K_8-(s^2+66)\,K_6+14K_4 && = -7,\\
n=8,\quad 9 \quad & 10\,K_{10}-(s^2+122)\,K_8+36K_6 && = 0,
\end{aligned}$$

the right-hand members being zero for 8 and all higher values of n It will be seen that one of the coefficients is arbitrary, providing the necessary undetermined element The problem would be so to choose it as to satisfy the condition at the pole

When s^2 may be treated as large, we may divide the system of equations by it, obtaining as the first approximation

$$K_0 = -s^{-2}, \qquad K_2 = 7s^{-2}, \qquad K_4 = -13\,s^{-2}, \qquad K_6 = 7s^{-2}, \qquad (93)$$

after which K_8, etc, vanish To obtain a second approximation we substitute the result of the first approximation in the smaller terms Thus

$$K_0 = -\frac{1}{s^2}\frac{1}{6}+\frac{2K_2}{s^2-6} = -s^{-4}+8s^{-4}$$

In like manner,

$$K_2 = 7s^{-2}-148\,s^{-4}, \qquad K_4 = -13\,s^{-2}+548\,s^{-4},$$
$$K_6 = 7s^{-2}-644\,s^{-4}, \qquad K_8 = 252\,s^{-4},$$

after which the K's are zero to this order. The approximation may be pursued, and at each step another K enters In this process the difficulty of satisfying the general condition at the pole is evaded

If we stop at the first approximation (93), we have

$$v_1 \sin\theta = +\frac{2\omega}{i\sigma}\frac{8\sin s\phi}{m\pi s^4}(1-7\mu^2+13\mu^4-7\mu^6)+ \qquad (94)$$

Using this value of $v_1 \sin\theta$ in (82), we find

$$\sigma-\sigma_0 = -\frac{16\omega^2\phi_1^2}{63\sigma_0} \qquad (95)$$

as the correction applicable for this mode of vibration of a narrow lune

From some points of view, there is advantage in the use of ν ($=\sin\theta$), rather than μ, as independent variable In place of (90) we have*

$$\nu^2(1-\nu^2)\frac{d^2V_m'}{d\nu^2}+\nu(1-2\nu^2)\frac{dV_m'}{d\nu}+6\nu^2V_m'-s^2V_m' = 2\nu^2-8\nu^4+7\nu^6; \qquad (96)$$

* See 'Theory of Sound,' § 338.

but for the moment we will take on the right the more general form $av^2+bv^4+cv^6$. Assuming

$$V_m' = H_0+H_2 v^2+H_4 v^4+ \ldots, \tag{97}$$

we find on substitution in the left-hand member of (96) as the coefficient of v^r

$$(r^2-s^2)H_r-\{(r-1)(r-2)-6\}H_{r-2} \tag{98}$$

Thus for the term depending on a,

$$H_0 = 0, \qquad H_2 = a/(4-s^2), \qquad H_4 = 0, \text{ etc},$$

so that

$$V_m' = \frac{av^2}{4-s^2}, \text{ simply} \tag{99}$$

For the term in b,

$$H_0 = 0, \quad H_2 = 0, \quad H_4 = \frac{b}{4^2-s^2}, \quad H_6 = \frac{5\ 4-6}{6^2-s^2}H_4, \text{ etc},$$

so that

$$V_m' = \frac{bv^4}{4^2-s^2}+\frac{b(5\ 4-6)v^6}{(4^2-s^2)(6^2-s^2)}+\frac{b(5\ 4-6)(7\ 6-6)v^8}{(4^2-s^2)(6^2-s^2)(8^2-s^2)}+ \tag{100}$$

In like manner, for the term in c,

$$V_m' = \frac{cv^6}{6^2-s^2}+\frac{c(7\ 6-6)v^8}{(6^2-s^2)(8^2-s^2)}+ \tag{101}$$

Introducing the numerical values of a, b, c, we find for the sum of the three contributions,

$$V_m' = \frac{2v^2}{2^2-s^2}-\frac{8v^4}{4^2-s^2}-\frac{7s^2v^6}{(4^2-s^2)(6^2-s^2)}\left\{1+\frac{4\ 9}{8^2-s^2}v^2+\frac{4\ 9}{8^2-s^2}\frac{6\ 11}{10^2-s^2}v^4 + \frac{4\ 9}{8^2-s^2}\frac{6\ 11}{10^2-s^2}\frac{8\ 13}{12^2-s^2}v^6+ \right\} \tag{102}$$

and there is also to be added the complementary function,

$$V_m' = Av^s\left\{1+\frac{(s-2)(s+3)}{2(2s+2)}v^2+\frac{(s-2)(s-1)(s+3)(s+4)}{2.4\ (2s+2)(2s+4)}v^4+ \right\}, \tag{103}$$

in which A is a constant

The expressions in (102), (103) vanish when $v = 0$ It is further necessary —and this is the condition determining A—that $dV_m'/d\theta$ should vanish at the equator ($v = 1$). Now

$$dV_m'/d\theta = \cos\theta \ dV_m'/dv,$$

in which $\cos\theta$ vanishes. So far as regards (99), dV_m'/dv is finite when $v = 1$, so that no further question arises here But for (100), (101), (103), dV_m'/dv is infinite when $v = 1$, and a further scrutiny is called for

As a first step we may examine (103), taking it in the more general form*

$$\psi_s = \nu^s \left\{ 1 + \frac{(\frac{1}{2}s-\frac{1}{2}n)(\frac{1}{2}s+\frac{1}{2}n+\frac{1}{2})}{1\ (s+1)}\nu^2 \right.$$

$$\left. + \frac{(\frac{1}{2}s-\frac{1}{2}n)(\frac{1}{2}s-\frac{1}{2}n+1)(\frac{1}{2}s+\frac{1}{2}n+\frac{1}{2})(\frac{1}{2}s+\frac{1}{2}n+\frac{3}{2})}{1\ \ 2\,(s+1)(s+2)}\nu^4 + \right\}. \quad (104)$$

In Gauss' notation for hypergeometric series,

$$\psi_s = \nu^s\, \mathrm{F}(\alpha, \beta, \gamma, \nu^2), \quad (105)$$

where $\alpha = \frac{1}{2}s-\frac{1}{2}n, \quad \beta = \frac{1}{2}s+\frac{1}{2}n+\frac{1}{2}, \quad \gamma = s+1$

Since $\gamma-\alpha-\beta = \frac{1}{2} > 0$, F is finite when $\nu = 1$, and accordingly so also is ψ_s. But for $d\psi_s/d\nu$ we have

$$d\psi_s/d\nu = s\,\nu^{s-1}\mathrm{F} + \nu^s\, d\mathrm{F}/d\nu,$$

of which the first part, being finite, need not be regarded. Thus when $\nu = 1$,

$$d\psi_s/d\theta = (1-\nu^2)^{\frac{1}{2}}\, d\mathrm{F}/d\nu. \quad (106)$$

Now

$$\frac{d\mathrm{F}}{d\nu} = \frac{\alpha\beta}{1}\frac{2\nu}{\gamma}\mathrm{F}(\alpha+1, \beta+1, \gamma+1, \nu^2),$$

in which,† when $\nu = 1$ nearly,

$$\mathrm{F}(\alpha+1, \beta+1, \gamma+1, \nu^2) = \frac{\Gamma(\gamma+1)}{\Gamma(\alpha+1)\,\Gamma(\beta+1)}(1-\nu^2)^{\gamma-\alpha-\beta-1}. \quad (107)$$

In the present case

$$\gamma-\alpha-\beta-1 = -\tfrac{1}{2},$$

and thus when $\nu = 1$

$$\frac{d\psi_s}{d\theta} = \frac{2\,\Gamma(\gamma)}{\Gamma(\alpha)\ \Gamma(\beta)} = \frac{2\,\Gamma(s+1)}{\Gamma(\frac{1}{2}s-\frac{1}{2}n)\ \Gamma(\frac{1}{2}s+\frac{1}{2}n+\frac{1}{2})} \quad (108)$$

If $n = 2$, as at present,

$$\frac{d\psi_s}{d\theta} = \frac{2\,\Gamma(s+1)}{\Gamma(\frac{1}{2}s-1)\ \Gamma(\frac{1}{2}s+\frac{3}{2})}, \quad (109)$$

when $\theta = \frac{1}{2}\pi$. Thus $d\psi_s/d\theta$ has a finite value at the equator, as was to be expected.

It may be proved without difficulty that (102) converges when $\nu = 1$ and s is not an even integer. Any finite number of terms which may have negative denominators being excluded, the remainder may be expressed as a hypergeometrical series. But the form is more complicated than before, and the evaluation of $d\mathrm{V}_n'/d\theta$ would be rather tedious, even if practicable.

* 'Theory of Sound,' *loc. cit.* Here $n = 2$.

† 'Infinite Series,' Bromwich, p. 171.

A question obtrudes itself as to what happens when s is an even integer When $s = 2$, there is synchronism between the primary and a derived vibration, and the occurrence of the infinite denominator $4-s^2$ is what might have been expected But in the case of other even integers no synchronism is apparent, and it would seem that the complication is of an analytical character only The solution compounded of (102) and (103) changes its form It would be of interest to follow out the process, say for the case $s=6$, which might roughly represent the circumstances of the Atlantic Ocean, but I am not prepared to undertake the task

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On a New Kind of Glow from Palladium in Vacuum Tubes

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(Communicated by Prof Sir J. J Thomson, F R S Received May 7,—Read May 13, 1909)

(Plate 1)

The phenomenon described in this paper was first noticed when making some preliminary experiments in an investigation suggested by Prof Sir J J Thomson The effect of the gases contained in metals on the nature of the discharge obtained when these metals are employed as electrodes in vacuum tubes has long been a question of interest. It was suggested that by employing palladium one might hope to obtain interesting results, owing to the property possessed by this metal of absorbing large quantities of hydrogen The result of these preliminary experiments has already been briefly stated,* but as the effect seems to be a new one, it seemed of sufficient interest to call for a fuller investigation

Apparatus—A tube was fitted up containing an ebonite plug through which two stout brass leads were passed To the extremities of the leads was attached, by means of screws, a piece of palladium foil about an inch in length and a tenth of an inch wide The leads and plugs were fixed in position by sealing-wax. The tube was sealed by the blow-pipe to a mercury pump and was in connection with a pressure gauge, drying-bulb, etc. The tube could be isolated from the rest of the apparatus by means of a barometric mercury seal By passing an electric current through the palladium, it could be heated to any required temperature. (See Plate 1, fig. 1.)

* 'Camb. Phil. Soc Proc.,' vol 14, No 6, p. 578, 1908

Appearance of the Glow—The pressure was reduced to 0·15 mm, and a current was sent through the palladium with the object of expelling some of the gases which it might contain. It was kept at a dull red heat for some time, and was then raised to an almost white heat.

At this point a purple glow was noticed around the hot palladium *. This glow resembled the negative glow in a vacuum tube, but as there was no electric field except that due to the current from the two storage cells used to heat the metal, it seemed to require an explanation. The experiments described in this paper were made with the object of investigating the nature and cause of this luminosity. As the matter has not been hitherto studied, it seems useful to describe the results of these experiments with greater detail than would otherwise be desirable.

The most satisfactory form of tube was that shown in the figure, in which the general appearance of the phenomenon is indicated. The colour of the glow is a rich purple-blue, when the foil is very hot, it appears bluer, but this is due to the light reflected from the walls of the tube. There is a "dark space" between the glow and the palladium, which is well marked when the foil is at a high temperature. The luminosity does not extend to the walls of the tube, especially when this is 4 or 5 inches in diameter. Its general appearance is like a purple halo around the hot palladium. The glow disappears when the current is stopped and begins again when it is sent through once more.

Material employed—The palladium foil used on the first occasion was a piece that had been in the laboratory for some time, and had all the appearance of having been heated up on former occasions. Some fresh palladium was obtained, and this, too, produced the glow. As palladium melts at a comparatively low temperature, it was found advantageous to use platinum foil into which some palladium black had been well rubbed instead. This gives almost as good results as the palladium foil, and has the additional advantage that there is a less dense deposit of disintegrated palladium on the walls of the tube. Experiments will be described showing the part played by the disintegration of the metal. In order to be assured that the phenomenon was due to the palladium itself and not to any dirt which might be on it, a piece was carefully cleaned as follows. It was heated for a few moments in strong nitric acid, which dissolved a thin layer. It was then washed and put into chromic acid, after which it was washed again. Finally, when it had been attached to the leads, it was once more put into chromic acid, washed in distilled water, and rapidly dried by pure alcohol. This piece of palladium produced the glow even better than other pieces which

* See figures on plate.

had not been carefully cleaned Other substances, such as platinum and carbon filament, were also tried, but the glow was not obtained

Electric Field.—It appeared possible that the phenomenon might be directly connected with the well-known ionisation effects produced in the neighbourhood of hot metals The glowing palladium was connected with the terminal of a battery of small cells, the other terminal being attached to an electrode in the glass tube at a distance of about an inch from the palladium. Various potentials up to 400 volts were applied without any effect on the glow, nor did it make any difference whether the palladium was attached to the positive or negative pole of the battery. Even the passage of a luminous discharge did not influence the general appearance of the glow.

Magnetic Field—The application of a magnetic field was also found to be without effect.

Effect of Heat—In the original experiment it was noticed that the glow soon faded away This happened in a narrow tube, which at the same time became very hot

It was subsequently found that the disappearance of the glow was caused by the heat of the walls of the tube If the tube is kept cool by placing the bulb in cold water, it is possible to keep the glow going indefinitely The effect of the temperature of the walls of the tube is not the least strange feature of the phenomenon If the tube be heated by means of a bunsen burner the glow fades away, and only returns when the tube has become cool again. If one side of the tube be heated and the other kept cool, it is found that the luminosity disappears on the hot side, but remains on the cooler one.

The Dark Space—The dark space surrounding the palladium seems to be due to the temperature A second pair of leads was introduced into the tube, to which was attached a small spiral of platinum wire. When this was placed well in the glow, which had been produced in the usual way, it was found that, if the spiral was heated to a high temperature, a dark space was produced around it This shows that the dark space around the palladium is due to its temperature This dark space is not a contrast effect, as it can be seen when an opaque object is held over the hot metal. So, too, the heating of the gas in the tube, the leads and walls, tends to diminish the intensity of the glow If the current be stopped for a short time so as to allow everything to grow cooler, it is found that the glow is brighter on re-heating the palladium.

Initial Pressure of the Gas—The phenomenon occurs within wide limits of initial pressure It was observed within a range of pressures from 0·2 to 0·003 mm. Generally the pressure goes up when the palladium is heated

This rise of pressure depends on the condition of the palladium On one occasion, when the pressure was rising owing to a slight leak in the sealing-wax, the glow was still visible when the pressure has gone up a centimetre or more The rise in pressure in normal cases is of the order of a millimetre

Action of Gases in the Tube—The fact that the glow could be obtained when the initial pressure of the tube was very low, so that the bulb gave green fluorescence when a discharge was sent through it before heating the foil, seems to show that the glow depends on the presence of gases which have come from the hot palladium When the tube had been filled with hydrogen before being exhausted, it was found impossible to produce the glow in the ordinary way When, however, the palladium was made the cathode of a coil discharge through the tube, it was observed that the glow suddenly appeared When the luminosity had once been produced, the cold discharge had no further effect In general, when the glow was slow in appearing, it was found that it was helped by passing a discharge through the tube for a short time The following experiment proves that the presence of gases in the tube which have been produced from the palladium is necessary. The glow was obtained in the usual way and allowed to continue for some time Air was now admitted to the tube and the tube was again exhausted to a pressure favourable for the phenomenon The palladium was again heated up, but the glow was not obtained In the same way, it was found that a piece of palladium which had been recently heated would not give the glow. Another similar result was obtained by attaching a side tube containing charcoal to the glow-bulb, when this was placed in liquid air, the glow disappeared, and returned again when the liquid air had been removed

Palladium recently heated—Various methods were tried to restore to palladium which had been heated for some time the power of producing the glow, without success It was saturated with hydrogen gas, and was made the electrode in a vacuum tube discharge, and was placed near the cathode in such a discharge, but the power of producing the glow was not restored. The only method found successful was to leave the palladium exposed to the air for a considerable time. Thus after six weeks the property of producing the glow was found to be restored to a moderate degree.

Disintegration.—The disintegration of the palladium has an important connection with the luminosity Either the particles of palladium are themselves the cause of the luminosity or they carry with them something which acts on the gases in the tube The black deposit on the sides of the tubes is soluble in cold nitric acid, and can be easily rubbed off the glass The following experiment shows the importance of the disintegration of the

palladium. A tube was fitted up with two pairs of leads, to the extremities of one pair was attached a piece of palladium foil, and to the other a piece of platinum foil A piece of glass tubing was placed over the extremities of the leads carrying the platinum foil The tube extended about an inch on each side of the platinum, thus shielding it from the deposit from the palladium The latter was now heated and the glow obtained This was allowed to continue for some time The current through the palladium was stopped, and the glass tubing removed from over the platinum by inclining the tube. The platinum foil was then heated by a current of electricity. No glow was obtained This result is important, as it shows that the glow is not due merely to the heat from incandescent metals. The platinum foil was then allowed to cool, and the palladium heated again The glow appeared in due course. As the platinum foil was now exposed to the disintegration of the palladium it was blackened by a deposit The current was stopped through the palladium, and the platinum was heated. This time, when the platinum was raised to a sufficiently high temperature, the glow appeared, and was almost as bright as in the case of the palladium This result, taken with those just described, proves that the luminosity is due to some reaction between the disintegrated palladium and gases in the tube It was observed that a small piece of mica placed near the palladium had the effect of preventing the glow in its shadow Behind a small opaque screen the glow was seen slightly

Mercury Vapour.—The experiments already described prove that the luminosity depends on the gases in the tube. In order to see if mercury vapour had any part in the phenomenon an experiment was made in which this was carefully excluded A glass spiral was carefully cleaned with nitric acid, and was then placed in a vessel of liquid air, one extremity was now sealed to the mercury pump and the other to a tube which had been fitted with fresh leads and which had been carefully cleaned away from mercury vapour The bulb was now exhausted to the required pressure, and was then sealed off from the pump As the spiral had been in the air during the whole operation, any mercury vapour that came from the pump was frozen out before it could reach the tube. That there was no mercury vapour present in the tube was shown by the absence of mercury lines in a discharge between the leads and an electrode which had been placed in the side tube of the bulb. When the palladium was heated the glow came in the ordinary way. Thus the presence of mercury vapour is not an essential condition

Phosphorus Vapour.—A similar experiment was made to see if the phenomenon had anything to do with any impurity arising from the

phosphorus pentoxide employed in the drying tube Crookes* showed that the presence of some such impurity accounted for the blue face often seen on striæ in vacuum tubes. This time the tube was first exhausted as far as possible by means of a water pump and then brought to the right pressure by means of a charcoal bulb in liquid air. The pressure could be judged by the thickness of the dark space of a discharge through the tube. In this experiment care had been taken to use nothing which had been exposed to phosphorus pentoxide or other known source of phosphorus vapour In this case also the glow was obtained as before

Water Vapour—The influence of water vapour was shown by the following results A small side tube was inserted into the bulb of the tube in which the glow was obtained This was so placed that its extremity reached into the purple glow This tube was like a test tube with the open end outside The palladium was heated and the glow produced A few drops of liquid air were now introduced into the side tube After a few seconds the glow began to fade away and finally disappeared At the same time a white deposit appeared on the extremity of the side tube When the liquid air had boiled off and the side tube had regained its ordinary temperature the glow gradually returned

In this way the glow could be made to go and come as often as was desired The same result could be obtained by allowing a few drops of liquid air to fall on the surface of the bulb This seemed to show that the glow depended on the presence of water vapour In order to ascertain that the absence of the glow was due to this cause a tube was fitted up in the usual way containing some phosphorus pentoxide. When the tube had been left long enough to enable the water vapour to be absorbed it was found that the glow could not be obtained. Thus the presence of water vapour is a necessary condition for the appearance of the glow This was also evident from other indications If the tube was very carefully dried the glow was not so easily obtained as when less care had been taken The best results were obtained when the apparatus had been filled with air which had not passed through a drying tube The result shows that the effect of the liquid air is to freeze out the water vapour from the bulb. The results already obtained by heating the bulb may also be connected with the effect of heat on the water vapour in the tube. This point will be considered when dealing with the probable cause of the phenomenon It might be thought that the precautions already described to keep out mercury would also abstract water vapour from the bulb. The spiral in liquid air was some distance from the bulb, and the whole operation of exhausting it only

* 'Roy Soc. Proc.,' vol 69, p. 399, 1901.

occupied a short time As will be shown further on, water vapour is also produced when the metal in the bulb is heated.

Spectrum of the Glow—The examination of the spectrum of the glow is rendered difficult owing to the amount of light reflected from the walls of the tube In spite of all precautions taken to do away with this inconvenience there was always a considerable quantity of reflected light when the glow was sufficiently bright to make satisfactory observations It was found, however, possible to obtain some interesting information by examining the spectrum When the spectrum is viewed through a spectrometer of low dispersion, the spectrum consists of a continuous portion extending from the red towards the blue end Just at the extremity of this region there is a blue-green band This band can only be observed when the glow is very bright and the dispersion of the spectrometer low Then there comes a dark space, which at first was thought to be an absorption band, and then a blue band When the spectrum of the reflected light was observed in the absence of the glow it was found that the two bright bands were absent, and that the continuous portion only extended to the beginning of the dark space When the glow was present these bright bands were present as an addition to the spectrum of the reflected light. No doubt other lines or bands could have been seen but for the presence of the reflected light Indeed, the green-blue band could not be seen as a rule, but was sufficiently well marked when the glow was very bright. This spectrum was then examined with a direct reading Hilgar spectrometer This instrument gave considerably greater dispersion than the other, and the band was therefore less well marked It was possible, however, by remaining in the dark for some time to distinguish the blue band sufficiently well to measure it It was not possible to measure the green band directly It was identified indirectly As the edges of the blue band are not well defined it was only possible to take approximate readings of its limits. Various sets of readings taken after intervals of some considerable time gave readings for the middle of the blue band which agreed very substantially The spectrometer had previously been standardised by being set at the blue mercury line, which is very close to the position of the blue band

The following are the wave-lengths of the various parts of the spectrum —

Beginning of the continuous spectrum	6220
Beginning of dark region	4697
Beginning of the blue band from glow	4419
Reading taken as middle of the blue band . . .	4380
End of blue band	4343

It was not possible to determine whether this blue band was composed of narrower bands or lines * It had all the appearance of a broad band

This band did not appear to correspond to any portion of the spectra involved in any of the substances one expected to meet with in the experiment It was thought that the examination of the spectrum of the gases in the tube made in the ordinary way might throw some light on the matter For this purpose a small spectrum tube was attached to the tube connecting the pump and the glow tube The whole was exhausted as usual and the glow obtained The spectrum tube was now sealed off from the rest and thus a specimen of the gas in the bulb was obtained This was connected to an induction coil and its spectrum examined The hydrogen lines were of course prominent In addition the spectrum of carbon monoxide was very strongly marked. All who have worked with hot metal cathodes have remarked the amount of carbon monoxide produced in the tube. At least the spectrum of carbon monoxide is always present A blue portion of this spectrum was found to correspond exactly with the blue band due to the glow The spectrum tube was placed on the far side of the bulb, so that it could be examined with the spectrometer through the latter The glow was then produced and at the same time a discharge was passed through the spectrum tube By this means it was ascertained that the bands in the spectrum of the glow corresponded to two bands in the spectrum of the gas in the vacuum tube The darker region in the spectrum of the glow also corresponded with a less luminous part of the spectrum of the gas in the discharge tube

The end of the continuous spectrum due to the reflected light just corresponds to the green-blue band The spectrum of the gas in the discharge tube was that of carbon monoxide. We have therefore sufficient evidence that the spectrum of the glow is due to carbon monoxide gas No doubt, were it not for the reflected light it would be possible to find other bands and lines of the spectrum due to that substance The blue band which was measured corresponds to a region of lines and bands close together. The other band corresponds to that of the carbon monoxide spectrum 4836.

The spectrum of carbon monoxide burning in air is a continuous one as far as can be seen. This spectrum includes the bands of the spectrum of the glow, but does not show the dark region, and also extends further towards the blue end

Polarisation absent.—It seemed just possible that the glow might be due to light scattered by particles of matter in the tube. To test this view the

* See note at end

glow was examined by means of Nicol's prisms to detect any sign of polarisation, but with negative result A strong beam of light from an arc lamp was also passed through the glow, for if the colour was due to reflected light this ought to make a difference, but no such difference could be observed. It seems justifiable to conclude that the luminosity is from the materials in the tube themselves, and from the spectroscopic results it seems that this luminosity is connected with the presence of carbon monoxide gas

Theoretical—In putting forward any theory as to the cause of the phenomenon it is necessary to take into consideration the properties of palladium A brief enumeration of the properties which seem to have a connection with this effect will be of assistance in arriving at some conclusion It is well to bear in mind that the electrical properties of hot metals are still known but very imperfectly, and any theory about the precise mechanism of the reactions taking place near hot wires will naturally depend much on our knowledge of such properties

The chemical behaviour of palladium is of a very peculiar and complicated character Various researches have been made on its properties of combining with hydrogen and other gases, to some of which it will be necessary to briefly refer That the phenomenon depends on these properties is evident, though the reaction which appears to be the one here involved is not that which would at first suggest itself

An analysis of palladium black was made by Mond, Ramsay, and Shields,* in which they showed that this material in its natural condition contains 1 6 per cent by weight or 138 vols of oxygen in the form of PdO It also contains 0 72 per cent of water Palladium goes on absorbing oxygen up to a dull red heat, and can absorb as much as 1000 vols When exposed to hydrogen gas some of this oxygen unites with the latter to form water Of the 873 vols. of hydrogen which palladium can occlude, 92 per cent. can be pumped off at ordinary atmospheric temperature, and nearly all the remainder came off at about 450° C No doubt palladium, like platinum, will keep giving off hydrogen for a very long time even when heated to a high temperature The most favourable temperature for the absorption of hydrogen at atmospheric pressure is 100° C At this temperature it parts with nearly all its hydrogen *in vacuo* These latter numbers appear to be also true of other forms of palladium.

It is a well-known fact that hydrogen which has been occluded by palladium and then driven out is much more active than ordinary hydrogen.

Many peculiarities in the behaviour of palladium have been noticed which

* "On the Occlusion of Hydrogen and Oxygen by Palladium," 'Roy Soc. Proc.,' vol 62, p. 290.

tend to complicate the study of its properties. There is one other reaction which is of the greatest importance in connection with the phenomenon under consideration, though it is one which did not seem to have any connection with the glow at first It is found* that palladium charged with hydrogen has the property of causing the combination of oxygen and carbon monoxide in the presence of water to form carbon dioxide. This would appear to be the reaction involved in the production of the luminosity, and it can be shown that the facts already stated in this paper can be reasonably interpreted on this view The series of reactions involved may very well be somewhat as follows

When the palladium is first heated the glow does not make its appearance until some little time has elapsed During this time the hot metal would be giving off hydrogen and oxygen, and water vapour, or would be bringing about the union of some of the gases to form water Also carbon monoxide would be being produced in the tube The help given by the passage of an electric current would probably consist in the formation of carbon monoxide or water vapour in the tube The advent of these gases in the tube gives rise to an increase of pressure Whatever be the origin of the carbon monoxide, its presence, as indicated by its spectrum, is always noticed in the discharge from a hot cathode At the same time the palladium is disintegrating, the higher the temperature to which it is raised the greater being the disintegration As we have seen, at the high temperature of the hot palladium the gases have been expelled, and thus the particles of metals are not charged with hydrogen when they leave the piece of foil. As these particles of metal travel out their temperature falls, and they may then absorb hydrogen. They are now in a condition to act on the oxygen and carbon monoxide and water vapour in the tube. If we suppose the union of the oxygen and carbon dioxide to take place with luminosity, we have all the facts about the glow accounted for What the nature of this union is cannot be discussed here That all such cases of combination are closely connected with electricity seems certain, and that the electrical properties of hot metals enter into this reaction would seem most probable However, too little is known as yet concerning the phenomena connected with hot metals to justify a discussion on this point.

The explanation just suggested seems to account for the facts which have been ascertained about the glow. According to this view, it is clear that a piece of palladium which has been strongly heated not long before could not produce the glow. Its recovery by exposure to the air is also accounted for. The presence of water vapour as a necessary condition

* Traube, 'Ber.,' vol. 15, pp. 2325, 2854, Remsen and Keiser, 'Ber.,' vol. 17, p 83

follows. So, too, the presence of carbon monoxide is explained. The peculiar effect of heat might, perhaps, be as follows Near the hot metal, as has been said, the particles of palladium are not charged with hydrogen, and hence not in a condition to bring about the union of oxygen and carbon monoxide So, too, the effect of strongly heating the tube might be accounted for Heating the tube may also have the effect of expelling hydrogen from the disintegrated palladium deposited on the sides of the tube, since an excess of hydrogen interferes with the reaction just described

Whether this luminosity is directly connected with the ionisation of gases by incandescent metals is a point which has not been investigated The writer hopes to make some experiments with a view to ascertaining if any special electrical effects can be detected in connection with this glow. The general laws concerning the electrical phenomena due to incandescent metals has already received a good deal of attention, but much remains still to be cleared up Many of the facts described in this paper suggest some connection with the phenomenon of ionisation, and it seems very probable that the presence of the ions from the hot palladium in the gas may be an important factor in the reactions which give rise to the luminosity Richardson* found that a wire which had lost the power of giving a positive leak owing to having been heated in incandescence can be made to regain it by being made a cathode in an electric discharge through gas at a low pressure, or by being placed near a cathode. This fact is recalled by the influence of the hot palladium on a piece of platinum placed near it, which we have described In the latter case the phenomenon appears to be either entirely due to the deposit of particles of the disintegrated metal or to be connected with such a deposit. The recovery of palladium owing to its exposure to air has also its counterpart in the case of the recovery of the power of ionisation In connection with these phenomena, a result may be mentioned here which is of interest, though no direct connection with the nature or cause of the glow has been found.

Another Effect—An experiment was made in which the piece of palladium foil mounted to the leads was made the cathode of an electric discharge The palladium was cold during the discharge When the discharge was stopped the current was sent through the foil Before the foil got red hot a slight blue-white luminosity was seen at its surface for an instant On stopping the current and turning it on a second time this did not appear The current was now regulated so that it was able to heat the foil just up to a red heat, but not strong enough to make it visible even in a very dark room. This current was stopped and a discharge from the coil sent through

* *Cf.* Richardson, 'Phil. Mag.,' (6), vol. 8, 1904, and (6), vol 9, 1905

the tube for a minute or two When this was stopped the wire was heated and at once the luminosity was seen This only lasted a few seconds, and did not reappear when the wire was heated a second time The phenomenon could be reproduced by again sending the discharge This result has evidently a connection with the phenomenon of thermo-luminescence, but the two do not seem to be identical The latter phenomenon is only known in connection with bodies called by Van't Hoff "solids and solutions,"* which are obtained by precipitating simultaneously two salts from a solution in which one is greatly in excess of the other Here there does not appear to be any salt in question, though it is possible that the effect is analogous and is due to the presence of impurities in the metal A piece of platinum foil which had been carefully cleaned in nitric acid did not produce this result On the other hand, it was observed when a "lime cathode" was used It is not necessary that the palladium should be itself the cathode The effect was also observed when it was in the luminous discharge from other electrodes Even when the palladium was itself the cathode the effect was not noticed except when the luminous discharge was about it The result was noticed also when the palladium was anode, but the coil discharge was not unidirectional although there was a spark gap in the circuit This effect has not yet been examined in any detail, and the subject is only introduced as being connected with palladium.

It has been assumed that carbon monoxide gas is present in the tube: in any case its presence is indicated by its spectrum The spectrum of carbon monoxide is always shown when carbon dioxide is introduced into the tube, and it appears to be due to the fact that the carbon dioxide is decomposed into oxygen and carbon monoxide, which then recombine again to form carbon dioxide.

This research was carried out in the Cavendish Laboratory, and the writer is very grateful to Prof Thomson for his many helpful suggestions and encouragement.

[*Note added May* 30.—Since this paper was read it has been possible, through the kindness of Mr J E Purvis, St John's College, to obtain a photograph of the spectrum of the glow The photographic record shows only one band, the position of which agrees with the measurements given in the paper. A photograph of the spectrum of a CO tube taken on the same plate shows that the blue band which constitutes the spectrum of the glow corresponds to the 4380 band of the spectrum of carbon monoxide When

* Van't Hoff, 'Zeit f Physik Chem,' p 322, 1895; E. Wiedemann and Schmidt, 'Wied Ann.,' vol. 54, p. 604, 1895

the negative is examined with a magnifying glass it is possible to distinguish several bright lines in the band which correspond to lines in the carbon monoxide spectrum With an exposure of nearly half an hour no record was obtained of the other band referred to in the text The single blue band seemed to constitute the spectrum under ordinary circumstances It may be possible to obtain further evidence by using extremely sensitive plates and longer exposure The photograph of the glow spectrum is not sufficiently distinct for reproduction, though quite clear enough for comparison with other photographs]

DESCRIPTION OF PLATE

FIG 1 —Sketch of tube employed to show the glow

FIG 2 —Shows general appearance of the glow The central white area is due to the glare of the hot palladium Owing to halation the outline of the piece of foil cannot be distinguished Around this is the "dark space" This is only relatively dark All round is a hollow shell of luminosity, so that the photograph is taken through this. Then comes the glow proper, which gradually reaches a maximum and fades away again The outline of the bulb is just visible below the glow.

FIG 3 —Shows the glow with an opaque object in front of it to remove halation effects The dark space can be seen

FIG 4 —Was taken with a larger aperture when the glow was more diffuse, and with a longer exposure The temperature of the palladium was lower than in the other cases, and the dark space is not seen

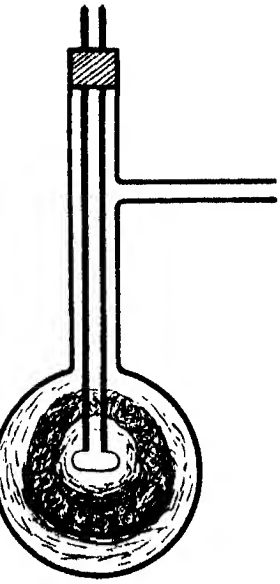


FIG. 1.

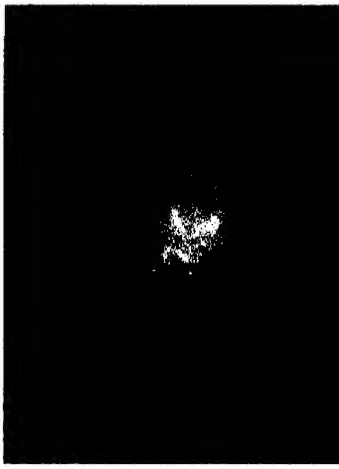


FIG. 2.

The Decay of Surface Waves produced by a Superposed Layer of Viscous Fluid

By W J Harrison, B A, Fellow of Clare College, Cambridge

(Communicated by Prof J Larmor, Sec R S Received May 13,—Read June 17, 1909)

§ 1 In a paper published recently by Prof R W Wood, on "The Mercury Paraboloid as a Reflecting Telescope,"* he notes the complete damping of ripples in the mercury surface produced by a thin superposed layer of glycerine It has been suggested to me by Prof Larmor that an interesting approximate estimate of the effect to be expected might be obtained from the results in my paper "The Influence of Viscosity on the Oscillations of Superposed Fluids"† In the case of a film of oil on the surface of water, in which the physical circumstances are different on account of the extreme tenuity of the film, Prof Lamb has made an approximate calculation of the effect in his Treatise on Hydrodynamics,‡ by supposing the surface to be inextensible for small wave-lengths

In my paper referred to, the approximations were obtained on the supposition that the viscosity in both liquids was small—the amplitude of the waves being as usual very small In the present case, although the viscosity in mercury is very small, that in glycerine is comparatively large Hence a new approximation is now required

§ 2 The characteristics of the wave-motion at the free surface of a single liquid of great depth have been obtained for the two cases of small and of great viscosity respectively In the second case there are two modes of vibration, both being dead-beat, for which the moduli of decay are $2k\nu/q$ and $(0\,91\nu k^2)^{-1}$, respectively § For application to glycerine the viscosity is not sufficiently great to make the approximation valid, except for wave-lengths less than 1 cm I have thought it instructive to give this case a special consideration, although it is a side issue, as it affords an example of a dead-beat type of vibration changing to a propagated type as the wave-length is increased

* 'The Astrophys Journ,' vol 29, No. 2

† 'London Math Soc Proc,' Ser 2, vol 6, p 396

‡ 3rd Edition, p 571

§ Lamb, 'Hydrodynamics,' 3rd Edition, p 567.

For the type of wave-motion with which we are concerned, the velocity system (u, v) is given by

$$u = \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y}, \qquad v = \frac{\partial \phi}{\partial y} - \frac{\partial \psi}{\partial x},$$

where

$$\phi = A e^{ky} e^{i(kx + \alpha t)},$$

$$\psi = H e^{\lambda y} e^{i(kx + \alpha t)},$$

$$\lambda^2 = k^2 + \alpha/\nu,$$

ν is the kinematical coefficient of viscosity and is equal to μ/ρ

Writing $\alpha/k^2\nu = \tan^2\theta$, the period equation can be put into the form

$$\tan^4\theta + 4\tan^2\theta + g/k^3\nu^2 = 4(\sec\theta - 1)\ * \qquad (1)$$

When ν is great the term $g/k^3\nu^2$ can be neglected in the determination of one mode, and the equation reduces to

$$\sec^3\theta + \sec^2\theta + 3\sec\theta - 1 = 0 \qquad (2)$$

We obtain an admissible root $\sec\theta = 0\,3$, whence $\alpha = -0\,91\nu k^2$

For glycerine, $\nu = 9\,64$, hence the term neglected above must be retained when the wave-length is greater than 1 cm

Writing $y = \sec\theta$, the equation (1) becomes

$$y^4 + 2y^2 - 4y + 1 + g/k^3\nu^2 = 0 \qquad (3)$$

It can be solved by Descartes' method, which leads to the cubic

$$K^6 + 4K^4 - 4gK^2/k^3\nu^2 - 16 = 0 \qquad (4)$$

The two values of y, which are admissible, are obtained from the equation

$$y^2 - Ky + (1 + \tfrac{1}{2}K^2 - 2/K) = 0, \qquad (5)$$

where K^2 is the positive root of (4), and K itself is taken positively

It will be seen from the table given below that the rate of decay is always great, but that waves of length greater than a certain limit, which lies between 2 and 3 cms, are propagated

When the wave-length is very great, one root of (4) is given by

$$K^2 = -2 + s^{\frac{1}{3}}, \quad \text{where} \quad s = 4g/k^3\nu^2,$$

whence

$$y^2 - \{-2 + s^{\frac{1}{3}}\}^{\frac{1}{2}} y + \tfrac{1}{2}s^{\frac{1}{3}} = 0$$

We obtain

$$\alpha/k^2\nu = -2 \pm \tfrac{1}{2}\iota s^{\frac{1}{3}},$$

or

$$\alpha = -2k^2\nu \pm \iota\sqrt{gk}$$

Thus α has the same form when νk^2 is small as we obtain in the case of a slightly viscous liquid.

* *Cf* Basset, 'Hydrodynamics,' vol. 2, p 312

Decay of Wave-motion on Glycerine.

L	sec θ	$\alpha/k^2\nu$	τ.	v
1	0 3	−0 ·91	0 ·003	
	0 ·99	−0 02	0 131	
2	0 43	−0 815	0 ·013	
	0 89	−0 208	0 ·051	
3	0 715±0 347i	−0 61± 0 496i	0 ·088	10 ·0
5	0 92 ±0 877i	−0 92± 1 61i	0 ·071	19 6
10	1 65 ±1 45i	−1 40± 6 ·05i	0 188	86 ·7
20	2 ·95 ±3 06i	−1 67±18 ·0i	0 ·629	52 ·9
30	4 06 ±4 18i	−1 96±38 1i	1 18	76 ·9

L is the wave-length in cms τ the modulus of decay in seconds
v the velocity of propagation in cms per second

§ 3 In the general problem we shall consider the motion of two fluids, both of depth large compared with the wave-length of the motion, in the two cases (1) both very viscous, (2) the lower one slightly viscous, the upper very viscous

The period equation for waves at the interface between a liquid of density ρ and viscosity ν, and a superposed liquid of density ρ' and viscosity ν', is*

$$4k^3(\nu\rho-\nu'\rho')^2(k-\lambda)(k-\lambda')+4k^2\alpha(\nu\rho-\nu'\rho')[\rho(k-\lambda')-\rho'(k-\lambda)]$$
$$+\rho^2(\alpha^2+gk)(k-\lambda')+\rho'^2(\alpha^2-gk)(k-\lambda)$$
$$-\rho\rho'[2k\alpha^2+\alpha^2(\lambda+\lambda')+gk(\lambda-\lambda')]=0,$$

where $$\lambda^2=k^2+\frac{\alpha}{\nu}, \qquad \lambda'^2=k^2+\frac{\alpha}{\nu'},$$

Viscosities both Large.

§ 4 (*a*) For the mode in which α is large, assuming that α is of the order ν or ν', the equation determining α is

$$4k^3(\nu\rho-\nu'\rho')^2(k-\lambda)(k-\lambda')+4k^2\alpha(\nu\rho-\nu'\rho')[\rho(k-\lambda')-\rho'(k-\lambda)]$$
$$+[\rho^2(k-\lambda')+\rho'^2(k-\lambda)-\rho\rho'(2k^2+\lambda+\lambda')]\alpha^2=0$$

The analysis is much simplified by putting $\nu=\nu'$. With this assumption, after writing $\alpha=\nu k^2\tan^2\theta$, we derive the equation

$$(\rho+\rho')^2\sec^2\theta+(\rho^2-6\rho\rho'+\rho'^2)\sec^2\theta+(3\rho^2-2\rho\rho'+3\rho'^2)\sec\theta-(\rho-\rho')^2=0. \qquad (6)$$

With this may be compared the equation (2) for a single liquid.

* 'London Math. Soc. Proc.,' Ser 2, vol 6, p. 398 I have quoted it again at length as there are two misprints as it has been previously printed.

Some numerical solutions of (6) are given in the table below:—

ρ'/ρ .	0	0·1	0 5	1
sec θ	0 29	0 27	0 097	0
$\alpha/\nu k^2$	−0·91	−0·93	−0 98	−1

When ν and ν' are different, we can safely say that the modulus of decay is not greater than $(0\,91\nu_1 k^2)^{-1}$, where ν_1 is the smaller of the two coefficients

(b) For the mode in which α is small, we assume that α is of the order ν^{-1} We easily find that

$$\alpha = -\frac{g(\rho-\rho')}{2k(\rho\nu+\rho'\nu')} \tag{7}$$

This corresponds to $\alpha = -g/2k\nu$ for a single liquid

Upper Liquid Viscous, Lower Slightly Viscous.

§ 5 (a) For the mode in which α is large, writing $\alpha/k^2\nu' = \tan^2\theta$ in the terms of highest importance, we obtain the equation (*cf* 6)

$$(\rho+\rho')\sec^3\theta+(\rho+\rho')\sec^2\theta+(3\rho'-\rho)\sec\theta-(\rho+\rho') = 0 \tag{8}$$

The nature of the solution of this equation is shown by the following table —

ρ'/ρ	0 1	0 5	0 9
sec θ	0·92	0 675	0 565
$\alpha/\nu k^2$	−0 15	−0 544	−0 68

(b) For the mode in which α is small we have

$$\alpha = -\frac{g(\rho-\rho')}{2k\nu'\rho'} \tag{9}$$

This can be obtained from (7) by putting $\nu = 0$

§ 6 *The Case of a Thin Layer of Glycerine superposed on Deep Mercury* — The period equation for this case is given in § 4 of my paper (in the first term of the fourth column ρ should be replaced by ρ'). The effect of capillarity can be included by writing $g+T_2k^2/\rho'$ instead of g in the second column, and $g+T_1k^2/\rho$ instead of g in the fifth and sixth terms of the fourth column, where T_1 is the surface tension of the glycerine-mercury interface, and T_2 is the surface tension of the glycerine free surface.

For glycerine at 17°, C. $\nu' = 9\,64$, and for mercury, $\nu = 0{\cdot}0011$, conse-

quently we shall approximate by putting $\nu = 0$ at once. This greatly simplifies the period equation, and does not affect its validity except when the modulus of decay is very large It will be further assumed that kh and $\lambda' h$ are small, so that squares of these quantities will be neglected.

With these assumptions the period equation becomes

$$\begin{aligned}&(\rho+\rho' kh)\,\alpha^4+4k^2\nu'(\rho+\rho' kh)\,\alpha^3\\&+[g\rho(1+kh)+k^2(T_1+T_2)+k^3hT_2\rho/\rho']\,k\alpha^2\\&+[g\rho+k^2(T_1+T_2)]\,4k^3\nu'\alpha\\&+k^3h(g+k^2T_2/\rho')[(\rho-\rho')g+k^2T_1]=0\end{aligned}$$

For wave-lengths less than a certain limit this equation has two negative roots and two complex ones. The two complex roots correspond to the mode in which the two liquids vibrate as a single liquid without slip at their common surface The two negative roots show that the waves of the other mode are dead-beat

It appears from the numerical calculation, in the case of $h = 1$ mm, that very approximately the two negative roots are obtained from the last two and the first two terms of the equation respectively Thus they are

$$\alpha_1 = -\frac{(g+k^2T_2/\rho')[g(\rho-\rho')+k^2T_1]}{4\nu'[k^2(T_1+T_2)+g\rho]},$$

$$\alpha_2 = -4k^2\nu'$$

These approximations are not, however, sufficiently good to be employed in the determination of the two remaining roots, as they necessarily make the real part of these roots to be positive, equal to $-\frac{1}{2}\alpha_1$.

For wave-lengths greater than the limit mentioned above all the roots are complex, and all the waves are propagated. This limit is given, in general, by the condition for equality of the roots of the last three terms of the equation equated to zero

For a glycerine-air surface, $T = 65$, for a mercury-air surface, $T = 540$ It is assumed for the purpose of an illustrative calculation that $T_1 = 300$ at the common surface, while $T_2 = 65$. In the table below the effect is exhibited of a layer of glycerine of 1 mm depth on the modulus of decay and velocity of propagation of wave-motion at the surface of mercury

The damping effect of a thin layer of glycerine is very noticeable from the table. The rate of decay of any arbitrary disturbance will depend, in general, on the rates of decay of those of the modes, into which it can be analysed, which are propagated. The other modes disappear instantaneously. As a consequence of this the curvature of the upper glycerine surface is always the same as that of the mercury surface, a fact which may be of importance in

the application to an optical instrument such as the one which gave rise to this investigation. Prof Wood made use of a depth of 4 mm of glycerine, a layer of this depth would give a much greater rate of decay.

L	τ	τ_c	τ_0	v	v_c	v_0
2	0 435	0 291	45 5			20 7
	0 ·0026*	0 0027*				
	10 ·87	5 09		17 5	19 7	
5	0 417	0 402	303			28 8
	0 017	0 ·017				
	12 ·99	10 03		28 1	28 ·9	
10	0 352	0 354	1100			39 8
	0 081	0 ·082				
	39 ·95	27 08		39 5	40 ·0	
20	0 518	0 518	5000	7 19	7 29	54 1
	†	†		54 1	54 1	

L is the wave-length in cms

τ is the modulus of decay when capillarity is neglected; τ_c that when capillarity is included; τ_0 that of waves at the surface of clean mercury (T = 540)

v, v_c, v_0 are the velocities of propagation in the corresponding cases

* In the case of these quantities the approximation, on the supposition that λ'/h is small, breaks down; but the rate of decay is in any case large

† The rate of decay cannot be determined without great labour

If the wave-length is small compared with the depth, the rates of decay can be derived from (8) and (9). These approximations are applicable to glycerine, as the wave-length is small The moduli of decay for the two modes are $(1\ 3k^2)^{-1}$ and $(491k)^{-1}$; both of these are extremely small.

In these investigations the damping effect of the air has been neglected. That this is very considerable in the case of water waves has been shown in my paper, to which reference has been made, but owing to the great density of mercury its effect will be small, especially for small wave-lengths, such as those tabulated. Its effect in the case of dead-beat modes is absolutely negligible

It may be mentioned that the mercury used by Prof Wood was of comparatively small depth, this would cause considerable damping in the case of the longer waves

[*Note added June* 19.—It will be noticed from the foregoing table that the modulus of decay, when the vibrations are influenced by the glycerine layer, increases much more slowly with the wave-length than in the case of a clean mercury surface. This fact may go a little way towards an explanation of the damping of waves on water by oil But a complete explanation is to be looked for in a totally different direction. Dr Houstoun notes an experimental result which is apparently contradictory to usual experience.*

* 'Phil. Mag,' January, 1909.

The modulus of decay (τ) for waves of length 5 cms. when the surface of the mercury is covered by 4 mm. of glycerine is 4·9 secs. All modes become propagated for wave-lengths greater than some value between 5 and 10 cms For a depth of 1 mm. of glycerine this limit lies between 10 and 20 cms., and thus we see that as the depth is increased this critical value of the wave-length approaches that for waves at the surface of deep glycerine, which lies between 2 and 3 cms]

The Elastic Limits of Iron and Steel under Cyclical Variations of Stress.

By LEONARD BAIRSTOW, A.R.C.Sc., Wh. Sch

(Communicated by Dr Glazebrook, F.R.S Received April 20,—Read May 13, 1909)

(Abstract)

An explanation of fatigue is developed in the paper which is in accordance with all the researches on the fracture of materials by the cyclical repetition of stress The theory was put forward by Bauschinger in 1886, when he suggested that the necessary condition of safety was that the repeated stresses applied should be within the limits of elasticity of the specimen, and that the least variation from this condition introduces fatigue and ultimately fracture occurs

In order for this to be true for Wöhler's well-known experiments, the elastic limits must be variable within very wide limits, and the present paper describes observations made during the adjustment of the limits of elasticity to any particular condition of experiment

It is now found that iron or steel is capable of adjusting itself to variations of stress, cyclically applied, after a sufficient number of repetitions When the adjustment is complete, the specimen under test is found to have become perfectly elastic throughout the whole cycle, and fatigue does not occur.

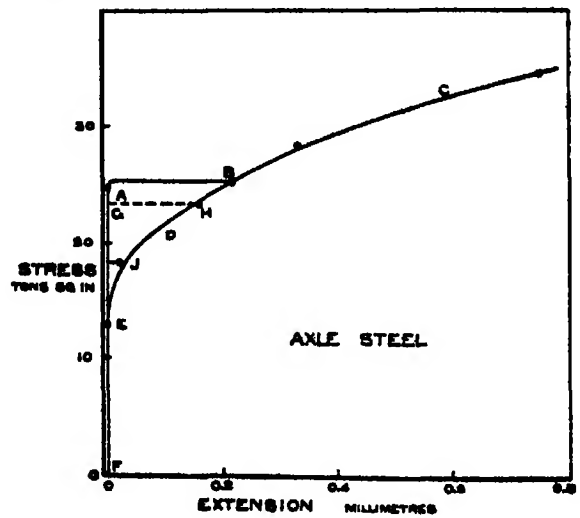
During the adjustment of the elastic limits to a given cycle of stress, a change of length occurs in the specimen, which corresponds to the extension observed in an ordinary tensile test when the yield stress is exceeded. For stress cyclically applied, this extension occurs even when the maximum stress in the cycle is less than the static yield stress

The greater the extension of the specimen during adjustment, the greater is the amount by which the elastic limits are raised.

Limits can be found to this power of adjustment, and if the cycle of stress imposed exceeds these limits, the specimen becomes or remains inelastic and work is absorbed during each cycle This work is expended in moving portions of the crystals of the material relatively to one another and is probably associated with microscopic slip lines, which gradually develop into cracks, and ultimately cause the fracture of the specimen

As the elastic limits of a new specimen are in suitable positions for reversals of equal and opposite stresses, and consequently do not need adjustment, fracture occurs without any appreciable extension

The figure illustrates the relation of the yield, produced by cyclical varia-



tions of stress, during adjustment of the elastic limits, to the extension in an ordinary tensile test The ordinates are the maximum stresses applied to the specimen and the abscissæ the corresponding permanent extension. Starting with a new specimen, the line FEA shows that, at a stress of 25 tons per square inch, no permanent extension was observed When the load was slightly increased, a sudden extension of about one-fifth of a millimetre occurred, this being the well-known yield Further increase of stress extended the specimen still further, the changes being represented by a line which cannot differ appreciably from BC.

In producing the curve FEABC, no cyclical variations of stress are concerned, and the curve is identical with the usual stress elongation diagram frequently taken during a tensile test.

An experiment on a specimen of axle steel showed that, under cyclical variations of stress, an extension which was not measurable at the first application of the load gradually appeared, due to repeated applications of a range of stress slightly greater than the safe range This extension continued for some time, any point on the line GH representing the extension at some particular time When the adjustment of the elastic limits was complete, H represented the final extension, and no further extension occurred due to further repetitions of stress

The point J was similarly obtained by repeating a cycle of stress having a less maximum value than that which produced the extension H. At E, which corresponds to the maximum safe stress during reversals, no extension occurred.

The points H, J, E are evidently on a continuation of the curve BC, and when cyclical variations of stress are considered, there is no break in the curve at B corresponding to the static yield point

Above this point the whole extension is produced by the maximum stress only, independently of the range of stress, which may be zero. As HJE is continuous with BC, it seems possible that an extension such as GH may be produced by the repetition of a cycle of stress in which the range is less than the safe range

Below the static yield point, iron and steel appear to be capable of maintaining their initial condition for a considerable time against cyclical variations of stress which ultimately produce a considerable change of length The first application of the maximum load in a given cycle of stress may show only a scarcely measurable extension, in spite of the fact that an extension two or three thousand times as great as the permanent extension in the first cycle is necessary before stability is reached

When the extension produced by a large number of cycles is itself small, *i e*, when the maximum stress in the cycle is not greatly in excess of the natural elastic limits, it does not seem surprising that the effect of the first cycle is not measurable and that ordinary determinations of the elastic limit fail to detect changes, which nevertheless are there

The Ionisation produced by an α-Particle.—Part I.

By H Geiger, Ph D., John Harling Fellow of the University of Manchester.

(Communicated by Prof E. Rutherford, F.R S Received May 19,—
Read June 17, 1909.)

Using an electrical method, Prof Rutherford and myself* were recently able to determine accurately the number N of α-particles which are expelled from a gramme of radium per second The final value of N obtained as an average of a great number of observations was $3\cdot4\times10^{10}$ α-particles per second from a gramme of radium itself, or four times this number if the radium is in equilibrium with its three α-ray products In another paper† the charge carried by an α-particle was measured by the same authors and found to correspond to $9\cdot3\times10^{-10}$ E S unit Since recent experiments have given conclusive evidence that an α-particle is identical with an helium atom carrying twice the ionic charge, it was necessary to take the ionic charge as $4\cdot65\times10^{-10}$ E.S unit.

The values of N and e as found from the above experiments enable us to determine the number of ions which are produced by an α-particle along its whole path with a greater accuracy than hitherto A determination of the number of ions produced by an α-particle emitted from radium itself was made in 1905 by Rutherford‡ in the following way. The ionisation current due to a thin film of radium was measured at its minimum activity, and the total number of α-particles fired off from this film was calculated from the total charge which the α-particles carried with them Taking the charge on an α-particle as equal to twice the ionic charge e, the number Z of ions produced by an α-particle from radium itself was found to be $1\cdot72\times10^5$. This number becomes $1\cdot18\times10^5$ if for N and e the latest values, referred to above, are introduced

It was thought advisable in the present experiments to use RaC as the source of α-rays The advantages of the active deposit of radium as a source of α-rays has been discussed in some detail in a previous paper. About a quarter of an hour after removal from the emanation the active deposit gives off homogeneous α-rays due to the radium C present and the number of α-particles fired off per second at any time after removal from the emanation can be calculated with great accuracy from the γ-ray activity The simplest way to determine the number Z of ions produced by an α-particle would be

* 'Roy Soc. Proc.,' A, vol. 81, p 141, 1908
† 'Roy Soc Proc.,' A, vol. 81, p. 162, 1908.
‡ 'Phil Mag.,' vol 10, p. 193, 1905

to measure the quantity of RaC deposited on a plate, and at the same time to measure the saturation current due to the complete absorption of the whole number of α-particles expelled from the active plate From these measurements the number Z could at once be deduced.

Preliminary experiments, however, showed that it was impossible to determine Z to the desired accuracy in this way Bragg and Kleeman* have already drawn attention to the difficulties of obtaining saturation currents when a gas is ionised by α-rays at atmospheric pressure Under conditions when practically complete saturation for ionisation due to β- or γ-rays is produced, a current of the same intensity, but due to the α-rays, may be 10 or 20 per cent below the saturation value. To explain the observed effect, Bragg and Kleeman assume that the ions newly formed by an α-particle are specially liable to recombine A much more intense field is therefore required to separate them. The effect of "initial recombination" is stronger in a complex gas than in air, and it decreases rapidly as the pressure is lowered Further, it depends upon the velocity of the α-particle which produces the ions. The smaller the velocity of the α-particle, the greater the tendency of the newly formed ions to recombine †

On account of the difficulties of obtaining complete saturation under the experimental conditions, it was found necessary to adopt an indirect method for the determination of Z This method is briefly described below

The ionisation due to the whole number of α-particles expelled from a known quantity of RaC was measured at a low pressure, allowing only a small definite portion of the range of each α-particle to be effective. The ratio of the ionisation produced within that small portion of the range to the ionisation produced along the whole path was then determined by another experiment.

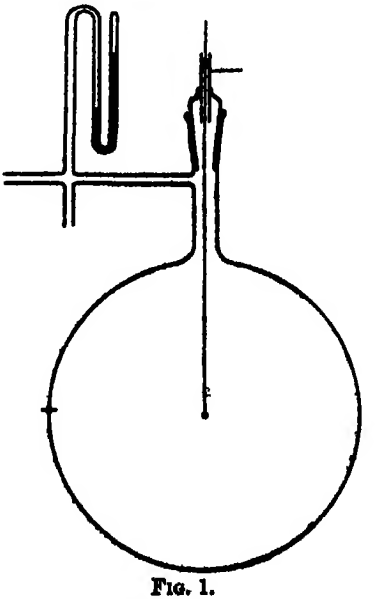


Fig. 1.

As regards the first part of the experiment, the measurements were carried out in the following way.—The amount of RaC deposited on a small metal plate (about 3 mm. square) was determined carefully by the γ-ray activity.

* 'Phil. Mag.,' vol. 11, p 466, 1906.

† Kleeman, 'Phil. Mag,' vol. 12, p. 273, 1906.

The plate was then suspended by a fine wire exactly in the centre of a glass bulb, as seen in fig 1 The internal diameter of the bulb was 15 9 cm., and the inside surface was silvered. By means of a thin platinum wire sealed through the glass, the inside could be charged to any desired potential. The wire and the plate attached to it were connected with an electrometer of the Dolezalek type A condenser of 0 1 microfarad was placed in parallel with the electrometer As soon as the active plate was fixed in position the pressure inside the bulb was reduced to a few centimetres and accurately measured. In most of the experiments the pressure was adjusted to 3 73 cm. of mercury. Since the range of an α-particle is inversely proportional to the pressure, and each α-particle expelled from the active plate in the centre had to travel through 7·95 cm of air at a pressure of 3·73 cm., only the first $\frac{7{\cdot}95}{76} \times 3\,73 = 0\,390$ cm. of the range of each particle was effective. The ionisation current was measured for different intensities of the electric field in order to test the degree of saturation. At such a low pressure and using only small amounts of active deposit on the plate, saturation was easily obtained This may be seen from the following figures :—

Silvered surface charged to—	Rate of movement of electrometer needle	
volts	div /sec	
40	1 51	The activity of the plate corresponded to $5\ 3 \times 10^5$ α particles per sec Corrections are made for the decay
80	1 56	
150	1 64	
330	1 65	
580	1 64	

After the measurements of the ionisation current had been taken, the activity of the plate was again determined as before A series of measurements was taken in this way Before, however, giving the numerical results, we shall first consider the method by which the ratio of the ionisation produced in the known small portion of the path of the α-particle to the ionisation along the whole path was determined.

The particular shape of the curve which represents the ionisation of an α-particle at different points of its path is well known from the experiments of Bragg and Kleeman* and Bragg.† Using the α-rays from a film of radium, the authors showed that the ionisation produced by an α-particle, per centi-

* Bragg and Kleeman, 'Phil. Mag.,' vol. 8, p. 726, 1904.

† Bragg, 'Phil. Mag.,' vol 10, p. 318, 1905.

metre of path, at first increases with the distance traversed, i.e. increases with decrease of velocity of the α-particle. After passing through a maximum, the ionisation diminishes rapidly The same result was obtained by McClung,* who used the active deposit from radium emanation, which gives off homogeneous α-rays Curves of the same character were obtained by Hahn† for the products of thorium and actinium.

For the present investigation, as has already been pointed out, it was necessary to determine quantitatively the change in ionising power along the path of the α-particle. In devising the experimental arrangement it was thought advisable to attempt to satisfy the following conditions —

(1) To use a practically parallel pencil of homogeneous α-rays

(2) To use an ionisation chamber of very small depth.

(3) To obtain saturation by taking the measurements of the ionisation current at reduced pressure

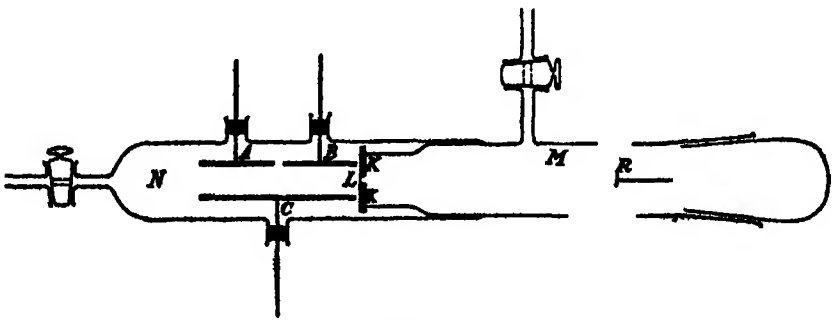


FIG. 2.

The details of the apparatus may be seen from fig. 2. An amount of RaC corresponding in γ-ray activity to about 2 milligrammes $RaBr_2$ was deposited on a polished glass disc of 0·6 cm. diameter. This disc R was placed in position in the centre of the glass tube M at a distance of 10 to 20 cm from the lead plate K, which covered the end of the tube. A fraction of the α-particles expelled from the RaC passed through the opening L of 1·5 mm. diameter, bored through the centre of the lead plate. The opening itself was made airtight by a thin sheet of mica, the thickness of which corresponded to 0·92 cm. of air After passing through the mica window the α-particles entered the ionisation chamber N. This consisted of two insulated plates A and B, both parallel to the plate C and distant 1 cm from it. The plate C and the lead plate K were charged to the same potential by means of a battery, the plate B being connected to the electro-

* McClung, 'Phil Mag,' vol. 11, p. 131, 1906.

† Hahn, 'Phil. Mag.,' vol. 11, p. 793, 1906, and vol. 12, pp. 69 and 244, 1906.

meter, while A served as guard plate to ensure that the current reaching B was due only to the ionisation between the plates B and C. In a few experiments the ionisation vessel was filled with air at a low pressure, but in most of the experiments hydrogen was used at a pressure varying from 10 to 20 cm, since saturation is obtained most easily in this gas Under these conditions a potential difference of 25 volts was sufficient for saturation. The depth of the ionisation vessel corresponded to 0·07 to 0 14 cm. of air at atmospheric pressure

Measurements were taken in two different ways

In some experiments the pressure in the tube M was adjusted to a certain noted value. Knowing this pressure and the distance of the disc R from the opening L, the exact portion of the path of the α-particles, which was producing ions between B and C, could easily be calculated After the ionisation current had been measured the pressure in M was changed and the current measured again Varying the pressure in this way within certain limits, the ionisation produced by an α-particle could be measured at different points of its path, in this case from 0 92 cm., which was the equivalent thickness of the mica window, to the end of the path

The second method of taking measurements was simpler A Bronson radio-active resistance was connected with the electrometer in order to get steady deflections The tube M was completely exhausted at the beginning of an experiment. The air was then allowed to run in slowly through a capillary tube The pressure of the gas at any time was found to be exactly proportional to the time of flow Thus the gas between B and C was ionised by successive parts of the path of the α-particles and the deflection of the electrometer needle varied as the ionising power of the particles In fact, the spot of light from the electrometer would trace out the ionisation curve on a photographic plate when moved with uniform velocity at right angles to the path of the light

Several curves were taken by the two methods, the gas pressure in the ionisation vessel being varied in the different experiments. The curves, however, differed only slightly up to 6·5 cm of the range. The maximum current corresponding to 6 5 cm. of the range varied somewhat, being in some experiments 10 to 15 per cent. higher than in others This difference, however, can only affect the final result to about 0·5 per cent.

Using the experimental arrangements described above, we cannot obtain the ionisation curve at the beginning of the path, since the initial 0·92 cm. of the range was taken up in traversing the mica window. The initial part of the curve, however, could readily be taken by using the vessel employed in the first part of the experiment and shown in fig. 1. A small amount of

active deposit was placed in the centre of the bulb and the ionisation current measured for different pressures Up to a pressure of about 15 cm. saturation was easily obtained For low pressures the ionisation current was found to be nearly proportional to the pressure, while for higher pressures the ionisation increased somewhat more rapidly than the pressure. The increase was found to be in agreement with the results obtained by the experimental arrangement as in fig 2 But from a pressure of about 20 cm (which is equivalent to about 2 cm of the range) the ionisation does not increase with the pressure so rapidly as the known ionisation curve would lead us to expect. This is obviously due to the lack of saturation at the higher pressures, even when large potentials are employed

The curve given in fig. 3 represents the average of all the measurements which have been taken It can readily be shown that the corrections to be

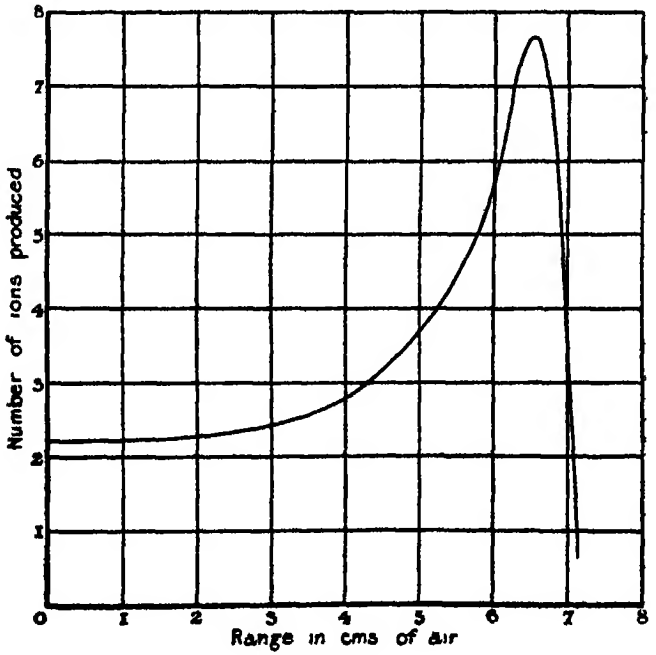


Fig 3

applied on account of the angle of the rays and on account of the depth of the ionisation chamber are exceedingly small, and do not appreciably affect the shape of the curve. It is thus clear that the ionisation due to a parallel pencil of α-particles travelling with identical velocity does decrease in the last 5 mm. of the range. Several possible explanations can be put forward to account for this diminution, but a discussion is reserved until some

investigations now in progress are completed. On the whole it appears probable that the effect is really due to the scattering of the α-particles in passing through the gas.

From the ionisation curve, fig 3, the ratio of the total ionisation produced by an average α-particle along the whole path to the ionisation produced within the first 0 390 cm of the range is found to be 27 1. This value is obtained from the ratio of the whole area of the curve to the area which is enclosed between the ordinates 0 and 0 390 From the ionisation produced within the first 0·390 cm. of the range, as measured in the first experiment, we can now calculate the whole number of ions produced by an α-particle from RaC. Since all the measurements were taken at practically the same room temperature, no correction for temperature was necessary The correction for the β-ray effect was found to amount to less than 0 5 per cent.

Special care was taken in the determination of the constants which were used in calculating the figures given in column 4 of Table 1 The condenser employed was compared with a standard condenser, the pressure gauge was carefully tested, and the sensibility of the electrometer frequently measured for different potentials

Table I

1 Activity of plate measured by γ rays	2 Number of α-particles expelled per sec	3. Ionisation current measured at a pressure of 3 ·78 cm	4 Whole number of ions produced by one α-particle
mg Ra		E S units.	
0 141	$2\ 40 \times 10^6$	9 88	$2\ 40 \times 10^5$
0 ·0326	$5\ 53 \times 10^5$	2 24	$2\ 36 \times 10^5$
0 ·0151	$2\ 56 \times 10^5$	1 ·04	$2\ 36 \times 10^5$
0 128	$2\ 18 \times 10^6$	8 88	$2\ 31 \times 10^5$
0 ·088	$1\ 41 \times 10^6$	5 ·88	$2\ 44 \times 10^5$

The average number of ions produced in air by an α-particle from RaC along its whole path may be taken, to the nearest figure, as

$$2{\cdot}37 \times 10^5.$$

The ionising power at different points of the path is illustrated by the following figures, which give the number of ions produced per millimetre at the respective points of the range All the figures refer to air at atmospheric pressure and temperature of 12° C.

The scale in fig. 3 is such that each square centimetre represents 10^4 ions. The number of ions produced within any part of the range can therefore be found at once.

Table II.

cm.	ions per mm.
At 1	2250
2	2300
3	2400
4	2800
5	3600
6	5500
6·5 (about)	7600
7 „	4000

There appears to be no simple relation between the ionisation and the velocity of an α-particle. Any attempt to connect them by a theoretical consideration must be delayed until further experiments have given an explanation of the end part of the curve

All the experimental evidence seems to show that the α-particles from the different radio-active substances are identical in mass and charge but differ only in their initial velocity * They all cease ionising when their velocity has diminished to the same value, *i.e* to $1{\cdot}5 \times 10^9$ cm/sec † It seems, therefore, justifiable to assume that all α-particles produce the same ionisation at the same velocity. Consequently the ionisation curves for different α-particles are identical for the same range of velocity Hence if the whole range of an α-particle in air is known, the total number of ions produced by it can be calculated from the curve given for RaC.

The correctness of the assumption was investigated for the α-particles from polonium by the following experiment. The ionisation current from a small disc coated with polonium was measured at a low pressure (3·73 cm) in the silvered glass bulb just in the same way as for RaC The number of α-particles emitted per second from the plate was determined by the scintillation method.‡ The plate was fixed in an exhausted glass tube about 10 cm. from a zinc sulphide screen and the number of scintillations produced on a square millimetre was counted by aid of a microscope. The efficiency of the screen was tested by counting the α-particles from a known quantity of RaC It was found that 92 per cent. of the α-particles which struck the screen produced scintillations. Applying this correction, the total number of α-particles expelled from the polonium plate per second was $4{\cdot}6 \times 10^4$.

The ionisation current due to the first 0·390 cm. of the range of all polonium particles was 0·120 E.S.U. The current due to a single polonium

* Rutherford, 'Phil. Mag,' vol. 12, p. 348, 1906; Rutherford and Hahn, 'Phil. Mag,' vol. 13, p. 371, 1906.

† Rutherford, 'Phil. Mag.,' vol 10, p. 183, 1905

‡ Regener, 'Verhdlg d. D Phys. Ges.,' vol. 10, p 78, 1908; Rutherford and Geiger, 'Roy. Soc. Proc.,' A, vol. 81, p. 141, 1908.

particle was therefore $0120/2\,3\times10^{4}=5{\cdot}2\times10^{-6}$ E S U The current produced by an α-particle from RaC measured under the same conditions is $4{\cdot}07\times10^{-6}$ E S U, and therefore the ratio of the two currents equals 1·28. Now from the ionisation curve (fig 3), this ratio ought to be 1·18, if the range of a polonium particle is taken as 3 86 cm The difference between the two values is within the experimental error, since the determination of the number of α particles from the polonium plate by the scintillation method involved an uncertainty of several per cent

In the following table the number of ions produced by the different α-particles from the radium family is calculated The calculations are based on the ionisation curve (fig 3) and the known range of the particles.

Table III

	Range	Total number of ions produced
	cm	
Radium	3 50	$1\ 53\times10^{5}$
Emanation	4 33	$1\ 74\times10^{5}$
Radium A	4 83	$1\ 87\times10^{5}$
Radium C	7·06	$2\ 37\times10^{5}$
Radium F	3 86	$1\ 62\times10^{5}$

It must be remembered that in calculating the above figures the charge on an ion is taken as $4\,65\times10^{-10}$ E S U If further investigation should lead to a more accurate value, these results can at once be corrected

The number calculated for radium itself [$1\,53\times10^{5}$] is in good agreement with the value obtained by Rutherford, considering that his number [$1\,18\times10^{5}$] must be increased by at least 10 per cent owing to the difficulties of obtaining saturation for an intense ionisation at atmospheric pressure

Note on the Determination of Small Quantities of Radium

The total ionisation current due to a gramme of radium at its minimum activity and spread out in an infinitely thin film on a plate so that one half of all α-particles are absorbed in ionising is :

$$1{\cdot}21\times10^{6}\ \text{E.S. units.}$$

This result may prove useful in estimating small quantities of radium But it must be remembered that the figures given refer to complete saturation.

Small quantities of radium or other radio-active substances may also be determined with great accuracy by measuring the ionisation current at a low pressure in a conducting bulb, which may be of smaller dimensions than the

one employed in these experiments. The number of ions produced by a single α-particle under the special conditions of the experiment is easily found from the curve given in fig 3 The determination of the ionisation current in the bulb then gives at once the total number of α-particles Care has to be taken to obtain saturation and to avoid ionisation by collision, which occurs when too large a voltage is applied

I wish to acknowledge the assistance which Mr E Marsden has given me in some of these observations.

In conclusion, I desire to express my gratitude to Prof Rutherford for his valuable suggestions and his kind interest in the experiments

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On a Diffuse Reflection of the α-Particles

By H Geiger, Ph.D., John Harling Fellow, and E Marsden, Hatfield Scholar, University of Manchester.

(Communicated by Prof E Rutherford, F R S Received May 19,—Read June 17, 1909)

When β-particles fall on a plate, a strong radiation emerges from the same side of the plate as that on which the β-particles fall. This radiation is regarded by many observers as a secondary radiation, but more recent experiments seem to show that it consists mainly of primary β-particles, which have been scattered inside the material to such an extent that they emerge again at the same side of the plate.* For α-particles a similar effect has not previously been observed, and is perhaps not to be expected on account of the relatively small scattering which α-particles suffer in penetrating matter †

In the following experiments, however, conclusive evidence was found of the existence of a diffuse reflection of the α-particles. A small fraction of the α-particles falling upon a metal plate have their directions changed to such an extent that they emerge again at the side of incidence. To form an idea of the way in which this effect takes place, the following three points were investigated :—

(I) The relative amount of reflection from different metals.

(II) The relative amount of reflection from a metal of varying thickness

(III) The fraction of the incident α-particles which are reflected

* See Schmidt, 'Jahrbuch der Radioaktivität und Electronik,' vol 5, p 471, 1908

† Rutherford, 'Phil. Mag.,' vol 12, p 143, 1906; H. Geiger, 'Roy Soc. Proc.,' A, vol 81, p. 174, 1908

For the observation of the reflected particles the scintillation method was used in all experiments. With regard to the details of the method we refer to the papers of Regener* and of Rutherford and Geiger†

On account of the fact that the amount of reflection is very small, it was necessary to use a very intense source of α-rays. A tube was employed similar to that which has been proved to be a suitable source in the scattering experiments of one of us‡ This source consisted of a glass tube AB (fig 1), drawn down conically and filled with radium emanation, the end B of the tube being closed airtight by means of a mica window. The thickness of the mica was equivalent to about 1 cm of air, so that the α-particles could easily pass through it.

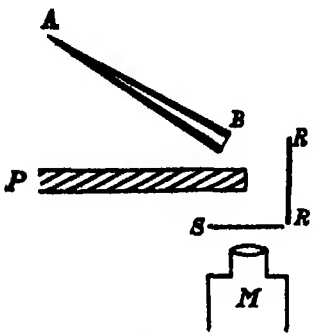

Fig 1

Since it is of importance that the gas pressure inside this tube should be as low as possible, the emanation was purified according to the methods developed by Prof Rutherford§ The tube contained an amount of emanation equivalent to about 20 milligrammes $RaBr_2$ at a pressure of a few centimetres. The number of α-particles expelled per second through the window was, therefore, very great, and, on account of the small pressure inside the tube, the different ranges of the α-particles from the three products (*i.e.* emanation, RaA, and RaC) were sharply defined

The zinc sulphide screen S (fig. 1) was fixed behind the lead plate P, in such a position that no α-particles could strike it directly When a reflector was placed in the position RR at about 1 cm from the end of the tube, scintillations were at once observed. At the same time the screen brightened up appreciably on account of the reflected β-particles.

By means of a low power microscope, the number of scintillations per minute on a definite square millimetre of the screen was counted for reflectors of different materials Care was taken that the different reflectors were always placed in exactly the same position

It is, of course, to be expected that the number of α-particles reflected from the plate would be different in different directions, and would also depend on the angle of incidence. In our arrangement, however, no appreciable difference was found for different angles This is due to the fact that,

* 'Verh d. D Phys. Ges.,' vol. 10, p. 78, 1908.
† 'Roy Soc Proc.,' A, vol. 81, p. 141, 1908.
‡ Geiger, 'Roy. Soc Proc.,' A, vol. 81, p. 174, 1908
§ 'Phil Mag.,' August, p. 300, 1908.

owing to the necessity of having the tube very near to the reflector, the angle of incidence varied very much An investigation of the variation of the effect with the angles of incidence and emergence would necessitate a parallel and very intense source of homogeneous α-rays, which can, however, not easily be realised

In the following table the number of scintillations observed per minute are given in column 3, in column 4 the ratio to the atomic weight is calculated, and it can be seen that this ratio decreases with decreasing atomic weight. The case of lead appears to be an exception which may be due to slight impurities in the lead.

1. Metal	2 Atomic weight, A.	3 Number of scintillations per minute, Z	4 A/Z
Lead	207	62	30
Gold	197	67	34
Platinum	195	63	33
Tin	119	34	28
Silver	108	27	25
Copper	64	14 5	23
Iron	56	10 2	18·5
Aluminium	27	3 4	12 5

Even in the absence of any reflector about one scintillation per minute was observed It was easy to show that this was due to a reflection from the air through which the α-particles passed. The numbers on the table are corrected for this effect

It is interesting to note here that for β-particles the number of reflected particles also decreases with the atomic weight of the reflector * But while for β-particles the number reflected from gold is only about twice as great as for aluminium, for α-particles the same ratio amounts to about twenty

(II) We have already pointed out that the diffuse reflection of the α-particles is a consequence of their scattering According to this point of view, the number of particles reflected must vary with the thickness of the reflecting screen. Since gold can be obtained in very thin and uniform foils, different numbers of these foils were used as reflectors. Each foil was equivalent in stopping power to about 0·4 mm. of air. It was necessary to mount the foils on glass plates, but the number reflected from the glass itself was found to be very small compared even with the number from one gold foil. The curve, fig 2, gives the result of the measurements.

* McClelland, 'Dublin Trans.,' vol. 9, p. 9, 1906.

The number of scintillations which were due to the reflection from the air is subtracted from each reading The first point on the curve represents the number of scintillations observed for a glass plate alone as reflector, the last point (marked 30) gives the number of scintillations when a thick gold plate was used

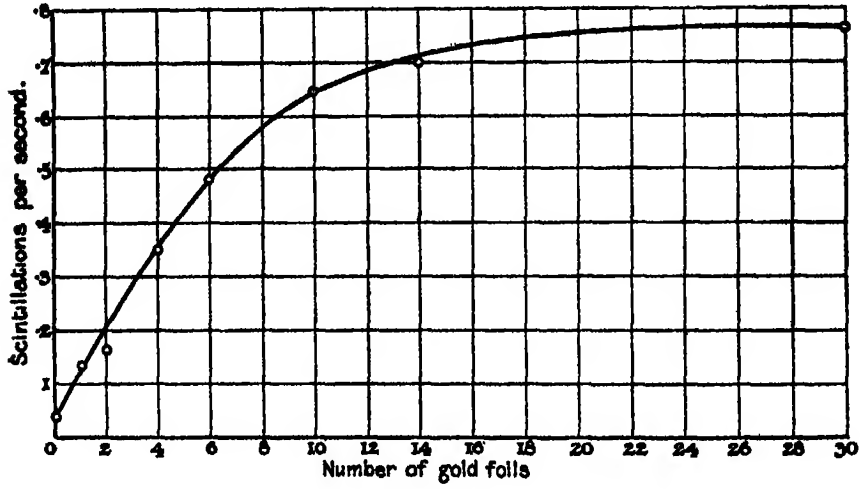

Fig 2

The curve is similar to those which have been obtained for the reflection of the β-particles.* It brings out clearly that the reflection is not a surface but a volume effect

Compared, however, with the thickness of gold which an α-particle can penetrate, the effect is confined to a relatively thin layer In our experiment, about half of the reflected particles were reflected from a layer equivalent to about 2 mm. of air. If the high velocity and mass of the α-particle be taken into account, it seems surprising that some of the α-particles, as the experiment shows, can be turned within a layer of 6×10^{-5} cm of gold through an angle of 90°, and even more To produce a similar effect by a magnetic field, the enormous field of 10^9 absolute units would be required.

(III) In the next experiment, an estimate of the total number of particles reflected was aimed at For this purpose the emanation tube used in the previous experiments was unsuitable, firstly, on account of the difficulty of correctly ascertaining the number of α-particles emerging from the tube, and secondly, on account of the different ranges of the α-particles from the

* McClelland, 'Phil Mag,' vol 9, p 230, 1905, 'Ann. d. Phys.,' vol 18, p. 974, 1905; Schmidt, 'Ann d Phys.,' vol. 23, p 671, 1907, 'Phys. Zeit.,' vol. 8, p. 737, 1907

three products: emanation, radium A, and radium C Consequently, as radiating source, radium C, deposited on a plate of small dimensions, was used. The arrangement, which is sketched in fig 3, was such that the α-particles from the plate A fell upon the platinum reflector R, of about 1 square centimetre area, at an average angle of 90° The reflected particles were counted on different points of the screen S

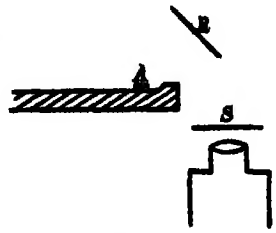

Fig 3

No appreciable variation of the number was found with different angles of emergence, the reason of which has already been explained above

The amount of radium C deposited on the plate was determined by its γ-ray activity Assuming that 3.4×10^{10} particles are expelled per second from an amount of RaC equivalent to 1 gramme Ra,* the number of α-particles expelled per second from the active plate was determined The number falling on the platinum reflector was then easily calculated from its known distance and area To find the whole number of reflected particles, it was assumed that they were distributed uniformly round a half sphere with the middle of the reflector as centre.

Three different determinations showed that of the incident α-particles about 1 in 8000 was reflected, under the described conditions

A special experiment conducted at low pressure showed that in the case of grazing incidence the number of particles reflected at a very small angle to the reflector is largely in excess of the number calculated from the above ratio. This tangential scattering is of considerable importance in some experiments, for instance, if α-particles from a radio-active source are fired along a glass tube of appreciable length the conditions are very favourable for this effect. The number of scintillations counted on a screen sealed to the other end of the tube is made up not only of the particles striking the screen directly, but also of those which have been reflected from the glass walls of the tube.

The correction for the latter effect may be appreciable, and would be still greater in the case of a metal tube. In the counting experiments of Rutherford and Geiger this effect did not influence the final result, the arrangement being such that the reflected particles were prevented from entering the opening of the ionisation vessel by the narrow constriction of a stopcock.

It appears probable that the number of reflected particles depends also upon the velocity of the α-particles falling on the reflector In our case

* Rutherford and Geiger, 'Roy. Soc. Proc.,' A, vol. 81, p. 162, 1908

the particles from the radium C had to travel through a little over a centimetre of air before reaching the reflector. The reflected particles had still an appreciable velocity, since, by interposing an aluminium foil of thickness equivalent in stopping power to ½ cm. of air, the number of scintillations counted was not changed This might be expected from Experiment (II), which showed that the α-particles are reflected from a relatively thin surface layer of the reflector

We are indebted to Prof. Rutherford for his kind interest and advice throughout this research.

The Passage of Electricity through Gaseous Mixtures.

By E M. Wellisch, M A (Sydney), Emmanuel College, Cambridge

(Communicated by Prof. Sir J. J. Thomson, F R S Received June 2,—Read June 17, 1909)

Introductory

According to the current theory with regard to the production of ions in a gas subjected to the action of Rontgen rays, the act of ionisation consists in the expulsion of one or more corpuscles (*i e* negatively charged units of electricity) from each of a certain number of molecules constituting the gas The residual portion of each of these molecules is then said to be positively charged, although the nature of this charge is not in any way specified There are thus present in the gas negatively charged nuclei (*i e.* the expelled corpuscles) and positively charged nuclei (the residual portions of the ionised molecules); owing to the forces due to electrostatic induction these nuclei attract several of the gas molecules, and the resulting molecular aggregates constitute the gaseous ions, both negative and positive

Suppose, now, that a mixture of two gases, *e g*, sulphur dioxide and oxygen, is subjected to the action of Rontgen rays, the positive nucleus would be of greater volume and mass in the case of sulphur dioxide than of oxygen, and in consequence it is quite possible that the resulting ions should show similar differences. Accordingly, if the two groups of positive ions move in the same electric field, a difference in velocities might thus reasonably be expected.

The object originally proposed in the present series of experiments was to

ascertain whether the ions of each sign produced by Rontgen rays in a mixture of two gases, or of a vapour and a gas at normal temperatures, possessed two distinct mobilities when influenced by an electric field

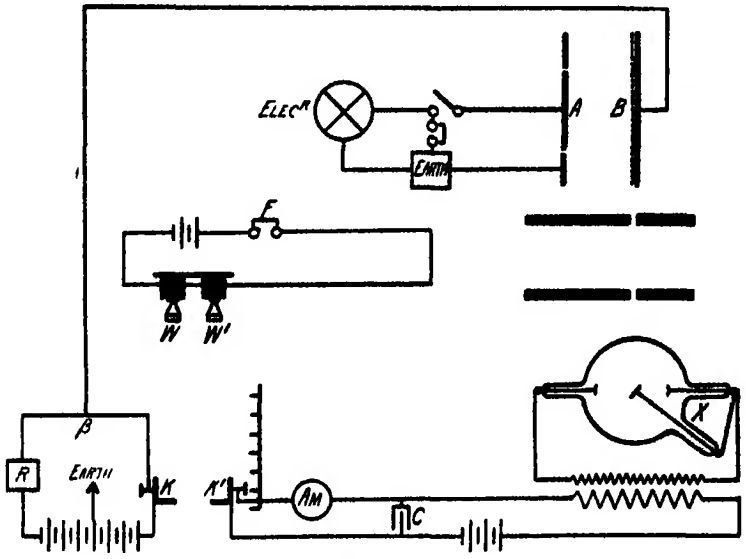

FIG. 1.

Experimental Method

It was found that Langevin's method as ordinarily applied, and of which a description is given in a recent paper by the writer,* was not exactly suited for the purposes of the present investigation On this account a modified form of Langevin's method was devised The ionisation chamber consisted of a cylindrical brass vessel with an aluminium bottom, inside the chamber were two rectangular aluminium electrodes B and A (fig 1), which were connected respectively to a source of potential and to an electrometer by means of wires led out through ebonite plugs, the electrode A being surrounded by an aluminium guard ring The dimensions of B were 23 by 6 cm, and of A were 7 by 4 cm, the two electrodes were placed in a vertical position, and at a distance from one another of 31 mm The inside of the chamber was lined with paper so as to minimise effects due to secondary radiation. A single flash of Röntgen rays of very short duration was sent through a slit between two rectangular plates of thick lead, after passing through another similar slit the rays then passed into the ionisation chamber, grazing the electrode B A narrow layer of ions was thus formed in the gas or gaseous mixture in the vicinity of the electrode B The lead

* 'Phil Trans,' A, vol. 209, p. 249, 1909

plates were adjustable, so that both the position and the width of the slit could be readily altered Suppose, now, that the electric field is such that the force on a positive charge is from B to A; then immediately after the passage of the rays the positive ions will commence moving towards A, while the negative ions will commence to be absorbed in the electrode B Let now the electric field be reversed at any definite interval t after the passage of the Röntgen rays, the positive ions which have not reached A will be turned back, while the negative ions which have not been absorbed in B will proceed towards the electrode A Suppose, on the whole, a quantity Q of electricity is received by A

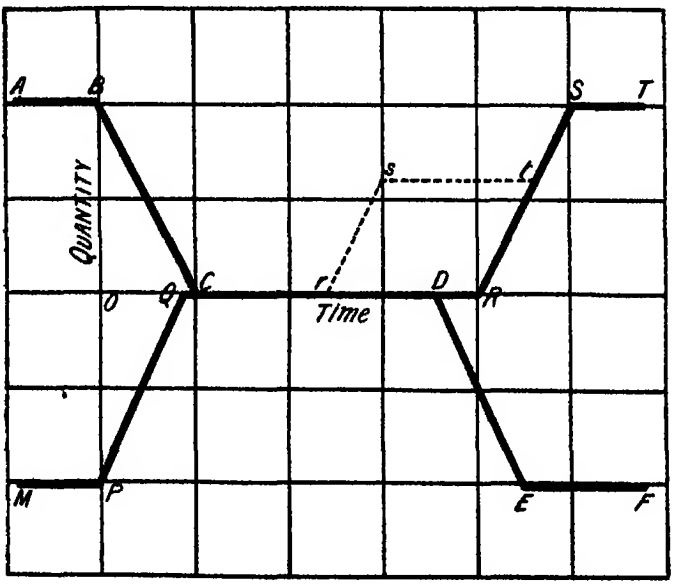

FIG. 2.

The curves connecting Q and t will be of the character given in fig 2, the curve MPQRST refers to the case when B is positive before reversal, and the curve ABCDEF to B negative before reversal; the oblique portions of the curves correspond to the finite width of the layer of ions, while the part that lies along the time axis occurs when the field is reversed after all the negative ions have been received at B, and before any of the positive ions have reached A. The distance between the points on the time axis corresponding to the points P and S represents the time taken for a positive ion to traverse the known distance from B to A, hence the actual velocity can be calculated. If there were two distinct mobilities, *e.g.*, as might be expected for the ions in a mixture of two gases, then the ions would begin to separate

as soon as the rays had passed, and the actual curve obtained would be of the form MPQ*rst*ST, in which the point *s* corresponds to the receipt by A of all the positive ions of larger mobility, and the point S to the receipt of all the more slowly moving ions

Thus an experimental realisation of the curve expressing the relation between Q and *t* should decide the question as to whether there are two distinct mobilities for the ions of any one sign in a mixture of gases

Experimental Procedure

The diagram of connections is represented in fig 1, W and W′ are two iron weights, which are supported by means of an electromagnet, on breaking the circuit through the magnet these weights fall simultaneously and break the platinum contacts at K and K′ respectively The breaking of the contact at K′, which is in the primary circuit of a Marconi induction coil, gives rise to an induced E M F in the secondary, and causes a momentary discharge to pass in the Rontgen bulb X, when the contact at K is broken the potential of the electrode B of the ionisation chamber is reversed, as is manifest from the diagram, R being a water resistance of the order of 1 megohm, and B at the moment of breaking being at the potential of the point β The time interval between the Rontgen ray flash and the subsequent reversal of potential could be adjusted by suitably fixing the position of K′ with regard to a graduated vertical scale, the actual values of the time interval were calculated by means of the ordinary formula for a body falling freely under gravity The bottom of the falling weight W was at a height of 54 8 cm above the contact K C is a capacity of about 7 microfarads inserted in parallel with the primary of the induction coil in order to prevent sparking at the contact K′, when the spark was entirely eliminated, and when the current through the primary of the induction coil was kept constant, the intensity of the Rontgen ray flash was sensibly constant at each discharge. A Dolezalek electrometer with a fine platinum suspension was employed in order to measure the ionisation produced, with the needle charged to a potential of 80 volts, the electrometer afforded a deflection per volt of 620 mm on a scale about 1 metre distant.

In the previous paper (*loc. cit*) a brief discussion is given with regard to the theoretical assumptions and experimental difficulties in connection with Langevin's method, for a more detailed account the reader is referred to Langevin's original memoir.*

A single reading was taken in the following manner:—

(1) K′ is adjusted to the required height on the scale

* 'Ann. de Chim. et de Phys.,' VII, vol. 28, p. 495, 1903.

(ii) The electrode A and the quadrants are earthed and the contact K is made

(iii) The magnet circuit is completed at F, the weights W and W′ are placed in position, the electrode A is insulated, the quadrants remaining earthed

(iv) The contact K′ is made, the magnet circuit is broken, allowing the weights to fall and break the contacts K and K′

(v) After the weights have fallen, the ions will have had sufficient time to be all received at the electrode, the contact at K is then remade

(vi) The total quantity received by A is then measured on the electrometer

The gases employed were all carefully dried before passing into the ionisation chamber The chamber was exhausted by means of a water pump and a Topler mercury pump, and the gas or vapour passed into it in the manner described in the previous paper.

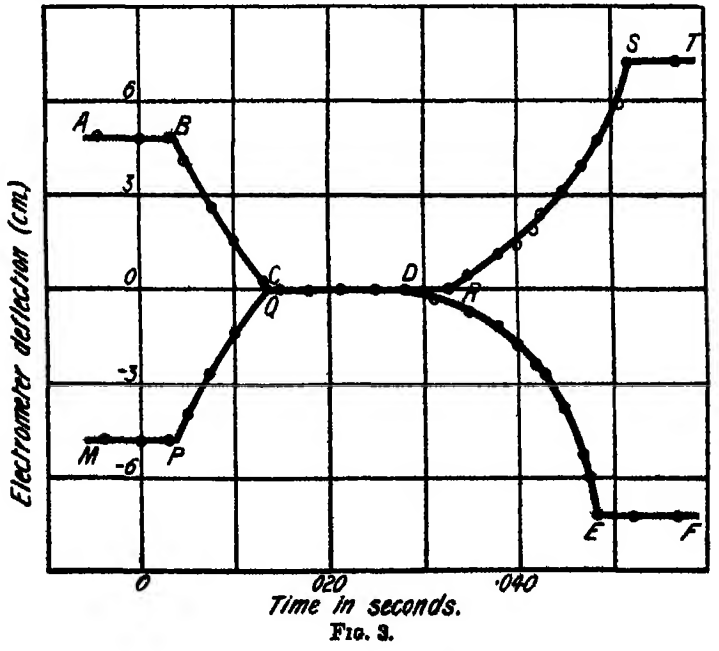

FIG. 3.

Experimental Results.

Sulphur Dioxide and Oxygen.—The first mixture of gases investigated was that of sulphur dioxide (pressure 131 mm.) and oxygen (629 mm.); the pressures being chosen so that each constituent gas afforded practically the same ionisation from the flash of Rontgen rays. The curves obtained are

given in fig 3; these correspond to positions of the slit between the lead plates, so that the layer of ions produced by the Rontgen ray flash did not graze the electrode B, the width of the slit between the plates was about 3 mm It is noticeable that there is no evidence of the existence of a portion of the curve corresponding to the part *rst* in fig. 2 The curvature of the oblique portions is due probably to non-uniformity in the distribution of the ions In the present instance, the fall of potential across the electrodes was 242 volts, and the times taken for the positive and negative ions to traverse a distance equal to that (31 mm) between the two electrodes were 0·0551 and 0·0519 sec respectively, these time intervals corresponding to the sum of the abscissæ of the points B and S in the case of the positive ion and P' and E in the case of the negative ion. The resulting mobilities are 0·72 cm./sec for the positive ion and 0·76 cm/sec for the negative ion

The curves in fig 3 are typical of other curves which were obtained for the same mixture corresponding to different positions and different widths of the slit

Ethyl Ether and Air.—Fig 4 represents similar curves obtained for a mixture of ethyl ether (122 mm) and air (508 mm), the pressures being

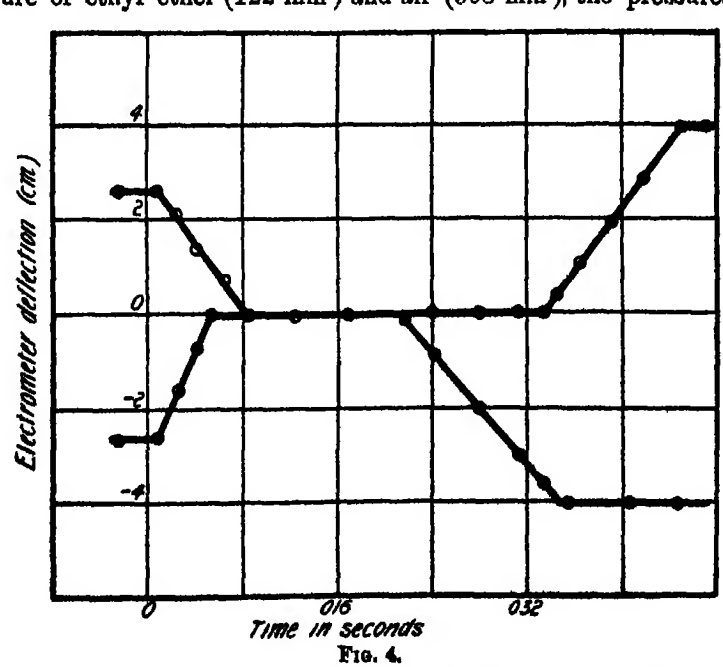

Fig. 4.

again so chosen that the ether and the air afforded practically the same ionisation from the flash of Röntgen rays. In this case, again, there was no evidence of two distinct mobilities either for the positive or for the

negative ions. The fall of potential across the electrodes was 202 volts, and the calculated times required for the positive and negative ions to traverse the distance between the electrodes were 0·0457 and 0·0353 sec. respectively. The mobilities of the positive and negative ions at the actual pressure of 630 mm were 1·04 and 1·35 cm./sec. respectively. Several other series of curves were drawn with different widths and different positions of the slit, but in no instance was there evidence of two distinct mobilities either for the positive or negative ions.

In connection with the experimental curves, it is worthy of notice that the ordinates of the points P and B are each less than the ordinates of the points S or E, whereas in the theoretical curve the ordinates are represented as being of equal length. This is due to the distortion of the electric field which occurs on the reversal of potential, the electrode A being now no longer at zero potential, and some of the lines of force in consequence passing over to the guard ring, which is earth connected. The electrode A will therefore receive only a fraction of the total number of ions which are formed near B, the remainder being received by the guard ring. The mobility values are, however, unaltered, as the abscissæ of the points P and B are not affected.

Another point worthy of mention in connection with the experimental curves is that the actual width of the layer of ions was invariably greater than the width of the slit between the lead plates. This effect is probably to be ascribed to secondary radiation from the gas and is without influence on the mobility values.

If we regard these curves as typical for any mixture of two gases or of a gas and a vapour, we are led to the conclusion that with the motion of the ion through the medium there must be associated some phenomenon of such a character as to produce a statistical average, so that as a result we are unable to detect the existence of two distinct mobilities in the mixture. Such an averaging might be explained in the following manner (for brevity we will consider only the case of the positive ion) —

The positively charged nuclei obtained from ionisation of the molecules of the two gases build up molecular aggregates which constitute the positive ions, but during the passage of the ion through the medium there is a continual exchange of the molecules constituting the ionic cluster with the molecules in the medium. On this account the differences in the size and mass of the two original nuclei would not of necessity result in the positive ions having two distinct mobilities. This is the conclusion at which M. Blanc* arrived as a result of his experiments on mobilities in gaseous mixtures, in particular the experiment in which an ion formed in carbon

* 'Journ. de Phys.,' vol. 7, p. 838, 1908.

dioxide and passed through air was shown to have the same mobility as an ion formed in air and passed through air

The experimental results to be described later lead, however, to an entirely different explanation of the statistical average.

There is an interesting feature in connection with the curves (figs 3 and 4) which suggests at first sight the existence of an averaging effect. It is noticeable that the distance along the time axis corresponding to the points R and S is greater than the corresponding distance for the points B and C; and similarly that the distance along the time axis for the points D and E is greater than the corresponding distance for the points P and Q, in other words, the ions require a longer time after their arrival at A before they are all received by the electrode A than what is necessary for them to be all received by B after their arrival. It appears, therefore, that the original layer of ions broadens out in its passage through the medium In order to test this point more completely, curves were drawn with oxygen alone in the ionisation chamber These curves are given in fig 5, from which it can be readily seen that the broadening out of the layer of ions occurs markedly for

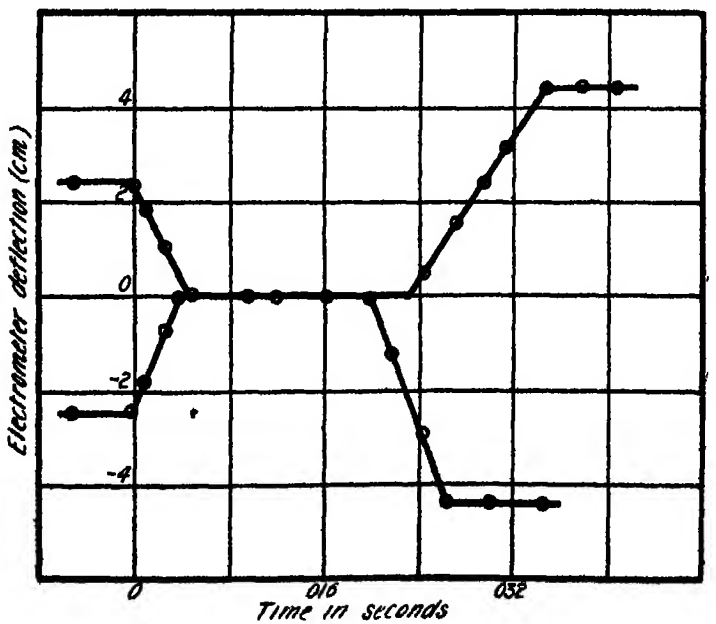

FIG. 5

the positive ions and to a very small extent for the negative ions, in fact, the small extent in the latter instance is ascribable to the effects of diffusion occurring during the motion of the ions. The writer has hitherto been unable

to assign any definite reason to account for the broadening out of the layer of positive ions, but in this connection it is of interest to note that Franck and Westphal* have recently drawn attention to the existence among the positive ions of a certain percentage which carry a double charge of electricity

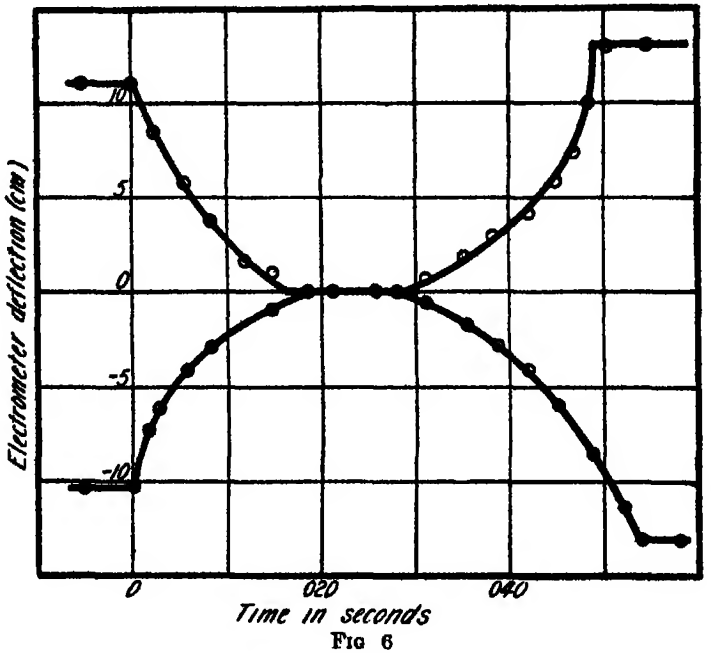

Fig 6

In figs 6 and 7 are given the curves obtained for sulphur dioxide and ethyl ether respectively They were obtained under the following experimental conditions

Fig 6—Sulphur dioxide (formed from sodium sulphite and sulphuric acid), dried by passing through calcium chloride Pressure, 331 mm Potential of B before reversal, 202 volts Time taken to traverse 31 mm positive ion, 0·0486 second, negative ion, 0·0541 second Calculated mobility values at pressure of 331 mm : positive ion, 0 98 cm /sec , negative ion, 0 88 cm./sec.

Fig. 7—Ethyl ether (Kahlbaum's preparation fractionally distilled and dried with metallic sodium) Pressure, 122 mm Potential of B before reversal, 121 volts. Time taken to traverse 31 mm . positive ion 0 0404 second, negative ion, 0 0404 second. Calculated mobility values at pressure of 122 mm. . positive ion, 1·97 cm./sec , negative ion, 1·97 cm./sec.

It is to be observed that in the case of sulphur dioxide the mobility of the

* 'Verhand. Deuts Phys. Gesell.,' No. 6, p. 146, 1909.

positive ion is greater than that of the negative, and that in this instance it is the layer of negative ions which appears to broaden out during motion through the gas.

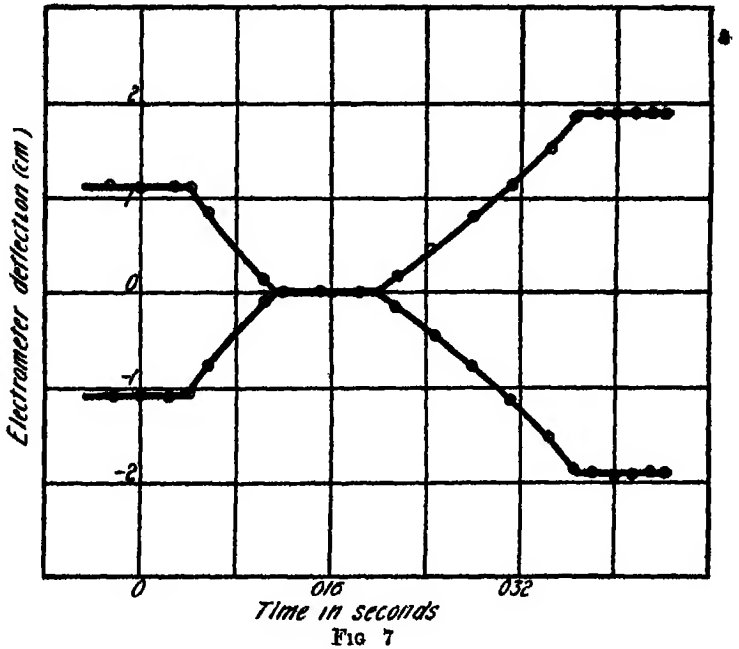

Fig 7

Effect of Small Traces of Vapours on the Values of the Mobilities of the Ions formed in Air

It has been shown by Zeleny and by Rutherford that the vapours of water or alcohol when present in small quantities in air produce a marked diminution in the mobility of the negative ion There was very little effect on the value of the mobility of the positive ion in air, in carbon dioxide the positive ion increased in mobility when water vapour was added and in hydrogen it decreased The following table exhibits the values of the positive and negative mobilities in the dry gases and also in gases saturated with water vapour—

Gas.	k_+.	k_-	Authority
	cm./sec	cm /sec	
Air (dry) . . .	1 36	1 ·87	Zeleny
„ (moist). .. .	1 37	1 51	„
Carbon dioxide (dry) .	0 ·76	0 ·81	„
„ (moist) ..	0 82	0 ·75	„
Hydrogen (dry)	6 ·70	7 ·95	„
„ (moist) .	5 ·30	5 ·60	„

A series of experiments was conducted in order to ascertain the effect produced on the ionic mobilities in air by adding small quantities of different vapours, it was found that the vapours of methyl iodide or ethyl bromide had very little effect on the mobilities of the positive or negative ion, while, on the contrary, the vapours of alcohol or acetone produced a marked decrease in the mobilities of both the positive and the negative ions The results are given in the following table—

Gas	k_+.	k_-	Pressure
	cm /sec	cm /sec	mm
Air	1 37	1 80	737
Air and 6 mm of C_2H_5Br	1 32	1 80	737
„ 6 mm of CH_3I	1 37	1 80	737
„ 10 mm of C_2H_6O .	0·91	1 10	755
„ 9 mm of C_3H_6O	1 15	1 37	755

Mixtures in which the Ions are formed from one Constituent only

When the ionisation chamber was filled with hydrogen at 760 mm pressure and a very narrow slit (about 1 mm) was employed, it was found that even with large currents through the primary of the induction coil there was extremely little ionisation produced from the single flash of Rontgen rays, such ionisation produced a scale deflection of less than 1 mm

The ionisation chamber was then exhausted to a very low pressure and methyl iodide was passed in till a pressure of 6 mm was attained, owing to the ease with which this vapour is ionised, on the passing of the flash of Rontgen rays quite a workable deflection (about 30 mm) was obtained Hydrogen was then passed in till the total pressure was 760 mm The mobilities were then measured and the following results were obtained —

Negative ion . . . . 6·86 cm /sec.
Positive ion .. .. . . 5 07 cm /sec.

Another series of experiments in which 12 mm of methyl iodide was employed gave—

Negative ion..... ..... 5 96 cm./sec
Positive ion . ... 4 81 cm./sec

Zeleny's values for pure dry hydrogen are—

Negative ion..... .... 7 95 cm /sec
Positive ion ..... 6·70 cm /sec.

It appears, therefore, that when the ionisation is all due to the methyl iodide the resulting ions move through a medium which is not sensibly

different from hydrogen with a velocity which approximates to that which would ensue if the ionisation had been due to the hydrogen

If we consider for the moment the positive ion only and if we regard it as the molecule of methyl iodide with which is associated a charge (e) equal to that carried by the monovalent ion in electrolysis, it is easy to calculate a superior limit to the velocity with which such an ion would move under unit electric intensity through hydrogen at 0° C and at a pressure of 1 atmosphere The formula given by the writer (*loc cit.*) is

$$k = \frac{A\eta}{\rho_1 p} 4\sqrt{2}\left(\frac{m}{M}\right)^{\frac{1}{2}}\left(1+\frac{M}{m}\right)^{-\frac{1}{2}}\left(1+\frac{s'}{s}\right)^{-2}\left\{1+\frac{4(K-1)e^2}{\pi n m v^2 (s+s')^4}\right\}^{-1}.$$

where M denotes the mass of the methyl iodide molecule,

m	mass of the hydrogen molecule,
$\frac{1}{2}s'$	radius of the methyl iodide molecule,
$\frac{1}{2}s$	radius of the hydrogen molecule,
η	coefficient of viscosity of hydrogen,
K	specific inductive capacity of hydrogen,
p	pressure of hydrogen in c g s units,
ρ_1	density of hydrogen at 760 mm pressure,
v	mean velocity of agitation of a hydrogen molecule,
A	quantity $\rho_1 e/m$

The factor in the curling brackets corresponds to the diminution in mobility resulting from the inductive attraction between the ion and the neighbouring molecules

The experimental values of the quantities involved in the calculation are $A = 1{\cdot}30 \times 10^{10}$, $\eta = 85 \times 10^{-6}$, $\rho_1 = 9 \times 10^{-5}$, $p = 1{,}014{,}000$,

$$K-1 = 26 \times 10^{-5}$$

In order to calculate s'/s it was taken as equal to $\{M/m\ d/d'\}^{\frac{1}{3}}$, where d' and d denote the densities in the liquid state of methyl iodide and hydrogen respectively With $d' = 2{\cdot}33$, and $d = 0{\cdot}08$ we have $s'/s = 1{\cdot}34$ On making the necessary substitutions it is found that the mobility (under 1 volt/cm) of the methyl iodide molecule through hydrogen is 0·58 cm /sec if we neglect the diminution in the mean free path due to the attraction of the neighbouring molecules, while if this attraction be taken into account the value of k is 0·15 cm /sec Of course, any clustering round this molecule as nucleus would produce a further diminution in the value of the mobility The calculated mobility values, even under circumstances tending to increase the velocity, are thus considerably less than the actual observed mobilities. We are forced to the conclusion that the charge cannot have

been carried throughout by the methyl iodide molecule, or, in other words, there must have been, at least initially, a transference of the charge from the methyl iodide molecule to a hydrogen molecule

A similar train of reasoning leads to the necessity of supposing that for the larger portion of its path the negative charge must have been associated with a hydrogen molecule

Experiments were also performed in which the ions were produced from carbon tetrachloride and mercury methyl and then passed through hydrogen. Results similar to those above described were obtained both for the positive and negative ions

In the case when 3 mm of mercury methyl was mixed with 757 mm of hydrogen and the ionisation was, as before, wholly due to the vapour, it was found that the mobilities of the ions through the mixture were

Negative ion	7 18 cm /sec
Positive ion	5 16 cm /sec

Calculations similar to the preceding give for the mobility of a charged molecule of mercury methyl through hydrogen

0 33 cm /sec if the attraction of the neighbouring molecules be neglected,

0 10 cm /sec if this attraction be taken into consideration

It is evident that the above considerations apply *à fortiori* to this particular case

Experiments with regard to the Stability of the Vapour Ions

The results described in the preceding section suggest that the ions formed from the large vapour molecules are unstable in the presence of the hydrogen molecules, inasmuch as the charge appears to be immediately transferred to the molecules of hydrogen Experiments were then conducted to ascertain whether the vapour molecules could hold the ionic charge for any considerable period in the presence of hydrogen. For this purpose the chamber was exhausted by the mercury pump to a low pressure, and then methyl iodide was passed in and swept through by means of a water pump till the pressure was 7 cm. The ionic mobilities were then ascertained and a small quantity of hydrogen was added, the mobilities were again determined, further hydrogen was added, and so on till atmospheric pressure was reached. The chamber was again exhausted and a similar series of experiments was gone through, commencing with 51 mm. of methyl iodide. The mobilities were also measured in a similar manner when the initial pressures of the methyl iodide were 25 mm., 12 mm., and 6 mm. The results obtained are given in the following table.—

Pressure			k_+	k_-
CH_3I	H_2	Total		
mm.	mm	mm	cm /sec	cm /sec
70	—	70	2 67	2 ·67
70	59	129	2 52	2 ·52
70	183	253	2 18	2 18
70	315	385	2 06	2 ·06
70	687	757	2 ·00	2 ·00
51	—	51	3 95	3 95
51	16	67	3 80	3 80
51	61	112	3 51	3 61
51	334	385	3 ·00	3 12
51	714	765	2 65	2 75
25	—	25	7 51	7 55
25	85	110	5 90	6 ·03
25	360	385	4 ·00	4 20
25	732	757	3 50	3 85
12	373	385	5 95	6 41
12	751	763	4 45	5 62
6	370	385	9 50	—
6	757	763	5 20	6 81

The results for the positive mobilities are exhibited in the curves shown in fig 8, the curve for pure hydrogen is based on Zeleny's values, and several points on the curve for pure methyl iodide have been calculated on the assumption of the validity of the law pk = constant

In order to interpret the results, let us consider the particular curve corresponding to 7 cm of methyl iodide It is evident that the addition of a small quantity of hydrogen does not appreciably alter the mobility, had the vapour ions been unstable and transferred their charge to the hydrogen molecules we would have expected a marked increase in mobility due to the small mass of the new carrier When further hydrogen is added the mobility decreases only very slightly, in fact, although the pressure changes from 7 to 76 cm the mobility only decreases from 2 7 cm /sec. to 2 cm /sec From this result we deduce that as hydrogen is added to the methyl iodide the charge tends more and more to be carried as a hydrogen ion, although the methyl iodide molecule can accompany the charge to quite an appreciable extent even in the presence of a considerable quantity of hydrogen.

From an inspection of the curves it can also be seen that the mobilities approximate more quickly to those of pure hydrogen the smaller the initial quantity of methyl iodide.

Considerations of a similar kind apply to the negative ions.

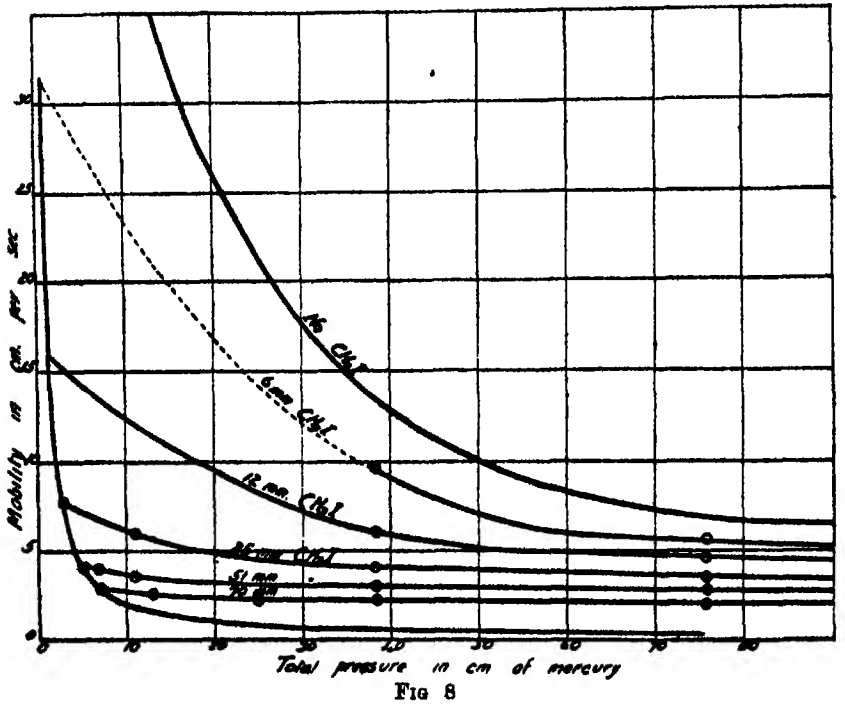

Fig 8

Another set of mobility values was obtained in which ethyl bromide was used in place of methyl iodide, the resulting curves were of a similar nature to those above described.

Discussion on the Mechanism involved in the Transference of the Positive Charge

It has been shown in the preceding sections that a positively charged vapour molecule when moving through hydrogen can transfer its charge to a hydrogen molecule. It is interesting to speculate on the different ways in which this might occur. On the one hand it is possible that, owing to the strong electric field surrounding the charged vapour molecule, a corpuscle might be dragged out of a hydrogen molecule, in this way the vapour molecule would become neutral, and the residual portion of the hydrogen molecule would on the ordinarily accepted theory carry a positive charge. On the other hand, the vapour molecule might transfer a positive unit of electricity to the molecule of hydrogen in a manner similar to what we may consider occurs in the case of the negative ion. The writer is more inclined to the latter view, especially as such a view appears to be necessary in order to explain the fact that in some gases, *e.g.*, sulphur dioxide, the positive mobility is greater than that of the negative.

The preceding considerations with regard to the transference of charge from the vapour molecule to the hydrogen molecule depend ultimately on the applicability of Maxwell's formula $L = \{\pi n\sigma^2\sqrt{1+M/m}\}^{-1}$ for the mean free path of a single uncharged molecule of mass M in a medium consisting of molecules each of mass m, n denoting the number of molecules per unit volume, and σ the sum of the radii of the two kinds of molecules. The application of this expression to the case of the diffusion of gases has been made by O Meyer,* who calculated the diffusion coefficients for a series of gases by a method involving this formula, and obtained a striking agreement with observation

If L denote the mean free path of the methyl iodide molecule through hydrogen at any pressure, and l the mean free path of the methyl iodide molecule through methyl iodide at this pressure, and if we regard the above formula as valid we deduce

$$\frac{L}{l} = \frac{8}{3}\frac{s'^2}{(s+s')^2},$$

where $\frac{1}{2}s'$ and $\frac{1}{2}s$ denote the radii of the vapour and gas molecule respectively Taking as before $s'/s = 1\,34$, we obtain $L/l = 0\,22$ approximately Now, the mobility at constant temperature is proportional to the mean free path, provided the ion is unaltered in mass and dimensions, as is readily seen from Langevin's expression for the mobility $k = eL/MV$, where M denotes the mass of the ion, L its mean free path through the gas, and V its mean velocity of thermal agitation It follows, therefore, that if the ion formed in pure methyl iodide at any pressure be unaltered or increased in mass and dimensions by the addition of any quantity of hydrogen, the curves for the mobilities shown in fig 8 would all lie nearer the axes than the mobility curve for pure methyl iodide The actual positions of these curves as determined experimentally imply that the ion diminishes in mass and dimensions as the hydrogen is added If the ion is then to be regarded as consisting of a cluster of molecules, it is necessary to postulate the original transference of charge by either of the methods indicated above and, in addition, the decrease of the ion in mass and dimensions as the hydrogen is added, this might possibly arise if we regard the addition of hydrogen as decreasing the time during which the ionic cluster involves a methyl iodide molecule, so that when a considerable quantity of hydrogen is present the ion would be approximately a cluster of hydrogen molecules Another explanation as to the nature of the passage of the electricity through the medium is given in the next paragraph, this view is capable of explaining in a simple manner all the preceding results and, moreover, is in harmony with other

* 'Kinetic Theory of Gases,' 2nd Edition (English), p. 274.

phenomena in connection with gaseous ionisation, *e g*, the formation of fresh ions by collision

The Mechanism underlying the Passage of Electricity through Gases at Ordinary Temperatures and Pressures.

It has been shown above that it is necessary either to postulate the existence of a unit of positive electricity or to suppose that there is some mechanism not involving the supposition of a positive electron by which a charged molecule can transfer its charge to a neutral molecule Either alternative renders it extremely probable that when the ions move through a medium consisting of a mixture of two gases there is a continual interchange of charges between the two sorts of molecules; the charge is associated for a fraction of its life in the gas with the molecules of one kind and for the remaining portion with the other kind of molecules. It is a natural inference to conclude that a similar process occurs in the simple gases such as oxygen, carbon dioxide, etc In the previous paper it was shown that the observed mobility values of the gaseous ions could be explained approximately on the supposition that the ion consisted of a single charged molecule if we took into account the diminution of the mean free path due to the attraction of the molecules by the charge on the ion This interchange of charge between the molecules in the medium occurs both for the positive and negative ions, the marked differences in the values of the mobilities of the two kinds of ions in certain gases, *e g.*, oxygen, hydrogen, etc, is explicable if we suppose that the charge which is in general associated with the molecule can exist in the free state for a certain fraction of its life On this view the negative mobility would be greater than the positive for those gases whose molecules have a greater affinity for the positive charge than for the negative, thus for gases such as oxygen and hydrogen it is necessary to suppose that the positive charge is associated with the molecules for a longer time than is the negative corpuscle. In addition it would appear from experiments on the mobilities in flames that the time during which the charge exists in the free state is a function of the temperature, so that as the temperature increases the ion approximates to a single detached electrical charge either positive or negative Such a view harmonises with recent experiments by Moreau,* who found that in a flame at 2000° C the mass of the negative carrier was $1{\cdot}1 \times 10^{-26}$ gramme, which is intermediate between that of a corpuscle (10^{-27}) and that of an atom of hydrogen (10^{-24}).

It follows also as a result of the experiments made with regard to the effect on the mobilities of small traces of vapours that we must suppose that

* 'Comptes Rendus,' vol. 148, p 342, 1909

the molecules of certain vapours (*e.g.*, water, alcohol, acetone) tend to hold the charge in association with them much more strongly than the molecules of the heavier vapours such as methyl iodide. Thus if a small quantity of water-vapour be mixed with carbon dioxide at one atmosphere the charge (both positive and negative) would be carried for a considerable portion of the path by the water molecules, owing to the relatively small mass of the molecule of water we would expect an increase in the mobility of the positive ion, but in the case of the negative ion this increase might be more than counteracted by the shortening of the period during which the corpuscle exists in the free state, so that on the whole the mobility of the negative ion would be decreased. Similar reasoning would explain, although merely descriptively, all the observed phenomena with regard to the effect of small quantities of vapours on the mobilities of gaseous ions.

It is of interest to record that Prof. Sir J. J. Thomson* has recently advanced the theory that the act of ionisation consists in the expulsion from the molecule of both a positive and a negative unit of electricity, such a view is quite in accordance with all the experimental results described in this paper.

I take this opportunity of expressing my gratitude to Sir J. J. Thomson for the inspiring interest he has manifested throughout the research and for many valuable suggestions.

* 'Phil. Mag.,' vol. 16, p. 685, 1908.

The Effect of Pressure on the Band Spectra of the Fluorides of the Metals of the Alkaline Earths.

By R Rossi, B.Sc

(Physical Laboratories, The University of Manchester)

(Communicated by Prof. Arthur Schuster, F R S Received May 19,—Read June 17, 1909)

The effect of pressure on arc spectra of several elements has been carefully examined in recent years by Humphreys* and Mohler† in America and by Duffield‡ in this country

All lines were found to widen out and to shift under pressure towards the less refrangible end of the spectrum, and for a given line this shift was found to be approximately proportional to the pressure of the gas surrounding the arc.

The cyanogen bands were also investigated by Humphreys and Mohler, but were found not to show any pressure-shift Their heads reverse§ under pressure, and the components in which the bands are resolved widen out but do not show the least trace of shift I have repeated these experiments up to pressures of 110 atmospheres, but even at that high pressure the centre of the heads of the bands at atmospheric pressure is seen to coincide with the centre of the reversals of the same heads under pressure

It has been suggested by Humphreys that lines which show a large Zeeman effect ought to show a large pressure-shift effect, but the experimental proof of such a connection is not yet complete.

No evidence that bands showed a Zeeman effect was obtained until Dufour,‖ about one year ago, observed that the bands of the fluorides of the metals of the alkaline earths are an exception to this rule It was therefore thought interesting, on account of Humphreys' hypothesis, to see if these particular bands which show a marked Zeeman effect would also be displaced by pressure I was fortunate in having the use of the 21½-foot Rowland concave grating of this laboratory, which has been previously carefully mounted by Dr W G Duffield and used by him for similar work.

* W. J. Humphreys, 'Astrophys. Journ.,' vol 4, p 249; vol. 3, p. 114; vol. 22, p. 217; vol. 26, p. 18

† J. F. Mohler, 'Astrophys. Journ.,' vol. 3, p. 114, vol. 4, p 175.

‡ W G. Duffield, 'Phil. Trans.,' A, vol 208, p. 111 (1908), and vol 209 p. 205 (1908).

§ J E Petavel and R. S. Hutton, 'Phil. Mag' [6], vol 6, p. 571 (1903).

‖ A. Dufour, 'Comptes Rendus,' vol. 146, pp. 118 and 229, 1908; 'Le Radium,' October, 1908.

Both the first and second order spectra were observed, giving on the photographic plate a dispersion of 2·6 and 1·3 Ångstrom units respectively per millimetre.

The source of light was an electric arc placed inside the pressure cylinder designed by Prof Petavel.* The fluorides examined were placed in the positive carbon of the arc, which was excited by direct current (about 12 amperes at 100 volts) Wratten and Wainwright panchromatic plates were found to be the most suitable, the maximum exposure being one hour at 40 atmospheres in the second order spectrum

The comparison spectrum, at atmospheric pressure, was photographed in the central strip of the plate both before and after the one taken under pressure, to show whether any accidental displacement had occurred during the exposure The spectrum taken under pressure appeared above and below the comparison spectrum The photographs were measured with Kayser's machine, from 8 to 16 readings being taken for each band

Each of the components into which a band was resolved was found to shift the same amount towards the red end of the spectrum Hence measurements were made on as many components as possible, but in some cases where the band was much weakened by pressure, measurements had to be confined to the first and most distinct component, viz, the head of the band.

Owing to the asymmetrical reversal and widening of the components of these bands under pressure it is difficult to make accurate measurements, and in some cases at low pressures, where the displacement is small, the error in the measured displacement may be a large one, amounting to even 20 per cent., for higher pressures, of course, the probable percentage error of the measured displacement is much reduced.

Calcium Fluoride.

This compound gives three bands fading towards the violet end of the spectrum with heads at λλ 6036·96, 6050·81, 6064·49. In the first experiments the powdered fluoride was introduced into an opening bored into ordinary carbon poles, but it was found that the arc did not burn as well under high pressures as with the carbons which are sold commercially for flame arcs and which contain large quantities of calcium fluoride. The latter were therefore used in the final experiments.

Each of the three bands was found to shift towards the red, the components of the same band shifting the same amount.

Table I gives the mean values of the displacements, in Ångstrom units, at

* R. S Hutton and J. E. Petavel, 'Phil. Trans.,' A, vol. 207, p. 421 (1908).

each pressure taken from the two sets of photographs in the first and in the second order.

Table I.

Band with head at λ	Pressure in atmospheres (excess above one atmosphere)					
	5.	10.	15.	20.	30	40.
6036·96	0·078	0 120	0 149	0 165	0 275	0 371
6050·81	0·051	0·092	0 112	0·138	0·201	0 261
6064 49	0·068	0·118	0 150	0 188	0 275	0 372

Table II gives the mean displacement per atmosphere and also the Zeeman separation in the magnetic field as found by Dufour. In the last column the direction of rotation is called normal when, as with ordinary metallic lines, it agrees with that deduced from the assumption that the electron giving rise to the observed oscillation is negative An abnormal rotation may, but does not necessarily mean, that the electron is positive as has been pointed out by Dufour in his latest publications

Table II

Band with head at λ	Mean displacement per atmosphere in Å U	Magnetic separation in Å U H = 10,000 (Dufour)	Direction of circular polarization (Dufour)
6036·96	0·0105	0 31	abnormal
6050 81	0·0078	0 22	"
6064 49	0·0103	0 30	normal

The relative intensity of the bands is not affected by pressure; the band with head at λ 6050 81 being always much less intense than the other two both at atmospheric and under high pressure

The components of the bands were always found to be reversed, at atmospheric and at high pressure

The band having the greatest tendency to reverse is, however, λ 6036. For in one photograph at atmospheric pressure taken in a preliminary experiment with a graphite pole containing small quantities of calcium fluoride, λ 6064 is an emission band; λ 6050 is faintly reversed; while λ 6036 is strongly reversed.

All components in which the bands are resolved widen out under pressure; but those of the band λ 6064 to a greater extent than those of the bands λ 6050 and λ 6036.

Strontium Fluoride.

Strontium fluoride gives five bands, fading away towards the violet, and with heads at λλ 6418·82, 6511·84, 6527·39, 6632·43, 6655·40.

The positive pole was a graphite tube about 1·5 cm. outside diameter and 1 cm. inside diameter.

Powdered graphite was mixed in a mortar with an equal amount of strontium fluoride and with a very small quantity of sugar solution in water, and a paste as homogeneous as possible was thus prepared. The graphite tube was then filled with the paste and baked in an oven at about 400° C. The sugar carbonised and a fairly hard and homogeneous mass was left inside the pole, such poles were found to burn under pressure far better than if the bored pole had been filled with the fluoride alone, or with mixed fluoride and graphite alone. Using the latter methods the fluoride is partly blown away by the arc and partly melted, falling to the bottom of the hole, with the result that the bands are weakened, and a much longer exposure is required.

Graphite was used instead of carbon, because it contains less impurities, burns slower, and is a better conductor.

Owing to the long wave-lengths of these bands, coloured screens would have been necessary to prevent overlapping if the work had been carried on in the second order spectrum, and also longer exposures would have been required. Photographs were therefore taken in the first order only.

Table III gives the displacements in Ångstrom units, Table IV gives the mean shift per atmosphere and the magnetic separation.

Table III

Band with head at λ	Pressure in atmospheres (excess above one atmosphere).			
	10.	15.	20.	25
6511·84	0·122	0·141	0·178	0·261
6527·39	0·094	0·109	0·146	0·192
6632·43	0·083	—	0·150	0·184
6655·40	—	—	0·137	0·171

These bands were emission bands at atmospheric pressure; but under higher pressures, λ 6511 and λ 6632, were found to be reversed.

As the pressure increases the intensity of the two bands at λ 6632 and λ 6655 decreases more rapidly than the intensity of λ 6511 and λ 6527.

Table IV.

Band with head at λ	Mean displacement per atmosphere in Å U	Magnetic separation in Å U H = 10,000 (Dufour).	Direction of circular polarisation (Dufour)
6511·84	0·0112	0 40	abnormal
6527·39	0·0079	0·40	,,
6632 43	0·0077	0 43	normal
6655 40	0·0068	?	probably abnormal

Barium Fluoride

The poles were prepared in the same way as those for strontium fluoride

The bands with heads at λλ 4050 85, 4992 25, and 5000 71 were studied, these bands fade away towards the red. The displacements in Ångstrom units are given in Table V, the mean displacement per atmosphere and magnetic separation in Table VI

Table V

Band with head at λ	Pressure in atmospheres (excess above one atmosphere)		
	5	12 5	15 5
4950 85	0 060	0·089	0 121
4992 23	0 093	0 145	—
5000·71	0·062	0 095	—

Table VI.

Band with head at λ.	Mean displacement per atmosphere in Å U	Magnetic separation in Å U H = 10,000 (Dufour)	Direction of circular polarisation (Dufour)
4950 85	0·009	0 24	abnormal
4992 23	0·015	0 12	,,
5000·71	0·010	0 24	normal

The intensity of these bands decreases rapidly with pressure. At 15 atmospheres the bands λ 4992 and λ 5000 have vanished, and at 20 the band λ 4950 has also disappeared At atmospheric pressure they were always found to be emission bands. At 5 atmospheres λ 4992 is still an emission band, λ 5000 is faintly reversed, and λ 4950 is completely reversed. Under pressure, the band at λ 5000 broadens out more than the other two.

It will be noticed that the highest pressure employed was only 15·5 atmospheres in the case of barium, and 40 in the case of calcium. Experiments at still higher pressures were not practicable for the following reasons:—The banded spectra of these fluorides are almost entirely due to light emitted by the outside layers or "flames" of the arc. Under pressure, the arc shortens very much, at 50 atmospheres, it is only a few millimetres long, it is very bright, but the flashing "flames" of a long arc which are obtained at atmospheric pressure seem to become more and more rare as the pressure increases. A long exposure would be required to obtain a good photograph under such conditions. This drawback was most marked when the barium fluoride was used; at 20 atmospheres, no sign of the bands appeared on the plate for the ordinary exposure. A still longer exposure would have fogged the plate owing to the continuous spectrum given by the incandescent poles whose images, in consequence of the shortness of the arc, could not always be kept away from the slit.

Although it is clear that these particular bands which show the Zeeman effect are also displaced by pressure, there seems to be no obvious relation between the magnitudes of the two effects. For example, in the case of the calcium fluoride bands the displacements due to pressure are proportional to the magnetic separation, while in the case of the barium fluoride the larger the Zeeman effect the smaller the pressure-shift. In the case of the strontium fluoride there does not seem to be any relation at all between the magnitudes of the two effects.

The amount of pressure-shift apparently does not depend upon whether the circular polarisation agrees in direction with that deduced from the motion of positive or negative electrons. The displacements found for these bands under pressure are of the same order of magnitude as those found by other observers on line spectra. The linear relation between displacement and pressure found for line spectra seems to hold roughly also for these bands.

Many thanks are due to Prof. Rutherford for placing the necessary apparatus at my disposal and for the great interest he has taken in this work.

To Dr. W. G. Duffield I must also express my thanks for the experience I gained under him during the last few years in high pressure work.

The Coefficients of Capacity and the Mutual Attractions or Repulsions of Two Electrified Spherical Conductors when close together.

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(Communicated by Dr C. Chree, F R.S. Received June 2,—Read June 17, 1909.)

Introduction

In connection with spark systems of wireless telegraphy, a knowledge of the electrostatic energy stored between spherical electrodes at the instant of the disruptive discharge is of great value to the engineer. In order to increase this energy, without unduly increasing the applied potential difference, the electrodes have been placed in compressed or highly rarefied gases and in oils or other liquid dielectrics having great electric strength In these cases the least distance between the electrodes may only be a small fraction of the radius of either, and so the computation of the electrostatic energy by the ordinary formulæ is so laborious that it is practically prohibitive

By extending a mathematical theorem first given by Schlomilch,* the author has succeeded in greatly simplifying the computation of this energy The formulæ given below are very easily evaluated when the spheres are close together, and hence, in conjunction with Kirchhoff's† final modification of his own formulæ, they give the complete practical solution of this important historical problem.

The formulæ obtained enable the attractive or repulsive forces between electrified spherical conductors to be easily calculated, however close the spheres are to one another They have been employed to recalculate the latter portion of the table published by Kelvin.‡ This table has also been extended so as to make it more useful to physicists and electricians

Mathematical Theorems.

By Schlomilch's method (*loc cit*) we can prove that

$$\sum_{s=1}^{s=\infty} \frac{1}{e^{(1+ns)x}-1} = \frac{F(n)-\log x}{nx} + \frac{1}{2}\left(\frac{1}{n}-\frac{1}{2}\right) - \frac{B_1 nx}{2!}A_1 - \frac{B_2 n^2 x^3}{4!}A_3 - \frac{B_{2m-1}(nx)^{2m-1}}{2m!}A_{2m-1}, \quad (1)$$

* 'Zeitschrift für Mathematik und Physik,' vol 6, p. 407, 1860

† 'Annalen der Physik,' vol. 27, p. 673, 1886

‡ 'Phil. Mag.,' April and August, 1853, or Reprint, p. 83.

approximately, provided that the last term is very small compared with unity

In this formula B_1, B_3, B_5, . ., are Bernoulli's numbers,

$$F(n) = \frac{n}{2} + \frac{B_1}{2}n^2 - \frac{B_3}{4}n^4 + \frac{B_5}{6}n^6 - \ldots, \tag{2}$$

and $$(-)^{m-1}A_{2m-1} = \frac{1}{2mn^{2m}} - \frac{1}{2n^{2m-1}} + \frac{(2m-1)B_1}{2!\,n^{2m-2}}$$

$$- \frac{(2m-1)(2m-2)(2m-3)B_3}{4!\,n^{2m-4}} + \ldots + (-)^{m-1}\frac{B_{2m-1}}{2m}. \tag{3}$$

It is easy to show by Stirling's theorem that

$$F(n) = -\Gamma'(1/n)/\Gamma(1/n) - \log n, \tag{4}$$

and hence, by the properties of gamma functions,

$$F(n) - F\left(\frac{n}{n-1}\right) = \pi \cot\frac{\pi}{n} - \log(n-1) \tag{5}$$

In many cases the value of $F(n)$ may be simply expressed. For instance, if γ denote Euler's constant, we have $F(1) = \gamma$, $F(2) = \gamma + \log 2$, $F(3) = \gamma + \pi/2\sqrt{3} + \frac{1}{2}\log 3$, $F(4) = \gamma + \pi/2 + \log 2$, etc In general, however, the value of $F(n)$ has to be computed by a series formula In our problem n is never less than unity, and so the following formula can always be used—

$$F(n) = \frac{n}{2} + \gamma + \frac{1}{n^2-1} + \frac{\pi}{2}\cot\frac{\pi}{n} - \log n + \frac{S_3-1}{n^2} + \frac{S_5-1}{n^4} + \ldots, \tag{6}$$

where $S_m = \frac{1}{1^m} + \frac{1}{2^m} + \frac{1}{3^m} + \ldots$. The values of S_3, S_5, . ., are given (*e.g.*) in Dale's 'Mathematical Tables.'

From (3) we can show that when n is unity $A'_{2m-1} = B_{2m-1}/(2m)$, and when n is 2, $A''_{2m-1} = -B_{2m-1}(1-2^{-2m+1})/(2m)$. We can also show that for any value of n, other than 1 or 2, A_{2m-1} lies in value between A'_{2m-1} and A''_{2m-1} Its value also is not altered when we write $1-1/n$ for $1/n$

Writing t for $(n-1)/n^2$, we easily find that

$$A_1 = \frac{1}{12} - \frac{t}{2}, \qquad A_3 = \frac{1}{120} - \frac{t^2}{4}, \qquad A_5 = \frac{1}{252} - \frac{t^3}{6} - \frac{t^2}{12},$$

$$A_7 = \frac{1}{240} - \frac{t^4}{8} - \frac{t^3}{6} - \frac{t^2}{12}, \quad \text{and} \quad A_9 = \frac{5}{660} - \frac{t^5}{10} - \frac{t^4}{4} - \frac{3t^3}{10} - \frac{3t^2}{20}$$

The author has verified by actual calculation that, to a seven figure accuracy at least, formula (1) is true, both when n is 1 and when n is 2, even when x is as great as 2 log 2, *i.e.* 1·386 nearly, and powers of x greater than the ninth are neglected.

Putting $n=1$ in (1) we get Schlomilch's Theorem. Schlömilch has verified by actual calculation that when x is log 2·5, *i e* 0 916 nearly, his formula, neglecting powers of x beyond the ninth, practically gives the numerical value of the series correctly to 10 significant figures He has also shown that when x is not greater than log (10/9), *i e* 0 105 nearly, the formula

$$\sum_{1}^{\infty}\frac{1}{e^{sx}-1}=\frac{\gamma-\log x}{x}+\tfrac{1}{2}-\frac{x}{144} \tag{7}$$

gives a seven figure accuracy. To obtain the same accuracy by Clausen's Theorem, which was utilised and extended by Kirchhoff (*loc cit*), 13 terms would have to be taken, and the calculation of the later terms is very laborious

The Capacity Coefficients of Two Spheres

Let us consider the case of two spherical conductors whose radii are a and b respectively, and let c be the distance between their centres. If the charges and potentials of the spheres be q_1, q_2 and v_1, v_2 respectively, we have

$$\left.\begin{aligned} q_1 &= k_{1,1}v_1+k_{1,2}v_2 \\ \text{and}\quad q_2 &= k_{2,2}v_2+k_{1,2}v_1 \end{aligned}\right\} \tag{8}$$

where $k_{1,1}$ and $k_{1,2}$ are the capacity coefficients of the two spheres The values of these quantities in terms of a, b, and c are given by the following equations* :—

$$k_{1,1}=\lambda\sum_{s=0}^{s=\infty}\frac{1}{\sinh(\alpha+s\omega)}, \tag{9}$$

$$-k_{1,2}=\lambda\sum_{1}^{\infty}\frac{1}{\sinh s\omega}, \tag{10}$$

$$k_{2,2}=\lambda\sum_{0}^{\infty}\frac{1}{\sinh(\beta+s\omega)}, \tag{11}$$

where $$4c^2\lambda^2=(c+a+b)(c-a-b)(c+a-b)(c-a+b), \tag{12}$$

$$\sinh\alpha=\lambda/a,\qquad \sinh\beta=\lambda/b,\quad\text{and}\quad \sinh\omega=\lambda c/ab \tag{13}$$

We also have

$$\omega=\alpha+\beta\quad\text{and}\quad \alpha=\log\left(\frac{\lambda}{a}+\sqrt{1+\frac{\lambda^2}{a^2}}\right) \tag{14}$$

When $c-a-b$ is small, it is necessary to alter the formulæ (9), (10) and

* *Cf* Maxwell, vol. 1, § 173.

(11) by means of (1) so as to lessen the labour involved in the computation. From (9) we have

$$\frac{k_{1,1}}{2\lambda} = \sum_0^\infty \frac{1}{e^{n+2\omega}-1} - \sum_0^\infty \frac{1}{e^{2n+2\omega}-1}$$

$$= \frac{F(n)-\log(\omega/2n)}{2\omega} + \frac{B_1\omega}{2!}A_1 + \frac{7B_3\omega^3}{4!}A_3 + \frac{31B_5\omega^5}{6!}A_5$$

$$+ \frac{127\,B_7\omega^7}{8!}A_7 + \frac{511\,B_9\omega^9}{10!}A_9, \qquad (15)$$

very approximately, where $n=\omega/\alpha$

Finding the corresponding formula for $k_{2,2}/2\lambda$ and subtracting it from (15), we get, by (5),

$$k_{1,1}-k_{2,2} = \frac{\pi\lambda}{\alpha+\beta}\cot\frac{\pi\alpha}{\alpha+\beta}, \qquad (16)$$

very approximately

When the spheres are so close together that λ is small compared with a or b, $\alpha=\lambda/a$ and $\beta=\lambda/b$, and hence

$$k_{1,1}-k_{2,2} = \frac{\pi ab}{a+b}\cot\frac{\pi b}{a+b} \qquad (17)$$

This formula* has been obtained previously for the case of spheres in contact.

From (10) also we find that

$$-\frac{k_{1,2}}{2\lambda} = \frac{\gamma-\log(\omega/2)}{2\omega} + \frac{\omega}{144} + \frac{7\,\omega^3}{86400} + \frac{31\,\omega^5}{7620480}$$

$$+ \frac{127\,\omega^7}{290304000} + \frac{511\,\omega^9}{3161410560}, \qquad (18)$$

very approximately.

The Attractions or Repulsions between the Spheres

If the potentials of the spheres be maintained constant, Kelvin† proved that when they alter their positions owing to their mutual electric actions they move in such a way that the electrostatic energy of the system is increased by an amount exactly equal to the work done on the conducting spheres by the electric forces. If W be the electrostatic energy, we have

$$W = \tfrac{1}{2}k_{1,1}v_1^2 + \tfrac{1}{2}k_{2,2}v_2^2 + k_{1,2}v_1v_2,$$

and therefore

$$\frac{\partial W}{\partial x} = \tfrac{1}{2}v_1^2\frac{\partial k_{1,1}}{\partial x} + \tfrac{1}{2}v_2^2\frac{\partial k_{2,2}}{\partial x} + v_1v_2\frac{\partial k_{1,2}}{\partial x} = F,$$

* See Maxwell, vol. 1, § 175 and the references given there

† Reprint, p. 466, Second Edition.

where $x=c-a-b=$ the least distance between the spheres, and F is the force between them If F is negative, W increases as x diminishes, and therefore the force is attractive, but if F is positive the force is repulsive The values of $\partial k_{1,1}/\partial x$, $\partial k_{2,2}/\partial x$ and $\partial k_{1,2}/\partial x$ can easily be found from the preceding formulæ.

Spheres at Microscopic Distances apart

In this case, $x/(a+b)$ being supposed negligibly small compared with unity, we have $\lambda^2 = \frac{2ab}{a+b}x$, and $\omega^2 = \frac{2(a+b)}{ab}x$, and hence

$$\left.\begin{aligned} k_{1,1} &= \frac{ab}{a+b}\left\{\mathrm{F}\left(1+\frac{a}{b}\right)+\tfrac{1}{2}\log\frac{2a(a+b)}{b}+\tfrac{1}{2}\log\frac{1}{x}\right\}, \\ k_{2,2} &= \frac{ab}{a+b}\left\{\mathrm{F}\left(1+\frac{b}{a}\right)+\tfrac{1}{2}\log\frac{2b(a+b)}{a}+\tfrac{1}{2}\log\frac{1}{x}\right\}, \\ -k_{1,2} &= \frac{ab}{a+b}\left\{\gamma+\tfrac{1}{2}\log\frac{2ab}{a+b}+\tfrac{1}{2}\log\frac{1}{x}\right\} \end{aligned}\right\} \qquad (19)$$

If the difference of potential produced between the spheres by giving a charge $+q$ to one and $-q$ to the other be V, then q/V is defined* to be the capacity between the spheres

This is the capacity that is generally considered by electrical engineers It is easy to show† that its value is

$$\frac{k_{1,1}k_{2,2}-k_{1,2}^2}{k_{1,1}+k_{2,2}+2k_{1,2}},$$

and hence it can be found by (19)

The joint capacity of the two spheres when at the same potential is $k_{1,1}+k_{2,2}+2k_{1,2}$ It therefore equals

$$\frac{ab}{a+b}\left\{\mathrm{F}\left(1+\frac{a}{b}\right)+\mathrm{F}\left(1+\frac{b}{a}\right)-2\gamma+\log\frac{(a+b)^2}{ab}\right\} \qquad (20)$$

When $a=b$, this capacity is $2a\log 2$, when $a=2b$, it equals $a\log 3$, when $a=3b$, it equals $\frac{3}{2}a\log 4$, etc

If the square of the difference of potential $(v_1-v_2)^2$ between the two spheres be not very small compared with $v_1^2+v_2^2$, the force F between them is attractive and is given by

$$\mathrm{F}=\frac{ab(v_1-v_2)^2}{4(a+b)x} \qquad (21)$$

* Sir J. J. Thomson, 'Electricity and Magnetism,' Third Edition, p. 84.

† Russell, 'Alternating Currents,' vol. 1, p. 93.

The Case of Equal Spheres

When the spheres are equal, $k_{1,1} = k_{2,2}$ and the formulæ become

$$k_{1,1} = \frac{\lambda}{\omega}\left\{2\,6566572 - \log\omega - \frac{\omega^2}{144} - \frac{49\,\omega^4}{345600} - \frac{961\,\omega^6}{121927680} - \frac{16129\,\omega^8}{18579456000} - \frac{261121\,\omega^{10}}{1618642206720}\right\}, \tag{22}$$

$$-k_{1,2} = \frac{\lambda}{\omega}\left\{1\,2703628 - \log\omega + \frac{\omega^2}{72} + \frac{7\,\omega^4}{43200} + \frac{31\,\omega^6}{3810240} + \frac{127\,\omega^8}{145152000} + \frac{511\,\omega^{10}}{3161410560}\right\}, \tag{23}$$

$$-\frac{\partial k_{1,1}}{\partial x} = -\frac{k_{1,1}}{\lambda^2}\left(\frac{c}{4} - \frac{\lambda}{\omega}\right) + \frac{1}{\omega^2}\left\{1 + \frac{\omega^2}{72} + \frac{49\,\omega^4}{86400} + \frac{961\,\omega^6}{20321280} + \frac{16129\,\omega^8}{2322432000} + \frac{261121\,\omega^{10}}{161864220672}\right\}, \tag{24}$$

and

$$\frac{\partial k_{1,2}}{\partial x} = -\frac{-k_{1,2}}{\lambda^2}\left(\frac{c}{4} - \frac{\lambda}{\omega}\right) + \frac{1}{\omega^2}\left(1 - \frac{\omega^2}{36} - \frac{7\,\omega^4}{10800} - \frac{31\,\omega^6}{635040} - \frac{127\,\omega^8}{18144000} - \frac{511\,\omega^{10}}{316141056}\right) \tag{25}$$

If a be the radius of each sphere, we have

$$\lambda^2/a^2 = x/a + x^2/(2a)^2 \quad \text{and} \quad \omega = 2\log\left(\lambda/a + \sqrt{1+\lambda^2/a^2}\right).$$

Formulæ (22)—(25) were used in computing the following tables:—

x/a	$k_{1,1}/a$		$-k_{1,2}/a$	
	Formula (22)	Kelvin	Formula (23)	Kelvin.
0·5	1·25302*	1·25324	0·52537	0·52537
0·45	1·27420		0·54682	
0·4	1·29316	1·29316	0·57202	0·57202
0·35	1·31330		0·60049	
0·3	1·34528	1·34527	0·63364	0·63395
0·25	1·38491		0·67321	
0·2	1·43131	1·43131	0·72378	0·72378
0·15	1·49328		0·78927	
0·1	1·58396	1·58396	0·88352*	0·88175

* Computed also by Kirchhoff (*loc cit.*).

x/a	$k_{1,1}/a$	$-k_{1,2}/a$	x/a	$k_{1,1}/a$	$-k_{1,2}/a$
0·09	1·60805	0·90633	0·02	1·96642	1·27181
0·08	1·63619	0·93619	0·01	2·13667	1·44246
0·07	1·66622	0·96795	0·0₂1	2·70920	2·01598
0·06	1·70234	1·00460	0·0₃1	3·28489	2·59124
0·05	1·74543	1·04852	0·0₄1	3·85998	3·16684
0·04	1·79864	1·10256	0·0₅1	4·43568	3·74249
0·03	1·86783	1·17253	0·0₆1	5·01124	4·31810

x/a.	$-\frac{1}{2}(\partial k_{1,1}/\partial x)$		$\frac{1}{2}(\partial k_{1,2}/\partial x)$.	
	Formula (24)	Kelvin	Formula (25)	Kelvin.
0·5	0·17424*	0·17432	0·20630	0·20630
0·45	0·20105		0·23107	
0·4	0·23158	0·23159	0·26464	0·26464
0·35	0·27319		0·30673	
0·3	0·32924	0·32917†	0·36325	0·36357
0·25	0·40848		0·44299	
0·2	0·52853	0·52852	0·56352	0·56350
0·15	0·73056		0·76603	
0·1	1·13844	1·13844	1·17489	1·17489

* Given also by Kirchhoff (*loc cit*).

† Kelvin had calculated the value previously by himself and made it 0·32926 (see Reprint, p 28)

x/a	$-\frac{1}{2}(\partial k_{1,1}/\partial x)$	$\frac{1}{2}(\partial k_{1,2}/\partial x)$	x/a	$-\frac{1}{2}(\partial k_{1,1}/\partial x)$	$\frac{1}{2}(\partial k_{1,2}/\partial x)$
0·09	1·2751	1·3111	0·02	6·1043	6·1410
0·08	1·4461	1·4823	0·01	12·340	12·377
0·07	1·6665	1·7028	$0\cdot0_21$	124·82	124·86
0·06	1·9608	1·9972	$0\cdot0_31$	1249·8	1249·8
0·05	2·3736	2·4101	$0\cdot0_41$	12500	12500
0·04	2·9938	3·0304	$0\cdot0_51$	125000	125000
0·03	4·0294	4·0661	$0\cdot0_61$	1250000	1250000

Simplified Formulæ for Equal Spheres.

When x/a is small, we find that

$$k_{1,1} = \frac{a}{2}\left(1+\frac{x}{6a}-\frac{x^2}{180\,a^2}\right)\left(1\cdot96351+\tfrac{1}{2}\log_e\frac{a}{x}+\frac{x}{72\,a}-\frac{213\,x^2}{43200\,a^2}\right), \quad (26)$$

$$-k_{1,2} = \frac{a}{2}\left(1+\frac{x}{6a}-\frac{x^2}{180\,a^2}\right)\left(0\cdot577216+\tfrac{1}{2}\log_e\frac{a}{x}+\frac{7\,x}{72\,a}-\frac{251\,x^2}{43200\,a^2}\right). \quad (27)$$

Putting $x/a = 1/10$ in these formulæ, we find that $k_{1,1} = 1\cdot58397$ and $-k_{1,2} = 0\cdot88355$ The true values are 1·58395 and 0·88352. Hence (26) and (27) can be used in all practical calculations when x/a is not greater than a tenth

When an accuracy of the hundredth part of 1 per cent suffices and x/a is not greater than 1/10, we may use the formulæ

$$\tfrac{1}{2}(k_{1,1}-k_{1,2}) = \frac{a}{2}\left(1+\frac{x}{6a}\right)\left(1\cdot2704+\tfrac{1}{2}\log_e\frac{a}{x}+\frac{x}{18\,a}\right), \quad (28)$$

and

$$2(k_{1,1}+k_{1,2}) = a\left(1+\frac{x}{6a}\right)\left(1\cdot3863-\frac{x}{12\,a}\right) \quad (29)$$

for the capacity between the two spheres and their joint capacity respectively.

It will be seen that this latter capacity is very little greater than $2a \log_e 2$, the value which it has when they touch.

Similarly, when the squares and higher powers of x/a are negligibly small compared with unity, the electrostatic force F between them is given by

$$F = -\frac{a(v_1-v_2)^2}{8x}\left\{1-\frac{12\gamma+1}{36}\frac{x}{a}-\frac{1}{6}\frac{x}{a}\log\frac{a}{x}+\frac{x^2}{90a^2}\log\frac{a}{x}\right\}$$
$$+\frac{\log 2-\frac{1}{4}}{12}(v_1^2+v_2^2) \qquad (30)$$

When x/a is small, and $(v_1-v_2)^2$ is not very small compared with $v_1^2+v_2^2$. the magnitude of the attractive force is approximately

$$\frac{a(v_1-v_2)^2}{8x} \qquad (31)$$

We see, therefore, that when the spheres are very close together the attractive force varies as the diameter of either sphere, as the square of their difference of potential, and inversely as the least distance between them

When the spheres are at the same potential, the formula given by (30) agrees with that found by Kelvin for the repulsive force between two spheres in contact As he pointed out, this force is independent of the size of the spheres

Finally, if the charges on the spheres be $+q$ and $-q$ respectively, we find by (28) and (31) that the attraction between them is given by

$$F = q^2/2ax\{1\,2704+\tfrac{1}{2}\log_e(a/x)\}^{-2},$$

which when 2 54 can be neglected compared with $\log_e(a/x)$ may be written

$$F = 2q^2/ax\{\log_e(a/x)\}^{-2}$$

On the Origin of certain Lines in the Spectrum of ε Orionis (Alnitam)

By Sir Norman Lockyer, K.C.B., F.R.S., F. E. Baxandall, A.R.C.Sc., and C. P. Butler, A.R.C.Sc., F.R.A.S., F.R.P.S.

(Received May 15,—Read June 17, 1909)

[Plate 2]

Although the great majority of the lines in the spectrum of ε Orionis—which is the type-star of the Alnitamian Group of the Kensington Classification—have previously been traced to their source, there remained a few outstanding lines for which no satisfactory origin has been suggested. Recent research here has revealed what appear to be, with little or no doubt, the laboratory equivalents of these lines, whose approximate wave-lengths are 4097·4, 4379·8, and $\left\{\begin{matrix}4647·6\\4650·8\end{matrix}\right.$, the latter being a close double, the mean position of which falls very near the strong oxygen line 4649·3. The first of these lines was given in previous publications* as being probably due to silicium, forming with lines at 4089·1, 4116·4, what were described as the Group IV. lines of that element. In a later publication by Mr. Lunt,† although the last mentioned lines were confirmed by him as being high temperature silicium lines, he concluded that the line 4097 could not be ascribed to Si, as he had found no trace of such a line in any of his silicium photographs. The results now arrived at here show that his conclusions were correct, this line having been traced to another origin.

In the course of an inter-comparison of the spark spectra of various metals, a set of four lines of peculiar behaviour was noticed by one of us in the spark spectrum of chromium. These appeared as "pole" or "beaded" lines near one edge of the spectrum, and are shown in strip 10 of the Plate, where they are indicated by four small arrows. Although the spectrum contained several hundreds of lines, a search for other lines of similar appearance along the same edge of the spectrum failed to reveal any. Here, then, was a set of evidently associated lines—possibly all due to the same element—which occurred specially under the conditions of the vapour surrounding one pole of the spark. It was at once seen that the position of the lines was in the

* 'Roy. Soc. Proc.,' vol. 67, pp. 403—409, October, 1900; 'Roy. Soc. Proc.,' vol. 74, pp. 296—298, October, 1904.

† "The Spectra of Silicon, Fluorine, and Oxygen," 'Annals of Cape Observatory,' vol. 10, p. 153, 1906.

vicinity of Hδ, and the presence of strong lines in this region in the spectrum of ε Orionis suggested a careful investigation of the laboratory wave-lengths and a comparison with the wave-lengths of the stellar lines

As the chromium photograph contained lines of iron and calcium as impurities, several standard lines of these metals were available for use as fiducial lines Those adopted were 4005 408 (Fe), 4063 759 (Fe), and 4226 904 (Ca) From measures made on these and the "pole" lines, the wave-lengths of the latter were calculated by Hartmann's formula They resulted as follows —

4089 04	4103·54
4097 49	4116·29

The first and last of these were so near the wave-lengths of the two strong lines of silicium (Group IV), previously referred to, that a silicium origin at once suggested itself. A comparison of the chromium and silicium spark photographs confirmed this suggestion, the lines in the silicium spark agreeing exactly in position with the outside pair of "pole" lines of the chromium spark

To establish definitely the occurrence of silicium as an impurity in the chromium, the other groups of silicium lines were looked for in the chromium spectrum Those of Group II (4128 1, 4131 1) and Group III (4552 8, 4568 0, 4574 9), were unmistakably present. The lines of Group I (3905·8, 4103·2) were doubtfully present, but as these occur more prominently as arc than as spark lines, their possible absence could be understood It may be said here that the Group II and Group III lines of silicium occur in this spark spectrum of chromium as ordinary lines, and not as lines intensified at one edge of the spectrum similarly to the Group IV lines.

It thus became evident that at least two of the strange quartette (4089 and 4116) could be accounted for as being due to silicium occurring as an impurity in the chromium used

With regard to the other pair, the stronger line of the two (4097 4) was known to be so near the well-marked line of unknown origin in ε Orionis (mentioned at the beginning of this paper) that it was determined to enlarge the stellar spectrum up to the scale of the laboratory spectrum of chromium and make a direct comparison of the two to see if the stellar and laboratory lines agreed in position When this was done it was found that the three lines mentioned (4089, 4097, 4116) fitted exactly the three well-marked lines of ε Orionis in the neighbourhood of Hδ (see strips 10 and 11 of the Plate) The position of the fourth strange line (4103·5) is so near Hδ that the latter line, being strong and diffuse in the star, probably masks the stellar counterpart of the laboratory line.

Although, then, there appeared to be no doubt as to the identity of the stellar and laboratory lines, it remained to determine the chemical element to which the middle pair of abnormal lines could be attributed

Search for Origins of Lines 4097·4, 4103·5

From previous experience we knew that the Group IV lines of silicium—with which the above lines are associated in the chromium spark photograph—show more prominently under vacuum tube conditions than in the ordinary spark spectrum We determined, therefore, to test various vacuum tubes to see if any of them furnished such a pair of lines. As these lines are in the portion of the spectrum beyond the visual region, it was impossible to watch for their appearance, or, if they occurred in the spectrum, to watch their behaviour while the electrical conditions were being varied It was known, however, that in the Alnitamian stars there is invariably a strong double line of unknown origin near the strong oxygen line 4649·3 This being well inside the visible region of the spectrum, a careful comparison of the region near this oxygen line was made, using vacuum tubes containing various gases, with the object of determining whether any abnormality or intensification near the oxygen line mentioned was introduced while the spark discharge conditions were being varied

Using a vacuum tube which contained alcohol vapour, under the low-tension spark condition the oxygen line 4649·3 and other oxygen lines in the same region presented quite their normal appearance. With a strong disruptive spark, however, it was instantly seen that there was a conspicuous intensification in the spectrum at or near 4649, without any corresponding alteration in the intensity of the neighbouring oxygen lines This appeared so encouraging that it was decided to obtain at once a photographic record of the spectrum for more detailed examination The resulting photograph—portions of which are reproduced in strips 2 and 16—showed the spectrum under the disruptive spark conditions. As compared with the lines near 4649 as they appear in the normal oxygen spectrum (strip 18), it will be seen that there is a considerable modification in the appearance of the lines in the alcohol spectrum (strip 16), an extra line having appeared on the more refrangible side of the oxygen line 4649·3, and the oxygen line on the less refrangible side, 4650·8, being considerably strengthened, probably by the superposition of another strange line.

These strange lines will be referred to in detail in a later part of the paper, and their relation to ϵ Orionis lines discussed.

An examination of the alcohol photograph in the region near λ 4100 showed at once an outstanding pair of lines at λ 4097·4, 4103·5, which,

when directly compared with the pole lines of the chromium spark photograph, was found to agree exactly with the middle pair, the origin of which was under investigation. The alcohol spectrum contained, in addition to oxygen, hydrogen, and carbon lines, the strange lines referred to, and the ordinary lines of nitrogen. These latter may have been introduced by a slight leakage of air into the tube. A comparison of the alcohol photograph with various photographs of oxygen, carbon oxides, and hydrogen spectra, failed to show on the latter any lines corresponding to the pair under investigation. It was found, however, that a pair of lines corresponding exactly in position with the strange double existed in the ordinary spark spectrum of nitrogen, the components being of the same relative intensity, but the double in the nitrogen being quite insignificant in intrinsic intensity as compared with its appearance in the alcohol tube. This, then, tended to show that the lines were due to nitrogen, but under certain conditions of current were abnormally strengthened, relatively to other nitrogen lines.

Photographs were then taken of the spectra given by a nitrogen vacuum tube (Gallenkamp)—

(1) Using the large jar and large air-break.
(2) „ small „ small „

These spectra are respectively shown in strips 3, 5. Under the low-tension condition, although the strong nitrogen line 3995 of the ordinary spark spectrum is shown as a strong line, the double 4097—4103 is lacking. Under the high-tension condition, however, the latter double is very conspicuous, the stronger of the pair being now quite as strong as, if not stronger than, the nitrogen line 3995. The nitrogen tube spectrum of strip 3 shows the stronger oxygen lines (Note the triplet 4070—4076.) Strip 4, however, shows the nitrogen vacuum tube spectrum without any trace of oxygen, but still showing the abnormal double 4097·4—4103·5.

The following table shows the intensities of the ordinary strong nitrogen line 3995·1, and the abnormal lines 4097·4—4103·5 as they occur in the

λ.	Exner and Haschek	Kensington spectra.		
	Spark spectrum. Max. int 50	Normal spark at atmospheric pressure Max. int 10.	Vacuum tube. Low tension. Max. int 10	Vacuum tube High tension Max. int. 10
3995·1	50	10	8	10
4097·4	3	4	—	10
4103·5	3	2	—	7

normal spark spectrum of nitrogen, and as a comparison the relative intensities are given of the same lines in the Kensington vacuum tube photographs referred to and reproduced in the plate.

During the search for the origins of the pair of lines 4097—4103 the spectrum given by an amidogene vacuum tube was photographed This was found to contain four lines which correspond exactly in position with the four "pole" lines of the chromium spark A portion of this amidogene spectrum is reproduced in strip 12 Reference to this and strips 10 (chromium), 11 (ϵ Orionis), and 13 (nitrogen high tension spark), will show that the three lines 4089, 4097, and 4116 of the amidogene and chromium spectra are identical with three very prominent lines of ϵ Orionis, the fourth line 4103 being probably masked in the star by the adjacent strong Hδ line The middle pair of the amidogene (strip 12) and chromium (strip 10) quartette is also seen to be identical with the strongly developed pair of the nitrogen spectrum of strip 13 The presence of the two silicium lines 4089 1 and 4116 4 in the amidogene spectrum is probably accounted for by the presence of small detached particles of glass in the bore of the capillary of the vacuum tube, which was an old one

Reference to Nitrogen Lines in Stellar Spectra

Shortly after the discovery by Mr Frank McClean in 1897 of oxygen lines in some of the helium stars, the identity of other lines in similar types of spectra with the stronger spark lines of nitrogen was established by a comparison of the Kensington laboratory and stellar photographs, and these identifications were incorporated in the tabular matter in a Catalogue of 470 of the Brighter Stars.*

In a paper "On the Presence of Oxygen in the Atmospheres of certain Fixed Stars,"† Mr David Gill, after saying (p 205). "there remains not the slightest doubt that all the stronger oxygen lines are present in the spectrum of β Crucis, at least between λ 4250 and λ 4575," goes on to say "It is almost equally certain that there is no trace of true nitrogen lines in this spectrum" In this he was probably referring to the limited region of the spectrum which he investigated. That the strong nitrogen lines 3995·1, 4630 7 occur in the β Crucis spectrum there can be no question, as a reference to Mr McClean's tabular list of lines‡ and his reproductions will clearly show

Another line recorded by McClean in β Crucis at λ 4447 2 is doubtless

* Published by the Solar Physics Committee (1902).

† 'Roy Soc. Proc.,' vol. 65, p 205, April, 1899

‡ 'Spectra of Southern Stars,' 1898

the counterpart of the strong nitrogen line at the same wave-length (Exner and Haschek's λ 4447 23, intensity 20). This line occurs in the region investigated by Gill, but he does not record it in his list of stellar lines.

The wave-length of the line in ε Orionis near 4097 as recorded in a previous publication* was 4097·3 The wave-length of the abnormal nitrogen line with which this has now been identified has, from measures made on the Rowland grating photographs of the nitrogen spark, been found to be 4097 45 This is not far removed from the recorded wave-length of the stellar line, but the latter has, with others in the same region, been remeasured, and its position redetermined by use of Hartmann's formula.

The fiducial lines used were 4026 34 (helium), 4143 92 (asterium), and 4388 10 (asterium) The resulting stellar wave-lengths are given in the first column of the following table. The second column gives the wave-lengths of the corresponding lines as reduced from the Rowland grating photographs of the laboratory spectra The last column gives the origins:—

Stellar wave-length Reduced from photograph with one 6 inch Henry prism	Laboratory wave length Reduced from Rowland grating photograph	Origin.
4076 19	4076 08	Oxygen
4089 14	4089 09	Silicium (IV)
4097 59	4097 45	Nitrogen (abnormal)
4116 54	4116 51	Silicium (IV)

Taking into account the fact that the stellar photograph is of comparatively small dispersion there is very good accord in the wave-lengths. The differences between the two sets are within the limits of error in determining the stellar wave-lengths, and there seems no reason to doubt from this evidence that the identity of the stellar and laboratory lines given in the table is a real one

The wave-length 4096·9 recorded in the Harvard publication† for the ε Orionis line appears to be about half a tenth-metre too low Hartmann,‡ for what is undoubtedly the corresponding line in δ Orionis, gives 4097 49, which is in very good accord with the redetermination of the stellar wave-length (4097·59), and also with the wave-length of the nitrogen line identified with it (4097·45).

* 'Catalogue of 470 of the Brighter Stars,' Solar Physics Committee, 1902.

† "Spectra of Bright Stars," 'Annals. Harv. Coll. Obs.,' vol. 28, Part 1, Table IV p 53 (1897).

‡ 'Astrophysical Journal,' vol. 19, p. 272, 1904.

In the paper by Mr. J. Lunt,* in which he suggests that the stellar line 4096·9 cannot be ascribed to silicium, he points out that it is a very important stellar line, and gives the following extract from Cannon and Pickering's intensities —

Wave-length	Intensities.				
	29 Canis majoris.	τ Canis majoris	ϵ Orionis	β Centauri.	γ Orionis
4089·2	6	12	15	5	2
4096·9	18	6	4	2	1
4101·8	25	25	25	35	40
4116·2	3	6	10	2	0

In a footnote, Lunt says "The first and last of these are silicon lines Cannon and Pickering assign no origins" In a subsequent paper,† Miss Cannon ascribes the lines 4089·2 and 4096·9 to argon These identifications will be discussed in a later part of the present paper

From the preceding table it will be seen that in the star 29 Canis Majoris line 4096·9 is tremendously strengthened relatively to the silicium lines 4089·2 and 4116·2 In ϵ Orionis, although it is a well-marked line, it is considerably inferior in intensity to the two lines just mentioned

With reference to the origin of the stellar line 4096·9 (Hartmann's λ 4097·49), Lunt,‡ after saying "Some other origin than silicium must be sought for this line," goes on to say "There are both oxygen and nitrogen lines very close to this place, but neither of these elements accounts for the strong stellar line" That the nitrogen line he refers to is a line of abnormal behaviour was not, of course, then known to him, nor to us, and he probably based his opinion as to its non-identity with the stellar line on its insignificant intensity in the ordinary nitrogen spectrum.

It will be instructive to compare the intensities of the strongest *spark* lines of nitrogen, as they occur in various types of stellar spectra, with the intensity of the abnormal nitrogen line 4097·4 in the same types. The following table gives these comparative intensities in the Rigelian, Crucian, and Alnitamian groups of the Kensington classification.

The type-stars of these groups are respectively Rigel, Bellatrix, and ϵ Orionis. It may be said here that no nitrogen lines, either of the normal or abnormal kind, occur in any of the groups representing a lower stage of

* 'Roy. Soc. Proc.,' A, vol. 76, p. 122, February, 1905
† 'Annals Harv Coll. Obs.,' vol. 56, part 4, pp 66, 67
‡ 'Roy. Soc. Proc.,' A; vol. 76, p 124, 1905

temperature than the Rigelian. As the Kensington photographs of stellar spectra include nothing of a higher level than the Alnitamian group, it is impossible to say how the lines of nitrogen behave in the higher groups

Stellar group	Nitrogen lines.	
	3995·1 (Normal strong spark line)	4097·4 (Abnormal line)
	Intensity, max. 10	Intensity, max. 10
Alnitamian (ε Orionis)	1	4
Crucian (γ Orionis)	4	1
Rigelian (β Orionis)	2	—

This table shows that the ordinary spark lines of nitrogen (as represented by the strongest line of that class) come in as weak lines in the Rigelian stars, intensify and obtain their maximum intensity at the Crucian stage, and weaken again at the higher Alnitamian stage On the other hand, the abnormal nitrogen line 4097·4 is lacking in the Rigelian, occurs as quite a weak line in the Crucian, and has considerably developed at the Alnitamian stage As shown in a previous table abstracted from Cannon and Pickering's publication, this line becomes more intense still in other stars, having an intensity 6 in τ Canis Majoris, and an intensity of 18 in 29 Canis Majoris In the latter star it closely approaches the intensity of Hδ (25)

Reference to Argon Lines.

In a recent publication on the 'Classification of Stars by their Photographic Spectra,'* Miss Cannon gives a brief description of each class of spectrum. Under Classes Oe, B, and B1, of which the type-stars are respectively 29 Canis Majoris, ε Orionis, and β Canis Majoris, lines 4089·2 and 4096·9 are referred to as being due to argon. Abundant evidence has been given in previous Kensington publications† that the former line is due to silicium (Group IV), and this has been confirmed by Lunt at the Cape Observatory. The second line is undoubtedly identical with the ε Orionis line which has now been traced to the abnormal nitrogen line 4097·4.

No evidence is given in the Harvard publication on which the identity of these stellar lines with argon lines has been based, and it is difficult to

* 'Annals Harv. Coll Obs.,' vol. 56, No. 4.

† 'Roy. Soc. Proc.,' vol 67, pp. 403—409, October, 1900; vol. 74, pp 296—298, October, 1904.

understand how they have come to be recorded as argon Reference to Eder and Valenta's publication* on the argon spectrum shows that there are lines at λλ 4089 04, 4097 27. The respective intensities of these, however, are only 1 and 2 where the maximum is 10 As there are, between λλ 3900 and 4700, 114 lines of argon, varying in intensity from 1 to 10, the evidence for the stellar lines being argon is almost negligible unless it can be shown that the stronger argon lines also occur in the stellar spectrum, or, as an alternative, that this pair of weak argon lines has some special behaviour relatively to other argon lines when the laboratory conditions are varied

To put this alternative to the test, the argon spectrum was photographed under the high-tension spark conditions which produced the abnormal nitrogen lines previously referred to, but there was no evidence of any relative strengthening of the weak argon lines in question

Comparison has also been made of the wave-lengths of the strongest lines of argon with those of the ε Orionis lines and there appears to be no connection whatever between the two sets

Line 4379 8

After the stellar line 4097 had been identified with a nitrogen line of abnormal behaviour, the nitrogen spectrum was examined to see if there was an anomalous line which could account for a fairly well-marked and sharp line in ε Orionis whose wave-length had, from recent measures, been estimated as 4379·8, and whose origin was unknown In the ordinary spark spectrum of nitrogen there is a line of insignificant intensity agreeing in position with the stellar line Exner and Haschek's wave-length for this line is 4379·75 Its intensity is 1 as compared with 50 for the strong spark line at 3995 1, and 20 for the strong spark line at 4447 2

On turning to the spectrum given by the Gallenkamp vacuum tube of nitrogen when the high-tension spark is used, this line was seen to be enormously developed relatively to other nitrogen lines This is shown in strip 7, where the line in question is quite as strong as 4447 2, whereas in strip 9 although the 4447 2 line is very strong, 4379 8 is lacking.

In another spectrum of the Gallenkamp tube, not reproduced on account of the broad nature of the lines, the line 4379·8 is, without exception, the strongest line in the whole spectrum.

In the light of the existence of the other abnormal nitrogen line 4097·4 in the stellar spectrum, then, there is little doubt that this abnormal line 4379 8 is identical with the stellar line at the same wave-length.

* 'Beitrage zur Photo chemie und Spectralanalyse,' p 247, Wien, 1904.

Lines 4647·6—4650·8.

When a strange double in this region was found in the alcohol spectrum (strip 16) which showed the abnormal nitrogen lines previously discussed, no doubt was felt as to its identity with the well-marked unknown double in ε Orionis (see strip 14)

Pickering, although he only records the stellar line as single, gives its wave-length as 4649·2, which is very near the wave-length of the oxygen line (4649·27), which falls about midway between the components of the strange double in Kensington laboratory spectra, giving the appearance of an equal-spaced triplet (see strips 16 and 17)

To establish the identity more thoroughly, the wave-lengths of the lines were determined both from the laboratory and stellar photographs The resulting values are as compared below —

Stellar wave-lengths.	Laboratory wave-lengths
4647·6	4647·53
4650·8	4650·92

Taking into consideration the comparatively small dispersion of the stellar spectrum, their wave-lengths are in as good accord as could be expected

Reference to strips 14 and 16 will show that, although the stellar spectrum is enlarged 12 times, the double given by the alcohol vacuum tube and the stellar double are identical in position The two oxygen lines 4591 and 4596 are also seen to be in agreement with two weak stellar lines

Although the identity of the stellar and laboratory doubles had been practically established both by measurement and direct comparison, there was no certainty as to their exact origin As the alcohol spectrum in which the lines were first noted showed lines of oxygen, carbon, hydrogen, and nitrogen (the latter as impurity lines), it seemed reasonable to assume that the lines in question belonged to one of these elements.

With the object of tracing the lines to a definite origin, the spectra of various vacuum tubes were then investigated both visually and photographically, the same spark conditions being used as in the case of the alcohol tube which gave the lines The coil used was one which gives an 8-inch spark, and a large jar and large air-break were introduced

The vacuum tubes investigated were those respectively containing nitrogen, amidogene, oxygen, sulphur-dioxide, coal gas, and carbon bisulphide. The only three of these to reveal the lines were SO_2, CH_4, and CS_2. The fact that neither the oxygen nor nitrogen tubes gave the lines seems to preclude the possibility of their being due to either of these elements. The

SO_2 and CS_2 spectra, in which the double occurs, show no hydrogen lines, so hydrogen could be discarded as furnishing no clue. Sulphur clearly could not be the origin, as there is no trace of the lines of that element in either the alcohol or coal-gas spectra which give the strange double

This process of elimination left only carbon to be considered. That element could account for the presence of the lines in the alcohol, CS_2, and CH_4 spectra, but not in that of SO_2 if the sulphur dioxide was pure and free from carbon To test the latter point the strong characteristic line of carbon at 4267·3 was looked for in the SO_2 spectrum, and it was at once seen that it not only occurred there, but was one of the strongest lines in the whole spectrum

The fact was thus established that in every case where the strange double appeared, the spectrum also contained the strong carbon line 4267 3, and the collective evidence pointed to carbon or some modification of carbon as being the true source of the lines.

It was then determined to investigate the spectra given by the spark between carbon poles in various gases at atmospheric pressure, using exactly similar electrical conditions—with regard to coil, jar capacity, and size of air break—as for the vacuum tube experiments The first gas tried in this way was oxygen, and although visual observations failed to show the lines with certainty, the photographic record showed that the more refrangible component was present but weak; the less refrangible component, falling on the fairly strong line of oxygen at 4650·8, cannot be separately distinguished

The spectrum of the carbon spark in nitrogen and ammonia was then tried, but as the visual observations gave no indication of the lines under discussion, no photographs were taken.

The spark in hydrogen was then examined, and it was at once evident that this condition gave the double quite strong and isolated. A photograph of the spectrum was obtained, and the region 4550—4670 of this is reproduced in strip 15, where it will be seen that the double stands alone, there being no oxygen lines mixed up with it, as in the alcohol and SO_2 spectra of strips 16 and 17. The original photograph of the carbon spark spectrum in hydrogen shows, in addition to the double 4647·6, 4650·8, the strong carbon line 4267 3, the hydrogen lines, and a faint trace of the strongest nitrogen line 4630·7. The last named is shown in the Plate; the others are outside the region of spectrum reproduced.

As further evidence of the identity of the stellar and laboratory lines, it may be pointed out that the nature of the double, as seen isolated in strip 15 (carbon spark in hydrogen), is the same as in the stellar spectrum

(in the latter case this point is not so well shown in the reproduction as in the original negative), the more refrangible line 4647·6 being sharp on each edge, while the other component, 4650·8, is sharp on its more refrangible edge, but diffuse towards the red

The fact that the double occurs strongly in the spectrum of carbon poles in hydrogen, only weakly in that of the same poles in oxygen, and apparently not at all when nitrogen is the gas used, would tend to show that, if really due to carbon, the presence of hydrogen, although not absolutely necessary, is conducive to the production of the lines It must be borne in mind, however, that the SO_2 vacuum tube gave the lines quite strongly without any trace of hydrogen being present. It may be that, although in the presence of oxygen only along with the carbon the lines are not well developed, the presence of sulphur as an additional element has the same effect on the development of the lines as that of hydrogen.

The bulk of the evidence is certainly in favour of the strange double being due to carbon or some modification of that element, the lines only coming out strongly under particular conditions of current Even at this stage of the inquiry, however, the evidence is not absolutely conclusive, and a carbon origin must be accepted as only provisional. Further research will probably settle the origin more definitely

It may be said here that, so far as we are aware, this important double line has not previously been recorded in laboratory spectra, either of carbon or any other element The reason for this is probably that, in contradistinction to the carbon line 4267 3, which occurs under a comparatively great range of spark conditions, the new double is of a more fleeting nature, and has probably hitherto eluded the attention of spectroscopists.

Reference to Bright-line-star Spectra.

In a paper on "The Wolf-Rayet Stars,"* Prof. Campbell gives a comparison of the lines in "Bright-line Stars" with those of other types, amongst the latter being Orion stars (dark lines) In this he suggests that the strong line 4652 in stars of the Wolf-Rayet type is identical with the line 4652 of the Orion stars. The line he refers to in the latter case is undoubtedly the strong conspicuous line of ε Orionis, which has been found to be a double line in the Kensington spectra (4647·5—4650·9), and which has been shown to be, in all probability, due to carbon, the laboratory lines occurring prominently only under particular conditions of current.

The position of the middle of this double is, from recent Kensington measures of the best ε Orionis spectrum, 4649·2, this being also the wave-

* 'Ast. and Ast. Phys.,' vol. 13, p. 473, 1894.

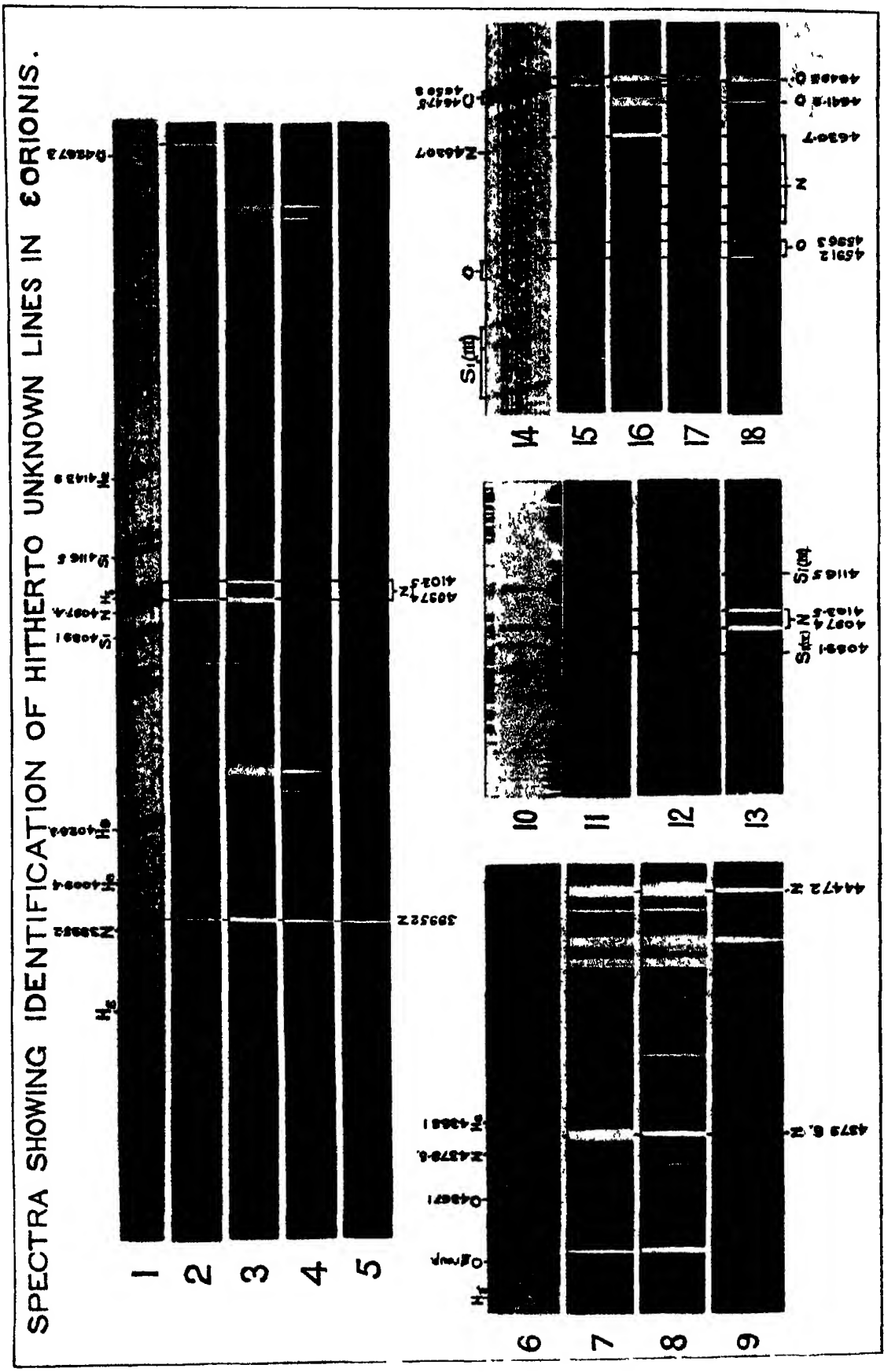
length of the single line recorded by Pickering If this be really identical with the Wolf-Rayet line, it would appear that Campbell's wave-length for the latter is somewhat high, but the difference is possibly within the limits of error in determining the wave-lengths of such broad, crude lines as those in Wolf-Rayet stars. The line 4652, for instance, is, in γ Argus, of such a width as to cover about 30 tenth-metres

Possibly the apparent agreement in position of the Wolf-Rayet line and the strong, dark double of ϵ Orionis, is only a fortuitous one, but as these spectra contain other lines in common, such as the stronger helium lines, and proto-hydrogen lines (4686, 4542, 4200), it is suggestive of a real identity

It seems necessary to obtain, if that be possible, more accurate wave-lengths of the lines in Wolf-Rayet stars before this point as to the identity of the two lines can be definitely settled

Further research will be made with the object of possibly tracing other conspicuous Wolf-Rayet lines to abnormal lines of known gases

[*Note added June*, 1909.—Shortly after the above paper was sent in, and before the reading, further experiments were made, giving conclusive evidence that the double line at 4647·6, 4650·8, could be ascribed to carbon. Three spectra were photographed on the same plate, the spark conditions being identical in every case, (1) carbon spark in hydrogen, (2) platinum spark in coal-gas, (3) platinum spark in hydrogen. While the platinum spark in hydrogen shows no trace whatever of the double, the platinum in coal-gas gives it, quite conspicuous and identical in position and nature with the double produced with carbon in hydrogen.]


SPECTRA SHOWING IDENTIFICATION OF HITHERTO UNKNOWN LINES IN ε ORIONIS.
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DESCRIPTION OF PLATE 2.

Strip No. 1.—Spectrum of ε Orionis (region 3900—4270). Directly enlarged 7 times.

" 2.—Spectrum given by alcohol vacuum tube (8″ coil, disruptive spark, large jar, large air-break). There is an impurity of nitrogen in this spectrum. The pair of N lines 4097—4103, which in the ordinary spectrum are very weak where 3995 is strong, are here shown as strong as, or stronger than, 3995. The stronger of the pair is identical with a well-marked stellar line. The stellar counterpart of 4103 is probably masked by strong Hδ.

" 3.—Spectrum of S P O vacuum tube of nitrogen (8″ coil, disruptive spark, large jar, large air-break), showing the ordinarily-weak N lines 4097—4103 very strongly developed, the former quite as strong as the well known N line 3995.

" 4.—Spectrum of Gallenkamp nitrogen vacuum tube (8″ coil, small jar, small air-break), showing 3995 strong, 4097—4103 weak.

" 5.—Spectrum of S.P O vacuum tube of nitrogen (8″ coil, small jar, very small air-break), showing 3995 strong, 4097—4103 lacking.

" 6.—Spectrum of ε Orionis (region 4340—4450). Directly enlarged 12 times.

" 7.—Spectrum of Gallenkamp nitrogen vacuum tube (8″ coil, disruptive spark, large jar, large air-break), showing line 4379·8—which is a weak, insignificant line in the ordinary nitrogen spark spectrum—quite as strong as 4447, which is, in the spark spectrum, one of the strongest lines.

" 8.—Spectrum of Gallenkamp nitrogen vacuum tube (8″ coil, small jar, small air-break), showing 4447 considerably stronger than 4379·8.

" 9.—Spectrum of Gallenkamp nitrogen vacuum tube (8″ coil, small jar, very small air-break), showing 4447 quite strong, 4379·8 being lacking.

" 10.—Spark spectrum of chromium (Spottiswoode coil, 42″ spark, large jar, large air-break), showing the four lines 4089, 4097, 4103, 4116 as "beaded" or "pole" lines near the bottom edge of the spectrum. They are indicated by four small arrows just above the "beads." The first and last are silicium (IV) lines, the middle pair abnormal nitrogen lines. The agreement of three of these with well-marked ε Orionis lines (Strip 11) will be seen. The remaining line 4103 is probably masked in the star by the strong Hδ line.

" 11.—Spectrum of ε Orionis (region 4050—4150). Directly enlarged 7 times.

" 12.—Spectrum of amidogene vacuum tube (8″ coil, large jar, large air-break), which gives four lines identical in position with the four "pole" lines of the chromium photograph (Strip 10). These amidogene tube lines are seen to agree also in position with lines in ε Orionis. The two silicium lines, 4089, 4116, in this spectrum are due to the presence of detached particles of glass in the bore of the vacuum tube.

" 13.—Spectrum of S P O vacuum tube of nitrogen (8″ coil, large jar, large air break), showing the abnormal pair of N lines 4097—4103 identical in position with the middle pair of the four lines given by the amidogene tube (Strip 12).

" 14.—Spectrum of ε Orionis (region 4550—4670). Directly enlarged 12 times.

" 15.—Spectrum of carbon spark in hydrogen (8″ coil, large jar, large air-break), showing strong double at 4647·5—4650·9, probably due to carbon, identical in position with a strong unknown double in ε Orionis (Strip 14). There is in this spectrum a trace of the strong N line 4630·7, which also occurs as a weak line in the stellar spectrum.

" 16.—Spectrum of alcohol vacuum tube (8″ coil, large jar, large air-break), showing the double carbon line of Strip 14 divided by the oxygen line 4649·3. This spectrum contains nitrogen lines, possibly due to a leakage of air.

" 17.—Spectrum of SO_2 Gallenkamp vacuum tube (6″ coil, small jar, small air-break), showing the double carbon line of Strip 14, divided by the oxygen line 4649·3. There is evidently an impurity of carbon in this vacuum tube, as the well-known high temperature carbon 4267·3 is very strongly shown in the SO_2 spectrum, but the line is outside the region reproduced in Strip 17.

" 18.—Spectrum of Gallenkamp oxygen vacuum tube (8″ coil, large jar, large air-break). The line immediately to the right of the strong 4649·3 line is a true oxygen line, but the lines in Strips 16 and 17, which apparently correspond to it, are there much too strong to be accounted for by the oxygen line alone. In those strips the line in question is probably compounded of the oxygen line (4650) and the less refrangible member of the carbon double of Strip 15.

On Pressure Perpendicular to the Shear Planes in Finite Pure Shears, and on the Lengthening of Loaded Wires when Twisted.

By J H Poynting, Sc D., F R S

(Received May 20,—Read June 24, 1909)

In the 'Philosophical Magazine,' vol. 9, 1905, p 397, I gave an analysis of the stresses in a pure shear which appeared to show that if ϵ is the angle of shear and if n is the rigidity, then a pressure $n\epsilon^2$ exists perpendicular to the planes of shear. That analysis is, I believe, faulty in that the diagonals of the rhombus into which a square is sheared are not the lines of greatest elongation and contraction, and are not at right angles after the shear, when second order quantities are taken into account, *i e*, quantities of the order of ϵ^2, I think the following analysis is more correct, and though it does not give a definite result, it leaves the existence of a longitudinal pressure an open question The question appears to be answered in the affirmative by some experiments, described in the second part of the paper, in which loaded wires were found to lengthen when twisted by a small amount proportional to the square of the twist

I —*Stresses in a Pure Shear*

Let a square ABCD (fig 1) of side a be sheared into EFCD by motion through $AE = d$, the volume being constant The angle of shear is $ADE = \epsilon$, and $\tan \epsilon = d/a$ exactly, neglecting ϵ^3, we may put $\epsilon = d/a$

To find which line is stretched most by the shear, consider the line r drawn from D to P and making θ with DC before stretching

Let it stretch to ρ, making θ' with DC; we have $r = a/\sin\theta$ and $\rho = a/\sin\theta'$,

also $$\rho^2 = r^2 + 2rd\cos\theta + d^2,$$

thus $$\rho^2/r^2 = 1 + 2d/r \,.\, \cos\theta + d^2/r^2,$$

$$= 1 + 2d/a \,.\, \sin\theta\cos\theta + d^2/a^2 \,.\, \sin^2\theta$$

Differentiating ρ^2/r^2 with respect to θ, it is a maximum when

$$2d/a \;\cos 2\theta + d^2/a^2 \,.\, \sin 2\theta = 0,$$

or $$\tan 2\theta = -2a/d = -2\cot\epsilon.$$

Put $\theta = 45° + \delta$, then $\tan 2\delta = \frac{1}{2}\tan\epsilon$,

or $\delta = \frac{1}{2}\epsilon$ to the second order, so that r makes $\frac{1}{4}\epsilon$ with the diagonal DB of the square through D, and on the upper side.

If the same shear is now made in the opposite direction ρ contracts to r, and the same directions of ρ before, and r after shear, give the maximum contraction It is almost obvious that ρ makes $\frac{1}{4}\epsilon$ with DB on the lower side, but it may be verified by putting

$$r^2 = \rho^2 - 2d\rho\cos\theta' + d^2,$$

and finding the maximum value of r^2/ρ^2 after putting $\rho = a/\sin\theta'$ on the right.

Hence the lines of maximum elongation and contraction are at $\frac{1}{4}\epsilon$ with the diagonals of the square, and are at right angles before and after the strain, to the order of ϵ^2 It is noteworthy that as the shear increases the fibres which undergo maximum elongation and contraction change.

To find r and ρ put $r = a/\sin\theta = a/\sin(45° + \frac{1}{4}\epsilon)$,

then $$r = \sqrt{2}\,a(1 - \tfrac{1}{4}\epsilon + \tfrac{3}{32}\epsilon^2);$$

and changing the sign of ϵ we get

$$\rho = \sqrt{2}\,a(1 + \tfrac{1}{4}\epsilon + \tfrac{3}{32}\epsilon^2)$$

It is easily seen that the elongation and contraction are respectively

$$e = (\rho - r)/r = \tfrac{1}{2}\epsilon(1 + \tfrac{1}{4}\epsilon), \qquad e = (r - \rho)/\rho = \tfrac{1}{2}\epsilon(1 - \tfrac{1}{4}\epsilon)$$

We shall now consider the stresses We shall assume that a pressure P is put on in the direction of maximum contraction and a tension Q in the direction of maximum elongation, these being, as we have seen, at right angles, and we shall consider the equilibrium of the wedge ABC (fig 2) when

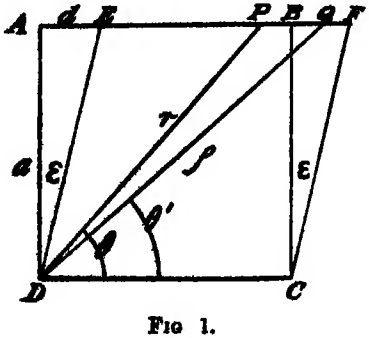

Fig 1.

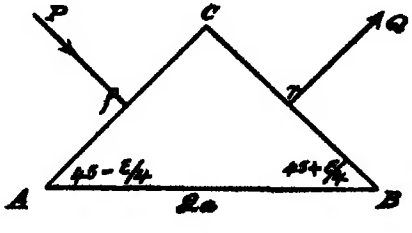

Fig. 2.

sheared, assuming that P and Q are the only forces on AC and BC Let AB = $2a$; AC = ρ, BC = r, these having the values just found

Resolve perpendicular to the base and let R be the pressure against the base, then

$$\begin{aligned} R\ 2a &= P\rho\cos(45°-\tfrac{1}{4}\epsilon)-Qr\cos(45°+\tfrac{1}{4}\epsilon) \\ &= Pa\cot(45°-\tfrac{1}{4}\epsilon)-Qa\cot(45°+\tfrac{1}{4}\epsilon) \\ &= Pa\frac{1+\frac{1}{4}\epsilon}{1-\frac{1}{4}\epsilon}-Qa\frac{1-\frac{1}{4}\epsilon}{1+\frac{1}{4}\epsilon} \\ &= Pa(1+\tfrac{1}{2}\epsilon+\tfrac{1}{8}\epsilon^2)-Qa(1-\tfrac{1}{2}\epsilon+\tfrac{1}{8}\epsilon^2) \\ &= (P-Q)a+(P+Q)\ \tfrac{1}{2}a\epsilon+(P-Q)\ \tfrac{1}{8}a\epsilon^2, \end{aligned}$$

where P and Q can only be taken as equal to the first order Proceeding to the second order, we must put

$$P = n\epsilon+p\epsilon^2,$$

then

$$Q = n\epsilon-p\epsilon^2,$$

where p is a constant to the second order

Thus $P-Q = 2p\epsilon^2$ and $P+Q = 2n\epsilon$,

and

$$R = (\tfrac{1}{2}n+p)\epsilon^2,$$

the third term being negligible If we resolve parallel to the base, it is easily found that the tangential stress is

$$T = \tfrac{1}{2}(P+Q) = n\epsilon.$$

If the shear is produced by a tangential stress T, then it requires the system P, Q, and R to maintain equilibrium with it.

It is possible that a stress exists perpendicular to the plane of the figure in fig 1 It can only be assumed that the changes of dimension in that direction neutralise each other to the first order when equal pushes and pulls are put on in the plane of the figure, when the dimensions perpendicular to the figure are constrained to remain the same to the second order—and this is our supposition—it may require a tension or pressure to effect this

Let us suppose that a pressure $S = q\epsilon^2$ is introduced, a tension if q is negative To make $R = 0$ we should require to have $p = -\frac{1}{2}n$, also P would then be less than Q If pressure perpendicular to AC is exerted alone, and then tension perpendicular to BC is exerted alone, it appears probable that for very large equal compressions and extensions P is greater than Q If we suppose that when they are simultaneous the tendency is in the same direction, then R should have a positive value, or the longitudinal pressure perpendicular to AB should exist

Let us examine the consequences of the supposition that both R and S exist. Let a thin tube of length l and of radius a be fixed at one end, and let

the other end be twisted through θ so that the angle of shear is $e = a\theta/l$ Let an end pressure $R = (\frac{1}{2}n+p)e^2$ be put on, and also a side pressure $S = qe^2$ so as to maintain constant dimensions

The side pressure S may be replaced by a uniform pressure S over the whole surface, and a tension S over the ends We have then an end pressure R−S and a pressure S all over

Now suppose that these forces are removed. Through the removal of R−S we shall have a lengthening dl_1 given by

$$(\tfrac{1}{2}n+p-q)e^2 = Y\,dl_1/l,$$

and a contraction δ_1 of the diameter given by

$$\delta_1/2a = \sigma\,dl_1/l,$$

where Y is Young's modulus and σ is Poisson's ratio

Through the removal of the pressure S we shall have a lengthening dl_2 given by $qe^2 = 3K dl_2/l$, where K is the bulk modulus, and an expansion δ_2 of the diameter given by $qe^2 = 3K\delta_2/2a$

The end lengthening is therefore

$$dl = dl_1+dl_2 = \{(\tfrac{1}{2}n+p-q)/Y+q/3K\}\,le^2,$$

or putting $1/3K = 3/Y-1/n,$

it is $\{(\frac{1}{2}n+p)/Y+(2/Y-1/n)q\}\,le^2 = sle^2 = sa^2\theta^2/l,$

where s is put for $(\frac{1}{2}n+p)/Y+(2/Y-1/n)q$

The diameter decreases by

$$\delta = \delta_1-\delta_2 = \{(\tfrac{1}{2}n+p-q)\sigma/Y-q/3K\}\,2ae^2,$$
$$= \{(\tfrac{1}{2}n+p-q)\sigma/Y-[(3+\sigma)Y-1/n]q\}\,2a^3\theta^2/l^2$$

It would not be easy to test this result with a thin tube But if we suppose that a wire extends by the amount equal to the average extension of the tubes into which it may be resolved, we get

$$dl = \frac{1}{\pi a^2}\int_0^a 2\pi r s r^2\frac{\theta^2}{l}\,dr = \tfrac{1}{2}sa^2\theta^2/l$$

I now proceed to describe some experiments which show that such an extension exists.

II.—*The Lengthening of Loaded Wires when twisted*

Experiments were made on several wires hung vertically from a fixed support, and loaded in order that kinks or remnants of the spiral due to the coiling to which they had been subjected might be taken out This was considered to be effected when the stretch for a given addition of load was

sensibly the same whether the wire was twisted or not. An account of the twisting of a steel wire before this stage was reached will be given later. Fig 3 represents the arrangement more or less diagrammatically. The upper end of the wire to be twisted was fixed to a stout bracket B near the ceiling. The wire was always about 231 cm. long, its lower end was clamped in jaws in the upper end of a turned steel rod *rr*, 51 cm long, which passed through a hole in the table T on which was the observing microscope M, and a parallel plate micrometer *m* One division of this micrometer was equal to 0·00974 mm. At the lower end of the rod was a horizontal iron cross-piece *cc*, 19 cm. long and 1 6 cm. square. From the lower end was suspended a carrier for the weights, or for the two stouter wires the weight itself, connected to the rod by a flat steel strip twisted in its middle, so that the upper and lower halves were in two vertical planes at right angles Below the weights was a set of vanes immersed in a shallow bath of oil to damp vibrations. This bath rested on a circular turntable *tt*, on which were two uprights *uu* at opposite ends of a diameter, with horizontal screws at their upper ends which could be brought to bear against the ends of the cross-piece as shown in the plan, fig. 4. The screws ended in small steel balls and the sides of the cross-piece were polished. On rotating the turntable the screws came against the cross-piece and turned it round, and so the wire was twisted by a couple with vertical axis. The axis of the turntable was made vertical by means of the levelling screws *ll*. To adjust this axis in the axis of the wire prolonged, the turntable could be moved over the base plate P by means of the horizontal screws *s*, of which only one is represented in fig. 3 All are shown in fig. 4. A horizontal microscope not represented in the figure was attached to one of the uprights and focussed on the edge of the rod *rr*. The adjustment by the screws *s* was

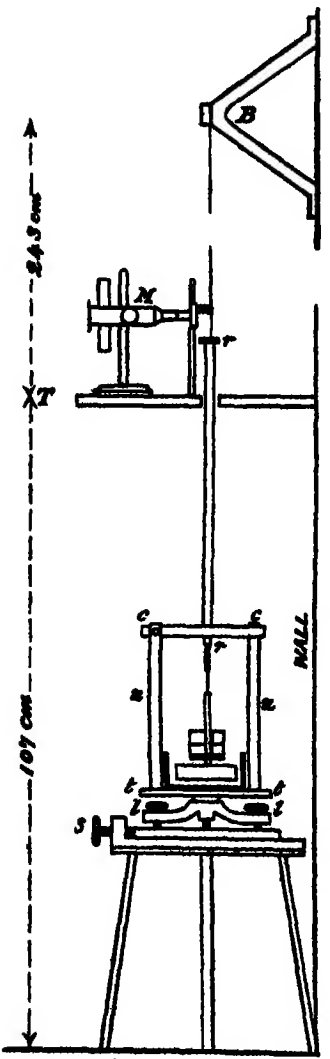

Fig 3.—Elevation of Arrangement for Twisting the Wires.

continued until the microscope always saw the edge of the rod in the middle of the field, however the turntable might be turned.

To give a definite point of view in the microscope M, in the earlier experiments starch grains were put on the wire about 1 cm from the lower end These were illuminated, and a suitable one was selected.

In the later experiments a needle, about 1 cm long, was fixed on the upper end of the rod, point upwards, close alongside the wire, and the needle point was viewed This was better than the starch grains.

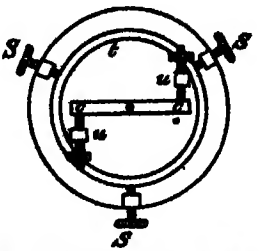

FIG 4.—Plan of the Cross-piece and Turntable

In the earlier work the temperature of the room was fairly steady, and the changes in length due to temperature variations were too slow to give trouble. But in some gusty weather occurring later there were such rapid and considerable variations in the temperature of the room that it was necessary to enclose the wire in a wooden tube After this was done temperature gave no further trouble, whatever the weather

In order to observe the effect of a twist the turntable was levelled and adjusted axially when the wire and cross-piece were free The turntable was rotated till the screws on the uprights just touched the cross-piece. Then chalk marks were made on the turntable and on the plate below, one just over the other The microscope was adjusted exactly to sight the upper or lower edge of a starch grain on its horizontal cross-wire, and the micrometer was read Then the turntable was rotated so many whole turns, and the micrometer plate was moved till the edge of the grain was again on the cross-wire and the micrometer was read again

With the exception of a wire stretched only by the weight of the rod and cross-piece, in some experiments described later, there was always a lengthening on twisting, of the same order whether the twist was clockwise or counter clockwise The lengthening was nearly proportional to the square of the twist put on. It was necessary to limit the twist to a few turns to avoid permanent set, and when such a small twist had been given and the wire was untwisted it returned sensibly to its original length

The lowering was entirely due to twisting and not to any giving of the support, for when a microscope was sighted on a point on the wire close to the upper end, no change in level could be detected, when the wire was twisted through 5 turns at its lower end. This was further verified by an experiment on a steel wire from the same piece as No. 3 below, which showed that the extension half-way down the wire was, within the limits of

experimental error, half that at the lower end. A microscope and micrometer were fixed on a table half-way up the wire and a needle point was fixed here as well as at the lower end At each twist and untwist both micrometers were read I give the observations in this experiment in full, as they will show the sort of accuracy attained

The lower end was twisted from a starting twist of $\frac{1}{4}$ turn to $4\frac{1}{4}$ turns

Micrometer Readings at Lower End.

$\frac{1}{4}$ turn	$4\frac{1}{4}$ turns	Lowering
22 3	18 6	3 7
22 5	19 0	3 5
23 0	19 2	3 8
22 6	19 4	3 2
22 9	19 6	3 3
22 6	19 5	3 1
23 0	19 5	3 5
23 0	18 9	4 1
22 7	19·2	3·5
22 9	19 6	3 3

Mean lowering, 3 50 divisions
One division of micrometer = 0 00974 mm
The lowering is 0·0341 mm

Micrometer Readings Half-way up the Wire

$\frac{1}{4}$ turn	$4\frac{1}{4}$ turns	Lowering
30 4	28·3	2 1
30 5	28 4	2 1
30 2	28 1	2 1
30 5	28 5	2·0
30 5	28·9	2·5
30 4	28 7	1·7
31·0	28 4	2·6
30 6	28 5	2·1
31 8*	29·9	1 9
31·6*	29·9	2 5

* Another point on the needle sighted.

Mean lowering, 2 16 divisions
One division of micrometer = 0·00751 mm
The lowering is 0·0162 mm

If the lowering at the end is accurate, that half-way up should be 0 0171 mm. The observed lowering is as nearly equal to this as could be expected.

With the first wire, determinations of extension due to an addition of 520 grammes were made both in the untwisted and twisted conditions, as it

was only when these became sensibly equal that the lowering on twist became equal for different loads. The extra load could be put on or taken off by lowering or raising a lever, not represented in fig 3 It is unnecessary to describe the details of this arrangement The experiments with the other wires were made with such loads that it was not considered necessary to observe the stretch due to addition of load

Results.

1*a* Steel piano wire, diameter 0 720 mm. (mean of 10 measurements at different points), length in this and all cases to observing point 230 cm Permanent set after putting on eight turns twist and then untwisting only a very few degrees Total load, 7081 grammes.

The twist is termed clockwise when the turntable as viewed from above is moved clockwise

Clockwise twist, 0—4 turns, lowering 0·0181 mm., mean of 10 observations.
„ „ 0—8 „ „ 0 0732 „ „ „

The ratio of these is 4 04 1

The extension due to an addition of 520 grammes was —

No twist on the wire 0·143 mm., mean of 10 observations
4 turns „ 0 141 „ „ „
8 „ „ 0 143 „ „ „

1*b* Same wire
Total load, 9081 grammes

Clockwise twist, 0—4 turns, lowering 0 0181 mm, mean of 20 observations.
„ „ 0—8 „ „ 0 0749 „ „ „

The ratio of these is 4 14 1

The extension due to an addition of 520 grammes was

No twist on the wire 0 142 mm, mean of 10 observations
8 turns „ 0 144 „ „ „

Taking the mean lowering for the two loads of 7081 and 9081 for eight turns twist, viz, 0 074 mm, and taking it as proportional to the square of the twist, the lowering for one turn is 0 00116 mm., and

$$s = 2l\,dl/a^2\theta^2 = 1\,043.$$

The moduli of elasticity of this wire were found to be

$$n = 0\,769 \times 10^{12}, \qquad Y = 2{\cdot}013 \times 10^{12},$$

whence $n/Y = 0{\cdot}382$. The value of n, found for loads of 1081 grammes and 9081 grammes respectively, was identical.

2 The same wire was raised to a red heat, by an electric current, with the load of 9081 grammes on it It lengthened about 3 cm., and this length was cut off The surface oxidised, and when the oxide was rubbed off the diameter was 0 696 mm (mean of 10)

The permanent set after twisting and untwisting was greater, and so only three turns were given

Total load, 9081 grammes

Clockwise twist, 0—3 turns, lowering 0·0129 mm., mean of five observations

The extension due to an addition of 520 grammes was —

No twist on the wire 0 155 mm, mean of 10 observations

3 turns „ 0 154 „ „ „

The lowering for one turn according to the square law is 0 00143 mm, and $s = 1\ 376$ mm

The five values of the lowering were · 1 5, 1 2, 1 2, 1 2, 1 5 divisions, mean 1 32 divisions With such small lowering no accuracy could be expected, and it would be difficult to verify the square law

The moduli of elasticity for the softened wire were —

$$n = 0\ 809 \times 10^{12} \quad \text{and} \quad Y = 2\ 06 \times 10^{12},$$

whence

$$n/Y = 0\ 393$$

3 Steel piano wire, diameter 0 970 mm. (mean of 10)

A needle point fixed at the side of the wire was viewed in the microscope. After twisting and untwisting, a slight permanent set threw the point out of focus if the start was from no twist A quarter turn was therefore put on initially, and the twisting was from this, and the untwisting was back to it

Total load, 19,504 grammes

Clockwise twist, ¼—2¼ turns, lowering 0 0087 mm, mean of 10 observations.

„ „ ¼—4¼ „ „ 0·0339 „ „ „

Counter clockwise twist, ¼—2¼ turns, lowering 0 0089 mm, mean of 10 obs.

„ „ „ ¼—4¼ „ „ 0 0340 „ „

Mean lowering, ¼—2¼, 0 0088 mm

„ „ ¼—4¼, 0 0340 „

By the square law the lowerings for 4¼, 2¼, and ¼ should be as 289 81 · 1, and the differences should be as 288 80 = 18 : 5.

The observed differences are as 18 9 · 5.

The lowering for one turn deduced from the difference between ¼ and 4¼ is 0·00189 mm., and $s = 0{\cdot}936$.

Comparing the lowerings for one turn of this wire with the hard wire No. 1, if the lowering is proportional to the square of the diameter, we ought to have for No 1 a lowering of $0\,00189 \times (72/97)^2 = 0\,00104$ mm The observed lowering was 0 00116 mm, which is as near the calculated value as could be expected

4 The same wire was then raised to a red heat by an electric current with the load on After being rubbed down its diameter was 0 947 mm (mean of 10 measurements). Same load as 3

Clockwise twist, $\frac{1}{4}$—$3\frac{1}{4}$ turns, lowering 0 0207 mm, mean of 10 observations

The deduced lowering for one turn is 0·00197 mm

The value of s is 1 017 mm.

Comparing the lowerings for one turn of this wire with the softened wire No 2, the square law for the diameter should give for No 2 a lowering $0\,00197 \times (696/947)^2 = 0\,00106$ mm The observed lowering was 0 0143 mm, a considerable divergence

5 Copper wire, diameter 0 655 mm (mean of 10) Load, 7081 grammes

Clockwise twist, $\frac{1}{4}$—$2\frac{1}{4}$ turns, lowering 0 0066 mm, mean of 10 observations

Counter clockwise twist, $\frac{1}{4}$—$2\frac{1}{4}$ turns, lowering 0 0083 mm, mean of 10 obs

It was not safe to give a greater twist owing to the largeness of the permanent set. With $2\frac{1}{4}$ turns the set was still small

The larger value of the lowering for the counter clockwise twist is almost certainly real, and not merely error of observation Some other observations showed an even greater excess, though they were very irregular owing to temperature variations, and are not worth recording The extension due to an addition of 520 grammes was—

No twist on the wire 0 268 mm, mean of 20 observations.
3 turns „ 0 269 „ „ „

Taking the mean for clockwise and counter clockwise twist, the lowering for one turn is 0 0015 mm, and $s = 1\,63$ mm

6. Brass wire, diameter 0 928 mm (mean of 10) Load 19,504 grammes

Clockwise twist, $\frac{1}{4}$—$2\frac{1}{4}$ turns; lowering 0 0169 mm., mean of 10 observations

„ „ $\frac{1}{4}$—$4\frac{1}{4}$ „ „ 0 0540 „ „ „

Counter clockwise twist, $\frac{1}{4}$—$2\frac{1}{4}$ turns, lowering 0 0135 mm., mean of 10 obs

„ „ „ $\frac{1}{4}$—$4\frac{1}{4}$ „ „ 0·0479 „ „

The difference between clockwise and counter clockwise twisting is too large for errors of observation.

For the square law the lowerings for $\frac{1}{4}$—$4\frac{1}{4}$, and for $\frac{1}{4}$—$2\frac{1}{4}$ should be in the ratio 18 : 5. They are in the ratios 16 : 5 for clockwise, and 17 7 5 for counter clockwise twisting. The lowering for one turn clockwise, as deduced

from ¼—4¼, is 0·0030 mm, and for one turn counter clockwise is 0 0026 mm.

The mean value of $s = 1\,537$ mm

Experiments with Smaller Loads

When the piano wire diameter 0 72 mm was loaded only with the rod and cross bar weighing 1081 grammes, there was a *rise* on twisting

Clockwise twist,	0—4 turns,	rise 0 041 mm
„ „	0—8 „	0 139 „
Counter clockwise twist,	0—4 „	0 023 „
„ „ „	0—8 „	0·108 „

The extension for an addition of 520 grammes was —

No twist on wire	0 137 mm,	mean of	6	observations.
4 turns clockwise	0 170 „	„	6	„
8 „ „	0 237 „	„	3	„
4 turns counter clockwise	0 156 „	„	6	„
8 „ „	0 238 „	„	3	„

If by means of the observed extensions we calculate the positions of the point viewed, when the load of 1081 grammes is taken off we find that the total rise would be for clockwise twist for four turns 0 110 mm, and for eight turns, 0 347 mm

The rise appears to be due to coiling up of the wire on twisting, through some remnant of the spiral condition in which it existed before suspension. This is confirmed by the very large increase in extension, due to addition of load as the twist on the wire is increased It may be a coincidence that the rise on twisting and the increase of stretch are both nearly proportional to the square of the number of turns

Experiments were then made with greater loads to find how the lowering and extension changed Only clockwise twist was observed.

Load 3081 grammes, the rise changed to lowering

Twist, 0—4 turns; lowering 0 0131 mm., mean of 20 observations.

„ 0—8 „ „ 0·0498 „ „ „

The extension due to an addition of 520 grammes was —

No twist on the wire	0·144 mm,	mean of	10	observations
4 turns	„ 0 143	„	„	„
8 „	„ 0 149	„	„	„

Showing a still slight excess of extension in the most twisted condition.

Load 5081 grammes.

Twist, 0—4 turns, lowering 0·0164 mm., mean of 20 observations.

„ 0—8 „ „ 0 0660 „ „ „

The extension due to an addition of 520 grammes was :—

No twist on the wire 0·141 mm., mean of 10 observations
4 turns „ 0·142 „ „ 15 „
8 „ „ 0·144 „ „ 15 „

The results for loads of 7081 and 9081 grammes are already recorded under 1*a* and 1*b*. There is obviously a tendency for the lowering to increase with load until the extensions under different twists become more nearly equal with equal added load.

When the same wire was softened and loaded with 3081 grammes the lowering for three turns was 0·0093 mm. (mean of 10 observations).

The extension due to an addition of 520 grammes was —

No twist on the wire 0·149 mm., mean of 10 observations.
3 turns „ 0·147 „ „ „

With load 9081 the same wire gave the results recorded under 2, which show a greater lowering for an equal twist but the same extension with added load.

The copper wire diameter 0·655 mm. (No. 5 above) with load 4081 grammes gave :—

Clockwise twist, 0—3 turns, 0·00965 mm., mean of 10 observations.
Counter clockwise twist, 0—3 turns, 0·0156 mm., mean of 10 observations.

Taking the mean of these, the lowering for one turn is 0·0014 mm.

The extension due to an addition of 520 grammes was —

No twist on the wire 0·268 mm., mean of 5 observations,
3 turns „ 0·270 „ „ „

extensions agreeing very nearly with those recorded above for a load of 6081 grammes on the same wire.

Remarks on the Results of Measurements.

The lowering was never so much as 0·1 mm. and was usually much less. The accuracy attained could hardly be expected to be great. The measurements, however, appear to show that when a wire is sufficiently loaded to be straightened, it is lengthened by twisting by an amount proportional to the square of the twist and with a given number of turns inversely as the length.

It might be thought possible that the effect observed was due to rise of temperature, either through adiabatic strain or through dissipation of strain energy as heat. But the observations give no support to this explanation. When the wire was extended by twisting, it remained extended, and when

untwisted it returned Temperature effects would be a maximum the instant after twisting, and would then gradually subside. It may be noted that the adiabatic change of temperature is proportional to $a^2\theta^2/l$, but it is a cooling, and its amount is such as to shorten the wire, in the case of steel, by something of the order of 1/100 of the observed extension. If we suppose that some definite fraction of the strain energy put in is dissipated, again the change, now a warming, is proportional to $a^2\theta^2/l$ The whole strain energy, in the case of steel, would only raise the temperature by an amount accounting for something of the order of 1/10 the observed extension, and, in fact, only an exceedingly minute fraction of the strain energy is dissipated.

A comparison of the wires (1) and (3) appears to show that the lengthening for a given twist is proportional to the square of the radius

If we put the lengthening

$$dl = sa^2\theta^2/2l,$$

s for steel is in the neighbourhood of 1 For copper and brass s is in the neighbourhood of 1 5 The lowering for the copper and brass wires tested for twists in opposite directions is not the same

With a hard steel wire with small load the end of the wire *rises* on twisting, probably through coiling

The value of $s = (\frac{1}{2}n+p)/\mathrm{Y}-(2/\mathrm{Y}-1/n)q$ appears to be measurable, but its value gives us no clue to the values of p and q.

If we could assume $q = 0$, then for steel we should have p about $2n$, but I see no justification for the assumption.

If we could measure the decrease in diameter, we should obtain the value of $(\frac{1}{2}n+p-q)\sigma/\mathrm{Y}-\{(3+\sigma)/\mathrm{Y}-1/n\}q$, and knowing n, Y, and σ we should be able to find p and q But a thin wire is quite unsuitable for this measurement The decrease is probably of the order of $2a/l \times$ lengthening. With the wires I have used this is of the order $1/1000 \times$ lengthening, and an accuracy of measurement of 10^{-6} inm would be required at least. With a shaft of considerable diameter it might be possible to measure the quantity, though the experimental difficulties are obviously very great.

The Effect of the Lengthening of a Wire on its Torsional Vibration.

If a wire is loaded with mass M having moment of inertia I, when M is set vibrating torsionally it falls and rises as it swings, its distance below the highest point being given by

$$x = \tfrac{1}{2}sa^2\theta^2/l.$$

The kinetic energy is $\mathrm{T} = \frac{1}{2}\mathrm{I}\dot{\theta}^2+\frac{1}{2}\mathrm{M}\dot{x}^2$.

The last term is easily found to be negligible.

The potential energy is

$$V = \tfrac{1}{4} n\pi a^4\theta^2/l - Mgx = \tfrac{1}{4} n\pi a^4\theta^2/l - \tfrac{1}{2} Mgsa^2\theta^2/l$$

The equation of motion is

$$I\theta + (\tfrac{1}{2} n\pi a^4/l - Mgsa^2/l)\,\theta = 0.$$

Whence

$$T^2 = \frac{8\pi^2 Il}{n\pi a^4(1 - 2\,Mgsa^2/n\pi a^4)},$$

and T is greater than it would be if s were 0 by the factor

$$1 + Mgs/n\pi a^2$$

If Y is Young's modulus and if e is the elongation of the wire due to the load Mg,

$$Mg/\pi a^2 = Ye,$$

so that the factor may be conveniently written as $1 + se\,Y/n$

If the vibrations are used to determine the modulus of rigidity n, then the value of n will be greater than that deduced by neglect of s, and by the factor $1 + 2\,se\,Y/n$.

To give an idea of the effect on the determination of the modulus of rigidity, let us suppose that a quite straight steel wire, diameter 0 7 mm., has a load of 2000 grammes For steel Y/n is about 2 6. For the given diameter e is about 2×10^{-4} We have found that s is about 1 The correcting factor is then about 1 001, or the true rigidity exceeds the value calculated in the ordinary way by about 1 in 1000 If the wire is not sufficiently loaded to be straight the value of s is less If very lightly loaded the sign of s may be changed and the true rigidity may be less than the value as ordinarily calculated The correction is hardly needful in practice, as the modulus of rigidity is probably not measurable to three figures.

Distortional Waves

In purely distortional waves in a medium of great extent it is evident that the pressure S perpendicular to the axes of shear, if it exists, will not produce any motion. To keep the waves purely distortional, *i.e.* with motion perpendicular to the direction of propagation only, a force must be applied from outside dR/dx per cubic centimetre in the direction of propagation. If this force is not applied then longitudinal motion must result, obviously of the second order, unless $\frac{1}{3} n + p = 0$ This is probably the condition for an incompressible medium. If $\frac{1}{3} n + p$ is not zero it appears possible that dispersion may exist. If the longitudinal motion is neglected the pressure in the direction of propagation is $(\frac{1}{3} n + p)\,e^2$ and all that we can say, at present, is that it is probably of the order of ne^2

The Wave Motion of a Revolving Shaft, and a Suggestion as to the Angular Momentum in a Beam of Circularly Polarised Light.

By J H Poynting, Sc D, F R S

(Received June 2,—Read June 24, 1909.)

When a shaft of circular section is revolving uniformly, and is transmitting power uniformly, a row of particles originally in a line parallel to the axis will lie in a spiral of constant pitch, and the position of the shaft at any instant may be described by the position of this spiral

Let us suppose that the power is transmitted from left to right, and that as viewed from the left the revolution is clockwise Then the spiral is a left-handed screw Let it be on the surface, and there make an angle ϵ with the axis Let the radius of the shaft be a, and let one turn of the spiral have length λ along the axis We may term λ the wave-length of the spiral We have $\tan \epsilon = 2\pi a/\lambda$. If the orientation of the section at the origin at time t is given by $\theta = 2\pi N t$, where N is the number of revolutions per second, the orientation of the section at x is given by

$$\theta = 2\pi N t - \frac{x}{a}\tan\epsilon = \frac{2\pi}{\lambda}(N\lambda t - x), \tag{1}$$

which means movement of orientation from left to right with velocity $N\lambda$.

The equation of motion for twist waves on a shaft of circular section is

$$\frac{d^2\theta}{dt^2} = U_s^2 \frac{d^2\theta}{dx^2}, \tag{2}$$

where U_s^2 = modulus of rigidity/density $= n/\rho$.

Though (1) satisfies (2), it can hardly be termed a solution for $d^2\theta/dt^2$, and $d^2\theta/dx^2$ in (1) are both zero But we may adapt a solution of (2) to fit (1) if we assume certain conditions in (1).

The periodic value

$$\theta = \Theta \sin \frac{2\pi}{l}(U_s t - x)$$

satisfies (2), and is a wave motion with velocity U_s and wave-length l. Make l so great that for any time or for any distance under observation $U_s t/l$ and x/l are so small that the angle may be put for the sine Then

$$\theta = \Theta \frac{2\pi}{l}(U_s t - x). \tag{3}$$

This is uniform rotation It means that we only deal with the part of

the wave near a node, and that we make the wave-length l so great that for a long distance the "displacement curve" obtained by plotting θ against t coincides with the tangent at the node We must distinguish, of course, between the wave-length l of the periodic motion and the wave-length λ of the spiral

We can only make (1) coincide with (3) by putting

$$\Theta/l = 1/\lambda \quad \text{and} \quad N\lambda = U_n$$

Then it follows that for a given value of N, the impressed speed of uniform rotation, there is only one value of λ or one value of ϵ for which the motion may be regarded as part of a natural wave system, transmitted by the elastic forces of the material with velocity $= \sqrt{(n/\rho)}$ There is therefore only one "natural" rate of transmission of energy

The value of ϵ is given by

$$\tan\epsilon = 2\pi a/\lambda = 2\pi aN/N\lambda = 2\pi aN/U_n = 2\pi aN\sqrt{(\rho/n)}$$

Suppose, for instance, that a steel shaft with radius $a = 2$ cm, density $\rho = 7\,8$, and rigidity $n = 10^{12}$ is making N = 10 revs per sec We may put $\tan\epsilon = \epsilon$, since it is very small The shaft is twisted through 2π in length λ or through $2\pi/\lambda$ per centimetre, and the torque across a section is

$$G = \tfrac{1}{2}n\pi a^4\,2\pi/\lambda = n\pi^2a^4N\sqrt{(\rho/n)},$$

since

$$\lambda = \frac{U_n}{N} = \frac{1}{N}\sqrt{\frac{n}{\rho}}$$

The energy transmitted per second is

$$2\pi NG = 2\pi^3a^4N^2\sqrt{(n\rho)}$$

Putting 1 H P $= 746 \times 10^7$ ergs per second, this gives about 38 H P.

But a shaft revolving with given speed N can transmit any power, subject to the limitation that the strain is not too great for the material When the power is not that "naturally" transmitted, we must regard the waves as "forced" The velocity of transmission is no longer U_n, and forces will have to be applied from outside in addition to the internal elastic forces to give the new velocity

Let H be the couple applied per unit length from outside Then the equation of motion becomes

$$\frac{d^2\theta}{dt^2} = U_n^2\frac{d^2\theta}{dx^2} + \frac{2H}{\pi a^4},$$

where $\frac{1}{2}\pi a^4$ is the moment of inertia of the cross section. Assuming that the condition travels on with velocity U unchanged in form,

$$\frac{d\theta}{dt} = -U\frac{d\theta}{dx} \quad \text{and} \quad H = \tfrac{1}{2}\pi a^4(U^2 - U_n^2)\frac{d^2\theta}{dx^2},$$

or H has only to be applied where $d^2\theta/dx^2$ has value, that is where the twist is changing

The following adaptation of Rankine's tube method of obtaining wave velocities* gives these results in a more direct manner. Suppose that the shaft is indefinitely extended both ways Any twist disturbance may be propagated unchanged in form with any velocity we choose to assign, if we apply from outside the distribution of torque which, added to the torque due to strain, will make the change in twist required by the given wave motion travelling at the assigned speed.

Let the velocity of propagation be U from left to right, and let the displacement at any section be θ, positive if clockwise when seen from the left. The twist per unit length is

$$\frac{d\theta}{dx} = -\frac{1}{U}\frac{d\theta}{dt} = -\frac{\dot{\theta}}{U}$$

The torque across a section from left to right in clockwise direction is

$$-\tfrac{1}{2}n\pi a^4\frac{d\theta}{dx} = \frac{n\pi a^4}{2U}\,.\,\dot{\theta}.$$

Let the shaft be moved from right to left with velocity U, then the disturbance is fixed in space, and if we imagine two fixed planes drawn perpendicular to the axis, one, A, at a point where the disturbance is θ and the other, B, outside the wave system, where there is no disturbance, the condition between A and B remains constant, except that the matter undergoing that condition is changing. Hence the total angular momentum between A and B is constant But no angular momentum enters at B, since the shaft is there untwisted and has merely linear motion At A, then, there must be on the whole no transfer of angular momentum from right to left. Now, angular momentum is transferred in three ways:—

1 By the carriage by rotating matter. The angular momentum per unit length is $\frac{1}{2}\rho\pi a^4\theta$, and since length U per second passes out at A, it carries out $\frac{1}{2}\rho\pi a^4\theta U$

2 By the torque exerted by matter on the right of A on matter on the left of A. This takes out $-n\pi a^4\dot{\theta}/2U$

3 By the stream of angular momentum by which we may represent the forces applied from outside to make the velocity U instead of U_n.

If H is the couple applied per unit length, we may regard it as due to the flow of angular momentum L along the shaft from left to right, such that $H = -dL/dx$. There is then angular momentum L flowing out per second from right to left. Since the total flow due to (1), (2), and (3) is zero,

$$\tfrac{1}{2}\rho\pi a^4\theta U - n\pi a^4\dot{\theta}/2U - L = 0,$$

* 'Phil. Trans.,' 1870, p. 277.

and $$L = \frac{\pi a^4 \dot{\theta}}{2}\left(\rho U - \frac{n}{U}\right) = \frac{\rho \pi a^4 \dot{\theta}}{2U}(U^2 - U_n^2) = -\frac{\rho \pi a^4}{2}\frac{d\theta}{dx}(U^2 - U_n^2),$$

and $$H = -\frac{dL}{dx} = \frac{\rho \pi a^4}{2}\frac{d^2\theta}{dx^2}(U^2 - U_n^2)$$

If $H = 0$, either $U^2 = U_n^2$ when the velocity has its "natural value," or $d^2\theta/dx^2 = 0$, and the shaft is revolving with uniform twist in the part considered

Now put on to the system a velocity U from left to right The motion of the shaft parallel to its axis is reduced to zero, and the disturbance and the system H will travel on from left to right with velocity U A "forced" velocity does not imply *transfer* of physical conditions by the material with that velocity We can only regard the conditions as reproduced at successive points by the aid of external forces We may illustrate this point by considering the incidence of a wave against a surface If the angle of incidence is ι and the velocity of the wave is V, the line of contact moves over the surface with velocity $v = V/\sin \iota$, which may have any value from V to infinity The velocity v is not that of transmission by the material of the surface, but merely the velocity of a condition impressed on the surface from outside

Probably in all cases of transmission with forced velocity, and certainly in the case here considered, the velocity depends upon the wave-length, and there is dispersion

With a shaft revolving N times per second $U = N\lambda$, and it is interesting to note that the group velocity $U - \lambda dU/d\lambda$ is zero It is not at once evident what the group velocity signifies in the case of uniform rotation. In ordinary cases it is the velocity of travel of the "beat" pattern, formed by two trains of slightly different frequencies The complete "beat" pattern is contained between two successive points of agreement of phase of the two trains In our case of superposition of two strain spirals with constant speed of rotation, points of agreement of phase are points of intersection of the two spirals At such points the phases are the same, or one has gained on the other by 2π Evidently as the shaft revolves these points remain in the same cross-section, and the group velocity is zero

With deep water waves the group velocity is half the wave velocity, and the energy flow is half that required for the onward march of the waves.* The energy flow thus suffices for the onward march of the group, and the case suggests a simple relation between energy flow and group velocity.

But the simplicity is special to unforced trains of waves Obviously,

* O. Reynolds, 'Nature,' August 23, 1877; Lord Rayleigh, 'Theory of Sound,' vol. 1, p. 477

it does not hold when there are auxiliary working forces adding or subtracting energy along the waves. For the revolving shaft the simple relation would give us no energy flow, whereas the strain existing in the shaft implies transmission of energy at a rate given as follows.

The twist per unit length is $d\theta/dx$, and therefore the torque across a section is $-\frac{1}{2}n\pi a^4 d\theta/dx$, or $\frac{1}{2}n\pi a^4\theta/U$, since $d\theta/dx = -\theta/U$. The rate of working or of energy flow across the section is $\frac{1}{2}n\pi a^4\dot{\theta}^2/U$.

The relation of this to the strain and kinetic energy in the shaft is easily found. The strain energy per unit length being $\frac{1}{2}$ (couple × twist per unit length) is $\frac{1}{4}n\pi a^4(d\theta/dx)^2$, which is $\frac{1}{4}n\pi a^4\dot{\theta}^2/U^2$. The kinetic energy per unit length is $\frac{1}{2}\rho\pi a^4\dot{\theta}^2$, or, putting $\rho = n/U_n^2$, is $\frac{1}{4}n\pi a^4\dot{\theta}^2/U_n^2$.

In the case of natural velocity, for which no working forces along the shaft are needed, when $U = U_n = \sqrt{(n/\rho)}$, the kinetic energy is equal to the strain energy at every point and the energy transmitted across a section per second is that contained in length U_n.

But if the velocity is forced this is no longer true,* and it is easily shown that the energy transferred is that in length $\frac{2U}{1+U^2/U_n^2}$, which is less than U if $U > U_n$, and is greater than U if $U < U_n$.

It appears possible that always the energy is transmitted along the shaft at the speed U_n. If the forced velocity $U > U_n$, we may, perhaps, regard the system in a special sense as a natural system with a uniform rotation superposed on it.

Let us suppose that the whole of the strain energy in length U_n is transferred per second while only the fraction μ of the kinetic energy is transferred, the fraction $1-\mu$ being stationary.

The energy transferred: strain energy in U_n: kinetic energy in U_n $= 1/U : U_n/2U^2 : U_n/2U_n^2$.

Put $U = pU_n$, and our supposition gives

$$\frac{1}{pU_n} = \frac{1}{2p^2U_n} + \frac{\mu}{2U_n} \quad \text{or} \quad \mu = \frac{2}{p} - \frac{1}{p^2} = 1 - \left(1 - \frac{1}{p}\right)^2$$

If the forced velocity $U < U_n$, we may regard the system as a natural one, with a uniform stationary strain superposed on it.

We now suppose that the whole of the kinetic energy is transferred, but only a fraction ν of the strain energy, and we obtain

$$\frac{1}{pU_n} = \frac{\nu}{2p^2U_n} + \frac{1}{2U_n} \quad \text{or} \quad \nu = 2p - p^2 = 1 - (1-p)^2.$$

* In the Sellmeier model illustrating the dispersion of light, the particles may be regarded as outside the material transmitting the waves and as applying forces to the material which make the velocity forced. The simple relation between energy flow and group velocity probably does not hold for this model.

It is perhaps worthy of note that a uniform longitudinal flow of fluid may be conceived as a case of wave motion in a manner similar to that of the uniform rotation of a shaft

A Suggestion as to the Angular Momentum in a Beam of Circularly Polarised Light.

A uniformly revolving shaft serves as a mechanical model of a beam of circularly polarised light The expression for the orientation θ of any section of the shaft distant x from the origin, $\theta = 2\pi\lambda^{-1}(Ut-x)$, serves also as an expression for the orientation of the disturbance, whatever its nature, constituting circularly polarised light.

For simplicity, take a shaft consisting of a thin cylindrical tube Let the radius be a, the cross-section of the material s, the rigidity n, and the density ρ. Let the tube make N revolutions per second, and let it have such twist on it that the velocity of transmission of the spiral indicating the twist is the natural velocity $U_n = \sqrt{(n/\rho)}$

Repeating for this special case what we have found above, the strain energy per unit length is $\frac{1}{2}n\epsilon^2 s$, or, since $e = ad\theta/dx = -a\theta/U_n$, the strain energy is $\frac{1}{2}na^2s\theta^2/U_n^2 = \frac{1}{2}\rho a^2 s\dot{\theta}^2$

But the kinetic energy per unit length is also $\frac{1}{2}\rho a^2 s\dot{\theta}^2$, so that the total energy in length U_n is $\rho a^2 s\theta^2 U_n$ The rate of working across a section is

$$ne sa\theta = na^2s\theta^2/U_n = \rho a^2 s\theta^2 U_n,$$

or the energy transferred across a section is the energy contained in length U_n

If we put E for the energy in unit volume and G for the torque per unit area, we have $$Gs\dot{\theta} = EsU_n,$$

whence $$G = EU_n/\theta = EN\lambda/2\pi N = E\lambda/2\pi$$

The analogy between circularly polarised light and the mechanical model suggests that a similar relation between torque and energy may hold in a beam of such light incident normally on an absorbing surface If so, a beam of wave-length λ containing energy E per unit volume will give up angular momentum $E\lambda/2\pi$ per second per unit area. But in the case of light waves $E = P$, where P is the pressure exerted. We may therefore put the angular momentum delivered to unit area per second as

$$P\lambda/2\pi$$

In the 'Philosophical Magazine,' 1905, vol. 9, p 397, I attempted to show that the analogy between distortional waves and light waves is still closer, in that distortional waves also exert a pressure equal to the energy per unit volume. But as I have shown in a paper on "Pressure Perpendicular to the

Shear Planes in Finite Pure Shears, etc." (*ante*, p. 546), the attempt was faulty, and a more correct treatment of the subject only shows that there is probably a pressure We cannot say more as to its magnitude than that if it exists it is of the order of the energy per unit volume.

When a beam is travelling through a material medium we may, perhaps, account for the angular momentum in it by the following considerations. On the electromagnetic theory the disturbance at any given point in a circularly polarised beam is a constant electric strain or displacement f uniformly revolving with angular velocity θ In time dt it changes its direction by $d\theta$

This may be effected by the addition of a tangential strain $fd\theta$, or the rotation is produced by the addition of tangential strain $f\dot{\theta}$ per second, or by a current $f\dot{\theta}$ along the circle described by the end of f We may imagine that this is due to electrons drawn out from their position of equilibrium so as to give f, and then whirled round in a circle so as to give a circular convection current $f\dot{\theta}$ Such a circular current of electrons should possess angular momentum

Let us digress for a moment to consider an ordinary conduction circuit as illustrating the possession of angular momentum on this theory Let the circuit have radius a and cross-section s, and let there be N negative electrons per unit volume, each with charge e and mass m, and let these be moving round the circuit with velocity v. If i is the total current, $i = Nsve$. The angular momentum will be

$$Ns2\pi a \,.\, mva = 2\pi a^2 im/e = 2Aim/e = 2Mm/e,$$

where A is the area of the circuit and M is the magnetic moment This is of the order of $2M/10^7$

It is easily seen that this result will hold for any circuit, whatever its form if A is the projection of the circuit on a plane perpendicular to the axis round which the moment is taken and if $M = Ai$. If we suppose that a current of negative electrons flows round the circuit in this way and that the reaction while their momentum is being established is on the material of the conductor, then at make of current there should be an impulse on the conductor of moment $2M/10^7$. If the circuit could be suspended so that it lay in a horizontal plane and was able to turn about a vertical axis in a space free from any magnetic field, we might be able to detect such impulse if it exists. But it is practically impossible to get a space free from magnetic intensity. If the field is H, the couple in the circuit due to it is proportional to HM. It would require exceedingly careful construction and adjustment of the circuit to ensure that the component of the couple due to the field

about the vertical axis was so small that its effect should not mask the effect of the impulsive couple. The electrostatic forces, too, might have to be considered as serious disturbers.

Returning to a beam of circularly polarised light, supposed to contain electrons revolving in circular orbits in fixed periodic times, the relations between energy and angular momentum are exactly the same as those in a revolving shaft or tube, and the angular momentum transmitted per second per square centimetre is $E\lambda/2\pi = P\lambda/2\pi$, where P is the pressure of the light per square centimetre on an absorbing surface.

The value of this in any practical case is very small In light pressure experiments, P is detected by the couple on a small disc, of area A say, at an arm b and suspended by a fibre. What we observe is the moment APb. If the same disc is suspended by a vertical fibre attached at its centre and the same beam circularly polarised in both cases is incident normally upon it, according to the value suggested the torque is $AP\lambda/2\pi$.

The ratio of the two is $\lambda/2\pi b$ Now b is usually of the order of 1 cm. Put $\lambda = 6 \times 10^{-5}$, or, say, $2\pi/10^{-5}$, and the ratio becomes 10^{-5}

It is by no means easy to measure the torque APb accurately, and it appears almost hopeless to detect one of a hundred-thousandth of the amount The effect of the smaller torque might be multiplied to some extent, as shown in accompanying diagram.

Let a series of quarter wave plates, p_1, p_2, p_3, ..., be suspended by a fibre above a Nicol prism N, through which a beam of light is transmitted upwards, and intermediate between these let a series of quarter wave plates, q_1, q_2, q_3, ..., be fixed, each with a central hole for the free passage of the fibre. The beam emerges from N plane polarised. If N is placed so that the beam after passing through p_1 is circularly polarised, it has gained angular momentum, and therefore tends to twist p_1 round. The next plate q_1 is to be arranged so that the beam emerges from it plane polarised and in the original plane. It then passes through p_2, which is similar to p_1, and again it is circularly polarised and so exercises another torque. The process is repeated with q_2 and p_2, and so on till the beam is exhausted. By revolving N through a right angle round the beam, the effect is reversed. But, even with such multiplications, my present experience of light forces does not give me much hope that the effect could be detected, if it has the value suggested by the mechanical model.

The Liquidus Curves of the Ternary System:—Aluminium-Copper-Tin.

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(Communicated by J. E Stead, F R S Received May 3,—Read May 27, 1909.)

[Plate 3]

The study of the constitution of alloys is of great theoretical interest, and the practical use of the knowledge so obtained can scarcely be overestimated, in fact, it may be said, that the heat-treatment of a given series of alloys cannot be correctly accomplished without an accurate knowledge of the structural changes which occur with varying temperature and concentration

So much work has been done on this subject, that we are now in possession of accurate data for a large number of the alloys containing only two metals, but with the exception of Mons. Charpy's* classical work on the lead-tin-bismuth alloys, comparatively little work has been published on mixtures of three or more metals. This is not surprising when we consider the increased analytical and experimental difficulties, and also the possible number of alloys in a series containing three elements.

If relatively the same number of determinations are required for a ternary as for a binary series of alloys, then $n \times \frac{1}{2}n$ will be the number required for the ternary when n is the number for the binary series In other words, if 30 alloys (which is a moderate figure) are necessary to determine the melting points and structural changes of a series of alloys containing two elements, 450 would be needed for a series containing three elements. At first sight these figures seem rather overwhelming, and the carrying out of such an elaborate set of experiments may appear somewhat out of proportion to the interest of the results, but the writers wish to draw attention to the following points:—

1. The properties which result from mixing two binary alloys having one metal common to each are seldom additive.

2. A large percentage of the non-ferrous alloys, used in the industries, contain more than two elements intentionally added for some particular purpose.

3. The following question is often raised: Can one element be advantageously replaced by another, and if so, to what extent?

* 'Étude sur les Alliages Blancs, dits Antifriction.'

4. Very little is known as to the effect of impurities on the non-ferrous alloys. This is very important, as many alloys are useful only when in a metastable condition, *e.g.*, an alloy containing 90 per cent of copper and 10 per cent. of aluminium, when cast in chill, gives the following mechanical properties :—

*Ultimate stress	Reduction of area.	Elongation on 2"
38·0 tons	31·0 per cent.	29·0 per cent.

If a bar of the same material is heated to 400° C for one hour, the mechanical properties are radically changed, viz. :—

Ultimate stress.	Reduction of area.	Elongation on 2".
33·0 tons	3·0 per cent.	3·0 per cent.

The structure of the alloy as cast consists of about equal proportions of α containing 7·35 per cent of aluminium, and β containing 12 per cent. of aluminium. The β-constituent is slowly decomposed between 400° and 500° C into α and γ, the latter being a very brittle body containing about 16 per cent of aluminium There can be little doubt that the rapid deterioration of properties brought about by heating the above alloy is due to the formation of the γ-constituent This alloy is therefore practically ruined by heating for an hour at 400° C, and it is quite possible that the presence of foreign elements may accelerate the $\beta = \alpha + \gamma$ conversion to such a degree as to render it mechanically unfit for use

A systematic investigation of the constitution of alloys containing three elements is therefore a matter of some importance, for only by this means can the questions mentioned above be elucidated Before commencing work of this description, a knowledge of the three binary alloys is essential, *e.g.*, if A, B, and C are the elements, then the systems A+B, B+C, and A+C should be known.

For this research the three metals aluminium, copper, and tin were chosen for the following reasons —

1. The constitutions of the three series. copper-tin,† copper-aluminium,‡ and aluminium-tin§ have been investigated. The liquidus curves are shown in figs. 2, 3, and 4.

2. There is an increasing tendency to substitute aluminium for tin in brasses and other industrial alloys

* 'Eighth Report to the Alloys Research Committee, Inst. of Mech. Engineers,' p. 179, Carpenter and Edwards.

† Heycock and Neville, 'Phil Trans.,' vol. 202, 1903; Shepherd, 'Journal of Physical Chemistry,' vol. 8, No. 6, 1904

‡ Carpenter and Edwards, 'Inst. of Mech. Engineers,' 1907

§ Shepherd, 'Journal of Physical Chemistry,' vol. 8, No. 4, 1904.

The simplest way of graphically representing a ternary series of alloys is by means of an equilateral triangle.* Each corner of the triangle represents a pure metal. The three sides correspond to the three binary mixtures A+B, A+C, and B+C, and any point inside the triangle indicates a certain mixture of the three. A point on any of the vertical lines drawn from each base to meet the corresponding corners indicates the percentages of each metal, consequently the length of lines drawn from any point in the triangle to meet the three sides at right angles is proportional to the percentage of each metal, and the sum of the length of the lines so drawn is equal to the height of the triangle.

Alloys occurring on lines drawn from the sides of the diagram to the apex have two components relatively constant, while the third varies. A line drawn parallel to one of the sides of the triangle represents a series of mixtures, in which the percentage of one component is constant The temperatures at which the liquid alloys begin to deposit solid are represented by drawing lines through identical freezing points, and thus obtaining isothermal lines for the whole series.

Preparation of the Alloys.

The metals from which the alloys were made had the following degree of purity —

Aluminium	99 57 per cent
Copper	99·98 „
Tin	99·98 „

The methods of making the alloys, although essentially the same as those usually employed, viz, of melting and mixing together the constituent metals, differed in some details, being modified according to the composition of the resulting mixture.

The copper-rich alloys were made by first melting the weighed quantity of copper, and then adding the necessary amounts of aluminium and tin, the aluminium being added always before the tin. Since all alloys made in this manner were analysed, no special precautions were taken to prevent any slight oxidation likely to occur But for reasons to be mentioned later, alloys containing approximately more than 12 per cent. aluminium and 16 per cent. tin were made by mixing together a binary alloy of copper-aluminium or copper-tin with the third metal, every precaution being taken to prevent oxidation by carefully melting at a low temperature under a layer of charcoal. Several alloys made in this way were duplicated, and were always found to

* G. G. Stokes, 'Papers,' vol. 5, p. 236 ('Roy. Soc. Proc.,' 1881).

give the same melting point within the limits of experimental error. The value of this method, *i.e*, of using binary alloys, may be expressed thus:—

1. Analyses of the binary alloys can be made.

2. A lower temperature can be employed in making the ternary alloys, and in this way less oxidation takes place than in the mixing of three pure metals.

In all cases the melt was thoroughly stirred in order to ensure perfect mixing. The crucibles used and found to be most suitable were the "Salamander" crucibles.

Chemical Analyses.

At the commencement of the research, it was intended to make a chemical analysis of each alloy. Indeed, for the copper-rich alloys analyses were made and were quite satisfactory. It was found, however, that analyses of the alloys containing more than 16 per cent. of tin and 12 per cent. of aluminium did not give concordant results. For example, different samples of the same bars of two alloys showed the following variations in tin contents.—

	Intended composition of the alloy.		
	Copper	Tin	Aluminium
No 1.	65·00	17 50	17 50
First result . ..	—	16 99	
Second result	—	14 90	
Third result	—	16·00	
No. 2.	52·00	24·00	24·00
First result	—	18·00	
Second result .	—	15 10	

It was obvious that these widely different results must be due to the lack of homogeneity of the alloys. In fact, this was quite apparent in alloys containing slightly greater quantities of tin and aluminium than the previous ones, the cast bars showing excessive segregation. So great was this in many cases that the outer part was quite brittle, and could readily be separated from the soft core by mechanical means. These mechanically separated samples were analysed, and although the results could only be taken as a rough measure of the segregation, they were convincing, showing that the inner core contained a large amount of tin, and the outer brittle layer was approximately a copper-aluminium alloy.

Experiments were made as to the possibility of producing a homogeneous solid bar for analyses, the idea being to freeze the whole mass before segregation had time to take place. But even with this procedure segregation occurred. Therefore, finding that chemical analyses were inapplicable synthetic mixtures made from binary alloys had to be relied upon.

Freezing Point Determinations

In order to avoid any error due to reheating, the freezing points of the alloys were determined directly after mixing by means of a platinum + 10-per-cent. iridium thermo-junction, the wires being fused together in an oxy-hydrogen flame The free ends were soldered to copper leads, these junctions being kept at zero, and the wires connected with a mirror galvanometer and balancing arrangement similar to that described by Messrs. Carpenter and Keeling* in their work on the iron-carbon alloys.

With most of the alloys no difficulty was experienced in finding the beginning of solidification, but with those alloys containing approximately over 50 per cent. of tin the amount of solid deposited at the initial freezing temperature was so small that great care had to be taken in order to detect the evolution of heat at this temperature. By cooling extremely slowly, and carefully timing the rate of fall of temperature of the melt, it was found possible to detect the slightest arrest of the movement of the galvanometer In many cases, determinations were repeated several times for verification The melt in all cases was cooled in the furnace, and except where the alloy froze at a very low temperature the furnace lid was not removed.

The freezing curves taken in the region H to J (fig. 1) present a certain peculiarity, in the form of a slight arrest which takes place at approximately a constant temperature, viz, 900° C. It was thought at first that this might be the freezing point arrest, but this idea is rendered improbable by the fact that such a value, if taken, would break the continuity of the isothermal curves, whilst the points actually taken conform to the general curvature of the lines The writers think this slight arrest is due to the mutual separation, in the liquid state, of the copper-aluminium mixture and the tin.

In order to locate the position of the isothermal curves, as shown in the plate, more than 400 alloys and melting point determinations were made.

* Carpenter and Keeling, "The Range of Solidification and the Critical Ranges of Iron-Carbon Alloys," 'Journal of the Iron and Steel Institute,' vol. 1, 1904.

The Diagram. (Plate 3)

The diagram is conveniently divided into areas, in which the direction of the isothermal curves is similar. Alloys in each area may be classed as a group or family, in which the influence on the liquidus of a certain phase predominates. As the compositions of the binary alloys only are known with any degree of certainty, the writers propose to speak of the alloys in

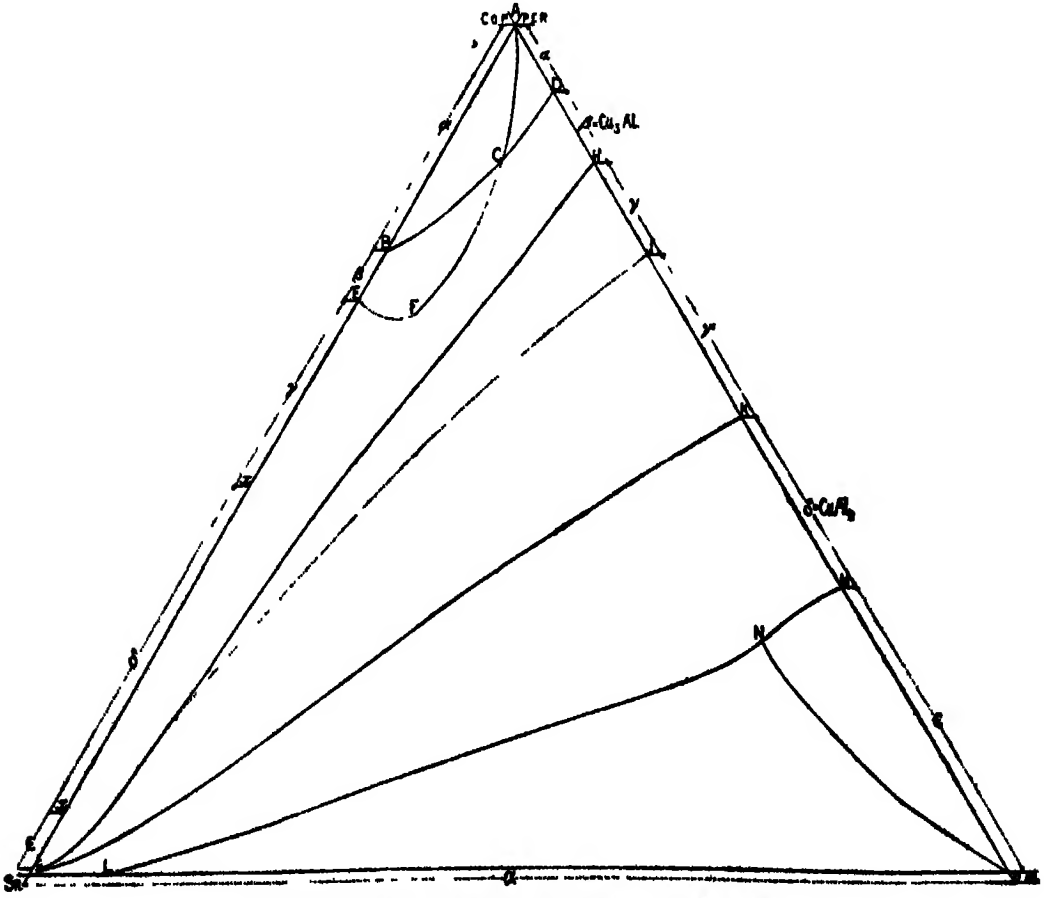


Fig. 1.—Key to Areas on Main Diagram (Plate 3).

groups, giving reference to the binary constituent which has the greatest influence on the liquidus of each group.

Alloys in the ABC area (fig. 1) begin to solidify with the formation of the copper-tin α-crystals, which also contain a certain amount of aluminium. The ACD group of alloys, on cooling, first deposit copper-aluminium α-crystals containing some tin.

Mixtures in the area BEFC commence to freeze with the deposition of copper-tin β-crystals, which also appear to contain some aluminium in solution. The lower part of this area is not well defined, but seems to merge into the adjacent one, the liquidus of which is formed by the freezing out of a chemically strong compound.

In the area DFExyGHI, the constituent which is the first to separate is the copper-aluminium β phase, which also holds tin in solid solution This

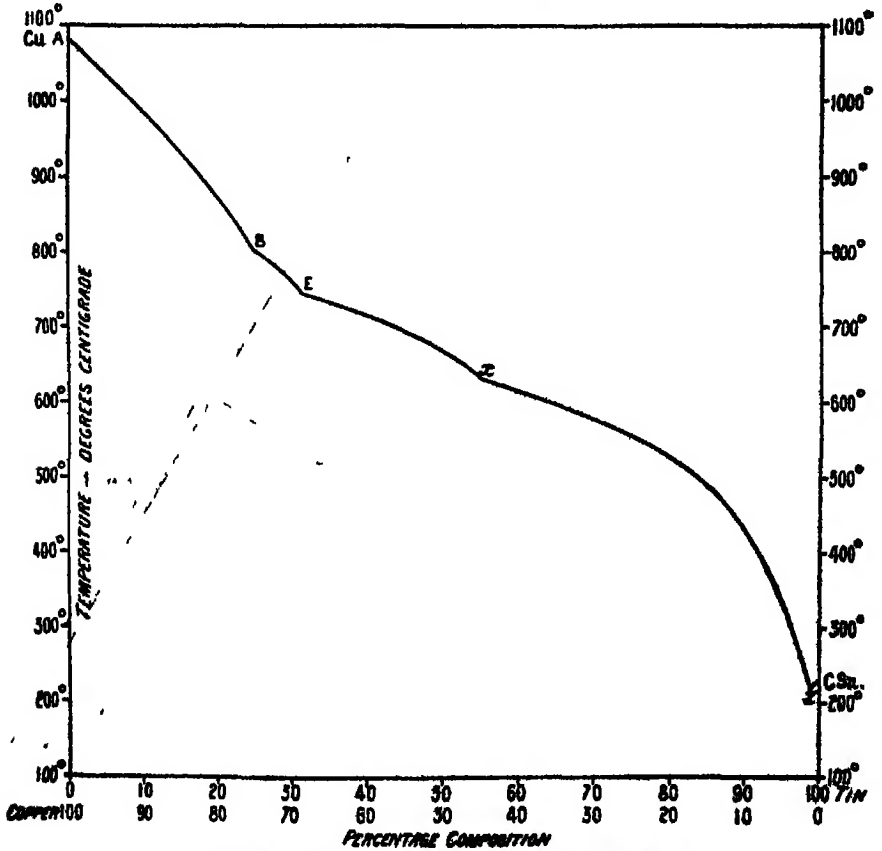


Fig 2.—Liquidus Curve of Copper-Tin Alloys.

constituent first appears in the binary alloys at 7·35 per cent. of aluminium, and is present in all alloys up to 16 per cent. An alloy with 12 per cent. of aluminium consists of pure β and closely corresponds to the compound Cu_3Al.

Whilst there is no indication on the liquidus diagram to mark the precise position of the lower HI boundary line of this area, the writers consider the area shown as being approximately correct. It was deduced by drawing

a line from the point H, which corresponds to the disappearance of the β-phase in the binary alloys, to the point I, which seems to be the extent to which this phase influences the formation of the curves. It is interesting to note that the phase which is present in the copper-aluminium alloys, only over a range of 8·65 per cent., viz., from 7·35 to 16 per cent. of aluminium is the constituent which influences so markedly the freezing point of these alloys even when as much as 95 per cent. of tin is present.

Further, the copper-tin γ and η phases which are deposited respectively from the liquid alloys containing from 32 to 57·5, and 57·5 to 99 per cent. of tin, do not crystallise from any of the liquid ternary alloys. This is obviously due to the added aluminium combining with the copper to form the copper-aluminium phase, thereby raising the freezing point of the alloy above that of either γ or η. Thus an alloy with 80 per cent of tin and 20 per cent of copper begins to freeze at 530° C., but if 0·50 per cent of aluminium is added to it, the mixture will commence to solidify at 730° C.

Alloys in the area HIJ and JIGK may be considered together. the first begins to deposit the copper-aluminium γ-phase, and the second the γ'-phase, both of which are pure. The tin separates and settles to the bottom in an almost pure state.

The area LGKMN encloses alloys which on cooling deposit pure crystals of the compound $CuAl_2$. The added tin has no influence on their melting points, but simply separates, forming a separate lower layer of tin.

Alloys in the area NMO on cooling deposit the solid solution of copper in aluminium which may also contain a small amount of tin.

The area LNO. The solid first formed on cooling alloys in this area is the solid solution of tin in aluminium which may also contain a limited amount of copper.

Summary.

1. The character of the liquidus curves, as seen on the diagram, indicates that no ternary compound is deposited from any of the liquid alloys, and throughout the whole system no true ternary phase appears to form above the solidus. The alloys around the point N cannot be said to consist of a ternary eutectic, but rather it would seem that the melting point of the copper-aluminium eutectic at M is slightly lowered by tin, which remains in the free liquid state. This absence of ternary phases may be accounted for by the fact that the copper-aluminium phases, in alloys containing more than 12 per cent. of aluminium, have very little or no affinity for tin. This behaviour of the copper and aluminium towards tin suggests that the system might almost be considered as a binary series of alloys. In fact, if the

freezing points of alloys containing a constant percentage of tin are plotted, liquidus curves are obtained of a similar character to that of the copper-aluminium alloys. (See figs. 5, 6, and 7)

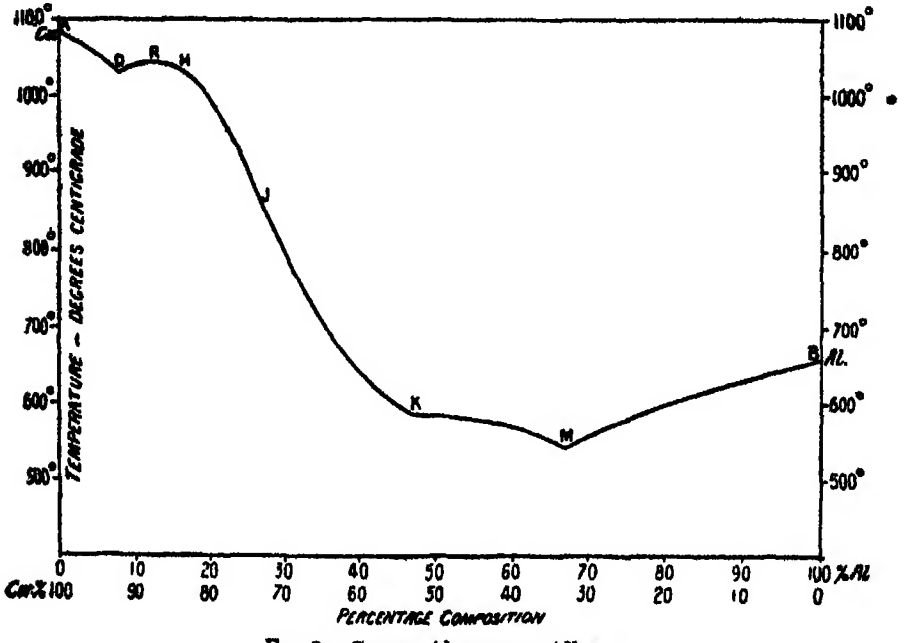


Fig 3.—Copper-Aluminium Alloys.

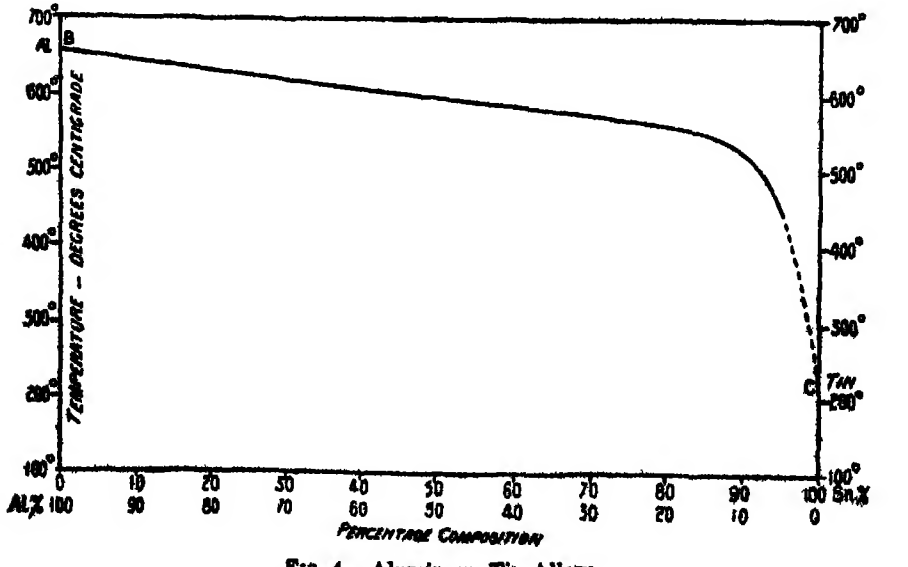


Fig 4.—Aluminium-Tin Alloys.

The liquidus curve of alloys containing 10 per cent. of tin, with the copper and aluminium varying from 0 to 90 per cent., is shown in fig. 5. The general character of this curve is almost identical with that of the pure copper-aluminium alloys (fig. 3). In both cases, the addition of aluminium first lowers the melting point from A to D, then produces a rise, to a

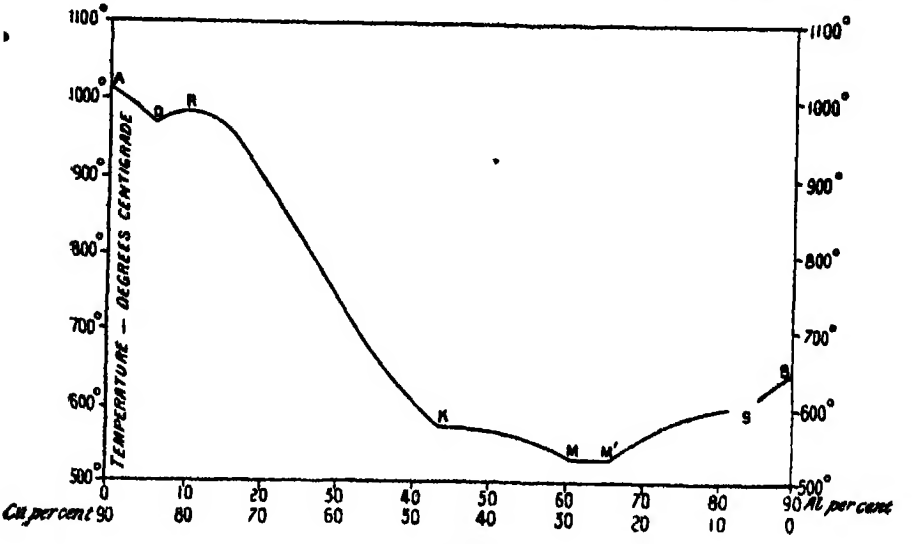


Fig 5

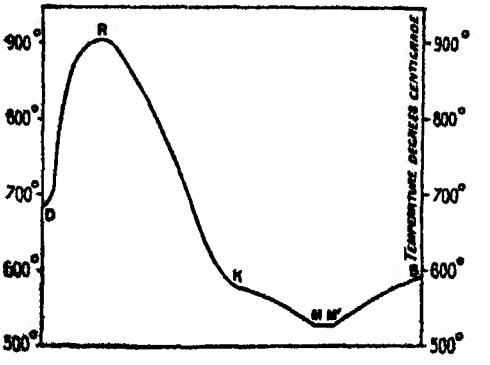


Fig 6.

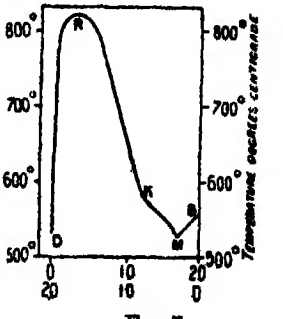


Fig 7

maximum at R, which corresponds to the compound Cu_9Al. This is followed first by a gradual, and then by a rapid fall in the melting point to K, where the compound $CuAl_2$ is in each case deposited from the liquid alloys. With more aluminium the melting point is gradually lowered, in the case of the binary alloys to the minimum point M, but with those containing 10 per cent. of tin to a minimum range M, M'. From here to B, fig. 3,

pure aluminium on the one hand, and 90 per cent aluminium with 10 per cent. tin on the other, B, fig 5, the melting points are raised, in the case of the latter curve there is a slight break at S, this being the point where it cuts the line NO in the key diagram, fig. 1.

Fig. 6 represents the liquidus curve of alloys containing 50 per cent of tin and from 0 to 50 per cent. of copper and aluminium. In this curve the branch AD, fig 3, which corresponds to the α-range, is absent, in other respects the curves are very similar Another curve of this character is shown in fig 7, for the alloys containing 80 per cent. of tin and from 0 to 20 per cent of copper and aluminium On this curve it is seen that the 80-per-cent. tin and 20-per-cent copper alloy begins to solidify at 530° C., and with the addition of up to 1 per cent. of aluminium the freezing point is rapidly raised to 800° C

Although there is 80 per cent of tin in this series, the curve from 1 per cent. to 20 per cent of aluminium is almost identical with fig 3 from the point D to B

Considered in another way, the phases formed in the copper-aluminium series from D to B, which cover a range of more than 90 per cent., are compressed into a range of only 20 per cent. in the presence of 80 per cent. of tin.

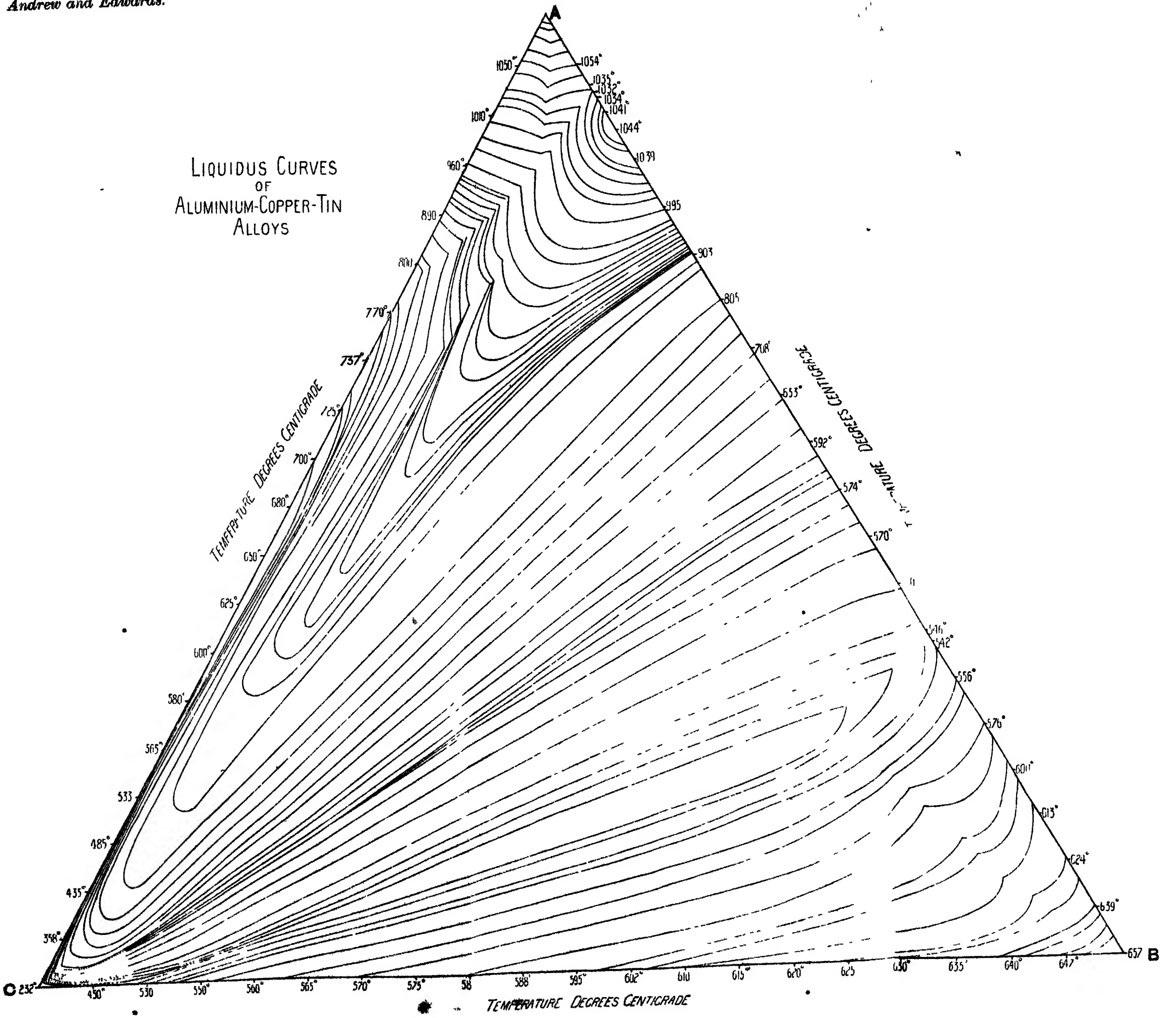
A curve plotted on a different system is shown in fig. 8 This curve is drawn through the melting points of alloys occurring on a line from the point R (Cu_3Al) through the maxima of the beak-like curves in the area DFExyGH to the point C which corresponds to pure tin From the curve it will be seen that the introduction of tin to the compound Cu_3Al steadily lowers the melting point of the latter to 945° C at B, from here to C the lowering is very slight, but afterwards there is a rapid fall in the melting point to D.

2 The remarkable stability of the compound Cu_3Al is clearly shown by the beak-like formation in the area DFExyGIH, which shows that the affinity of tin for either copper or aluminium is not sufficient to overcome the affinity of those two elements for each other

3 Alloys in the area KGLMN which contain a relatively constant proportion of copper and aluminium, *i.e.*, those which occur on lines drawn from the side of the triangle AB to the corner C, have identical freezing points This means that the introduction of tin to any of the copper-aluminium alloys between KM has no influence on the freezing points, which indicates that the two liquids, tin and the copper-aluminium mixtures, are mutually insoluble

4. Tin is insoluble in by far the greater number of the alloys This

LIQUIDUS CURVES OF ALUMINIUM-COPPER-TIN ALLOYS



phenomenon is most evident in all those on the lower side of the line HI with the possible exception of a very limited number containing upwards of about 94 per cent of aluminium

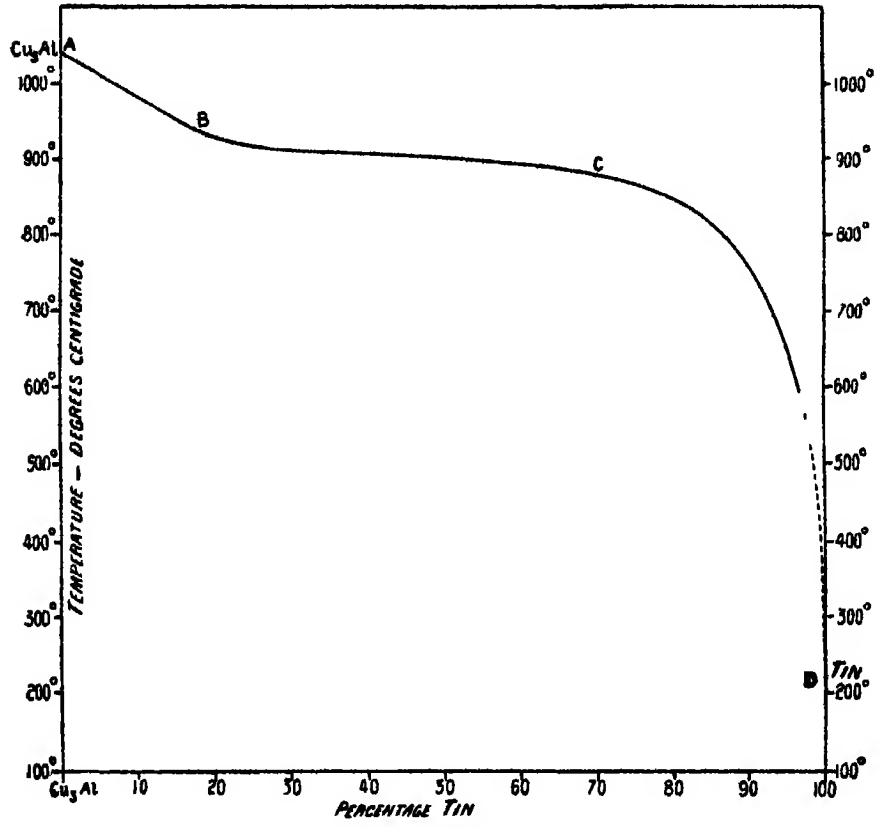


FIG 8

In conclusion, the writers wish to gratefully acknowledge the assistance given by Prof H. C H. Carpenter, in placing the apparatus in his laboratories, and the materials used, at their disposal, and for kindly criticism during the progress of the work

Sensitive Micro-balances and a New Method of Weighing Minute Quantities.

By Bertram D Steele and Kerr Grant.

(Communicated by Sir William Ramsay, K.C B., F.R S Received June 10,—Read June 24, 1909)

In the course of experiments undertaken by the authors with a view to establishing a possible relation between the amount of ionisation produced at the surface of certain heated metals and the amount of oxidation of the metal, it became necessary to be able to measure changes of weight of the order of one-thousandth of a milligramme (1×10^{-6} gramme)

A micro-balance of the Nernst type was accordingly constructed which possessed the requisite sensitiveness, but considerable difficulty was experienced in obtaining consistent readings with it, owing chiefly to the inconstancy of zero and the great variation of sensibility with load This latter defect is an inevitable consequence of the fact that a restoring couple due to gravity, the magnitude of which varies, as in the ordinary balance, with the position of the centre of gravity of the system relative to its point of suspension, is superposed on the restoring torque of the quartz fibre. Attempts to minimise this trouble led finally to the conclusion that for the purpose in view better results were to be expected from a gravity balance of the ordinary type in which the required degree of sensitiveness should be attained by making the beam very light. As the maximum load which it was intended to use on the balance was less than half a gramme, this could be done without loss of proportionate rigidity.

A beam, the weight of which was less than half a gramme, was accordingly made in the form of a plane frame-work of fused quartz rod of 0 6 mm. diameter. The ordinary knife-edges were at first replaced by pairs of very fine points ground on the ends of quartz rods, the central pair rested on a polished plane of quartz crystal, the ordinary pointer was replaced by a small concave glass mirror attached in line with the central axis.

With this beam it was found possible to weigh masses not exceeding one-fifth of a gramme with an accuracy of one-thousandth of a milligramme (1×10^{-6} gramme), and as it was clear that the sensitiveness of an instrument of this type could be still greatly increased, the possibility suggested itself of constructing an instrument sufficiently sensitive to detect and perhaps even to measure the changes of weight which radio-active substances are supposed to undergo For, according to Rutherford,* the

* 'Radio-active Transformations,' pp. 149 and 150

amount of radium emanation given off by 1 gramme of radium bromide amounts approximately to 2×10^{-11} gramme per second, *i.e* $1{\cdot}73 \times 10^{-9}$ gramme per milligramme per day. Assuming, then, that 10 milligrammes of radium bromide were available, the loss of weight per day would be approximately $1{\cdot}73 \times 10^{-8}$ gramme, and in order to detect this loss the balance would have to possess a sensibility of 1×10^{-8} gramme, and to be free from any wandering of zero which might mask or obliterate the real alteration of weight After long continued experimentation and the introduction of numerous successive improvements in the details of construction an all-quartz constant-load vacuum gravity micro-balance has been designed and constructed in which the above conditions are completely fulfilled.

Two types of micro-balance have been constructed —

Type A —A micro-balance designed for the measurement of small alterations in weight of any substance and sensitive to one two-hundred and fifty-thousandth of a milligramme (4×10^{-9} gramme)

Type B —A micro-balance for the absolute determinations in weight of masses not exceeding, in general, one decigramme, with an accuracy of one ten-thousandth of a milligramme (1×10^{-7} gramme).

Before proceeding to the description of the micro-balances a description of the balance-case and its attachments will be given. The same case has been used for both types of balance, and its construction will be made clear by reference to figs. 1, 2, and 3, which show respectively the front section, plan, and end section

In order to ensure rapid equalisation of temperature the case was made as small as possible and of brass The walls are about one-eighth inch thick and are carefully tinned both inside and outside to prevent the possibility of leakage due to the porosity of the brass. The case consists of the box-like base B and of the cover C and its inside dimensions are as follows — Length, 12 cm , height, 9·5 cm., and width, 6 cm. The base and cover are each provided with a flange 1 5 cm wide, and these two flanges are carefully ground together so as to make, when properly lubricated, a perfectly vacuum-tight joint. In order to separate the cover from the base without jolting the instrument, the flange on the base is provided with the lug *c*, that on the cover with a corresponding lug c_1, and the thumb-screw *d*

A hole 5 × 4 cm. is cut in the end of the cover and over this is cemented a window of good plate glass. The whole case is supported by three brass legs D, on a marble base E, which is in turn supported by three levelling screws (not shown in the figure). The knife-edge of the beam rests on a plate of ground and polished quartz crystal, for the preparation of which we are indebted to our colleague Mr. H. J. Grayson. This plate *f* is

cemented to the top of a brass pillar b, which is screwed into the solid block g in the base

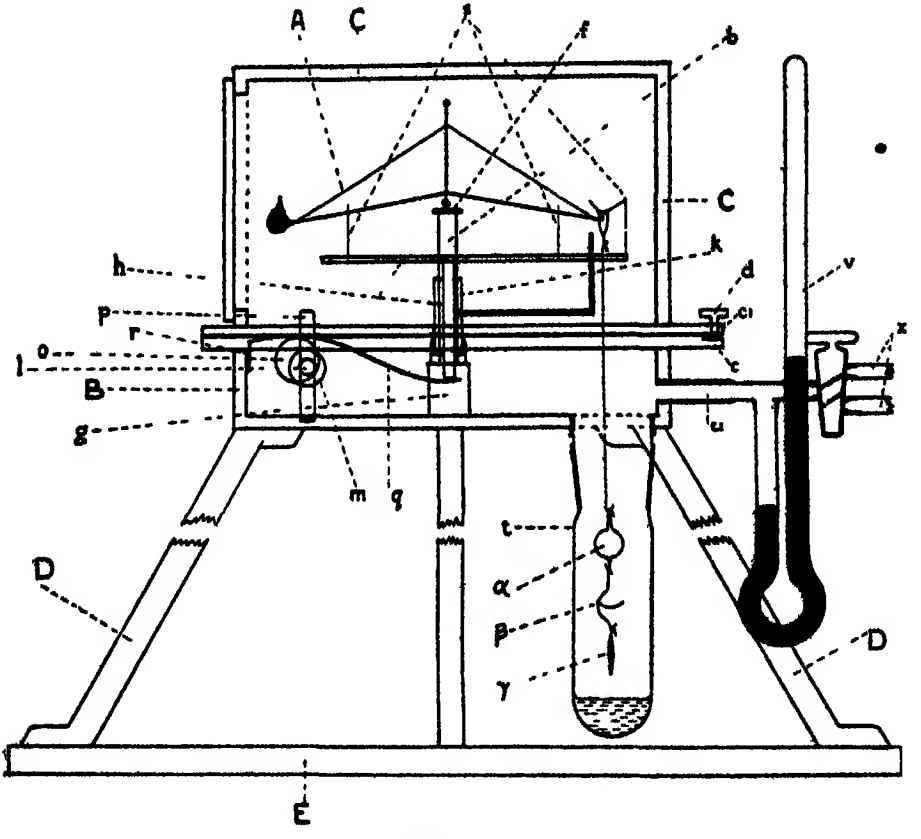


Fig 1

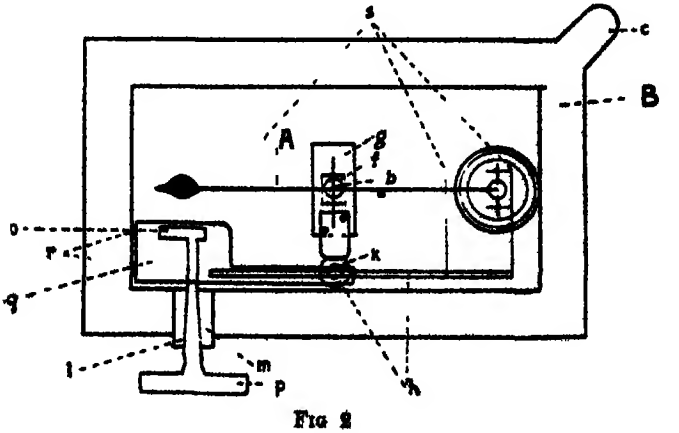


Fig 2

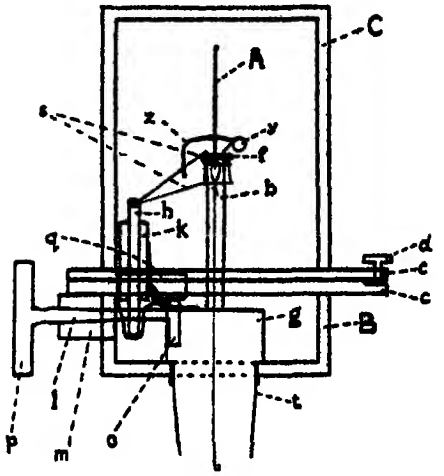


Fig 3

The attachments for the arrestment and release of the beam are as follows: The upright arm of a brass T-piece *h* passes through a hole along the axis of the rod *k*, an L-shaped piece of stout brass wire is attached to the rod *k*, and the vertical part of this passes through a hole in the horizontal part of the T-piece, which is thus free to move in a vertical direction only. The vertical movement is controlled by the excentric cam *o* on the end of the rotating piece *l*. The latter consists of a brass stopper carefully ground to fit the tapering tube *m*, and when properly lubricated makes a perfectly vacuum-tight joint and at the same time can be rotated freely. It is provided with a T-shaped vulcanite handle *p*, vulcanite being used to prevent as far as possible access of heat to the case when the handle is turned. The cam *o* works on a curved lever of brass *q*, which is hinged to the base at *r*, and by its rise or fall raises or lowers the T-piece *h*. The lever *q* is made broad at its hinged end so as to allow of the partial withdrawal of *l* for purposes of lubrication. To the T-piece *h* are cemented the V-shaped pieces of fine quartz rods which are adjusted so as to centre but not to raise the beam when the arrestment is lifted. The bottom of the base is provided towards one end with a hole 2·3 cm. in diameter into which is cemented the ground glass joint *t*. In the balance of type B the suspended parts hang into this tube, by removal of which access may be had to the scale-pan and counterpoise. A small quantity of calcium chloride in the bottom of the tube serves to dry the air within the case. The brass tube *u*, which is soldered to the case, has the manometer tube *v* cemented into it, connection with the atmosphere or with a Geryk vacuum pump is made through the two-way stop-cock *x*. The height of the mercury column in the

manometer is read in the usual manner by means of a telescope and scale to a tenth of a millimetre To prevent entrance of dust particles, the air entering the case through the stop-cock is made to pass through a tightly packed plug of cotton wool A thermometer is cemented into a narrow brass tube (not shown in the figure) which is soldered into the top of the cover.

It was found that a cement consisting of 95 per cent shellac and 5 per cent oil of cloves was capable of making perfectly vacuum-tight joints, and this was accordingly used wherever cement was required in the construction of the case

The capacity of the finished case to retain a vacuum was repeatedly tested and the following measurements are quoted as typical :—

February 1, 1909, 6 P.M. ..... P = 0·41 cm.
February 2, 1909, 10 5 A.M ... . P = 0 41 cm.

In order to damp accidental vibrations the levelling screws of the case are supported on glass plates lying on rubber corks 2 inches in diameter and 1 inch deep This device was found sufficient to obviate all trouble of this kind

The zero position of the instrument is determined by reading the position on a millimetre scale of the image of the filament of a Nernst lamp cast by the mirror attached to the beam In the more sensitive balance of type A it was found that at pressures greater than about 2 cm. the heat from the source of light created considerable disturbances, and to minimise these the lamp was screened by a metal case with a small hole in it. The light passed through this hole and then through a flat glass cell containing a solution of alum to absorb as much heat as possible, and in addition a hollow double screen of asbestos was suspended in the path of the beam, which screen, by a pulley arrangement, could be raised for the moment when it was desired to read the position of the image.

We will now give a detailed description of each type of micro-balance.

Micro-balance A. A differential micro-balance capable of measuring alterations of not less than one two-hundred and fifty-thousandth of a milligramme (4×10^{-9} gramme)

The case having already been described, the description of the instrument and the method of calibrating and using it will now be given under the following heads :—

(*a*) The Beam, (*b*) Theory and method of calibrating and using the instrument

(*a*) The beam consists of a framework of fused quartz rod of the form

shown in figs. 1 and 2, A. Its dimensions along with its other constants are given in the table at the end of this section.

The use of fused quartz as the material of the beam has the following important advantages:—

(1) It is under all probable conditions of usage absolutely incorrodible, does not, as most, if not all metals, probably do, occlude gases, and is only slightly hygroscopic.

(2) It is very light, and in this respect has a further advantage over all metals except aluminium.

(3) Its tensile strength is very great, and its elasticity perfect within the limits of possible strain.

(4) Its coefficient of temperature expansion is exceedingly small, and consequently no distortion of the beam, with consequent alteration of the sensibility of the balance, is to be anticipated as the result of small temperature changes. This property also confers on the beam immunity from fracture during process of construction or subsequent exposure to high temperature.

(5) It is readily and cheaply obtainable in a condition of perfect purity and practical homogeneity. This latter condition, as will be shown below, is absolutely essential in a sensitive vacuum-balance.

(6) It is easily and safely manipulated with the oxy-gas flame, so that the beam can be quickly constructed, and the adjustments for balance, stability, and sensibility readily made in the manner described below.

(7) The whole beam consisting entirely of quartz, can be thoroughly and easily cleaned in a manner which would be impossible if any metal whatsoever entered into its construction.

The disadvantages of quartz as compared with metals lie in its small conductivity for heat and electricity. No trouble, to our knowledge, has arisen from the former of these causes, and the irregularities of behaviour, which were at an early stage of the experiments traced to the persistence of the electrification acquired by the beam during handling, have now been entirely obviated by ionising the air inside the case either by an X-ray discharge or, as is more convenient, by placing on the floor a small quantity of uranium oxide.

The form of the beam, that of a double triangle, figs. 1, 2, and 3, A, is one well adapted for ensuring rigidity. The slight inclination of the rods forming the lower sides of the triangle to the horizontal enables the centre of gravity of the whole to be brought close to the central knife-edge without the use of a subsidiary mass. The balance thus forms a single rigid system oscillating about a central knife-edge. Where the adoption of such a system is feasible, its advantages over the ordinary balance with its double suspended scale-pans are obviously very great.

The central and only knife-edge is ground upon the end of a quartz rod 0·6 mm. diameter and about 2 mm long, which had been fused at the end to a blob 1 mm. in diameter The length of the knife-edge is consequently about 1 mm ; the angle between its two planes is about 90°. These two planes are very carefully ground and the angle of their intersection, when viewed under a high-power microscope, showed as a perfectly straight line with no irregularities.*

In the earlier beams constructed, a double-point axis was used instead of a knife-edge, but this was found unsuitable for balances of high sensibility, as also was a modification of it formed by holding the two points for a moment in the oxy-gas flame in the hope that they would assume a perfectly spherical form. In this case they would give an equivalent ideal knife-edge passing through the centres of curvature of the spheres, but as a matter of experiment it was found that the departure from uniformity of curvature was too great to permit of constant sensibility over any finite range of oscillation The double-sphere and double-point forms of knife-edge were consequently abandoned in favour of the above described knife-edge, which has been found to function in an entirely satisfactory manner.

In the earlier micro-balances constructed a very small concave glass mirror was attached by shellac to a projecting rod of quartz in line with the central axis. Small changes of zero which occurred were assigned with probability either to a steady evaporation of the shellac or to a viscous flow in it, producing a consequent shift in the position of the principal axis of the mirror In order to avoid the use of a cement, some concave quartz mirrors (diameter 4 mm, focal length 25 cm), with lug attached, were ground and polished By means of the lug it was found possible to fuse the mirror directly to the beam The position of the mirror relative to the knife-edge is shown in fig 3, *y*

* The grinding of the knife-edges is easily done with the aid of a small holder in the form of a piece of hard steel 2 × 1 cm, across the centre of which is riveted a piece of brass 1 cm square and about 2 mm in thickness Holes are drilled in the projecting end of the brass piece, and into these the quartz rods which are to be ground are cemented. The grinding is done on a piece of glass plate which has been ground with fine emery or carborundum, and the holder is moved on the plate in such a manner that it rests on the steel base and on the quartz rods. A gentle pressure is used in rubbing, and the holder is turned alternately so as to grind the ends to a chisel. The operation is finished off with very fine carborundum mud, and then a second piece of steel 2·5 × 1 cm. is secured on the back of the first piece so that it projects 2·5 mm. on either side of it. Resting the holder on this new edge, and again on the ends of the quartz rods, the latter are ground on a very finely ground glass plate without using any carborundum or emery, but with a little glycerine and water as lubricant. A gentle pressure is used in this final grinding. It is quite easy to make a set of three knife-edges in an hour or less.

Attached to the beam at its centre, and on the opposite side to the mirror, is a small piece of quartz rod with its end turned downwards at a right angle (fig. 3, *z*). This rod serves the dual purpose of balancing the mirror and of allowing the position of the centre of gravity of the beam to be adjusted. Attached to one end of the beam is a small quartz bulb of known internal volume, and containing air sealed up at known temperature and pressure. The object of this bulb will be shown immediately. At the other end is attached a quartz counterpoise of any desired shape, which will depend on the purpose for which the particular micro-balance has been constructed The difficulties of adjustment for balance and for the position of the centre of gravity, which at first seemed insurmountable, disappeared with the discovery that quartz is appreciably volatile in the oxy-gas flame, and consequently, after a rough adjustment has been made by fusing on or removing small quantities of thin quartz rod to the counterpoise for balance, and either to the apex of the beam or to the subsidiary arm *z* for centre of gravity, the final adjustment is made with extreme ease by holding a projecting point on the counterpoise or on the apex of the rod *z* in the hot flame of the oxy-gas for periods of time varying from half a minute to a fraction of a second. Before making this final adjustment the beam is cleansed by boiling it for about 10 minutes in aqua regia and then for short intervals in successive quantities of distilled water. Unless the beam is thoroughly clean, *no consistent readings are obtainable*

Finally, to accelerate any slow minute changes in the shape of the beam it is annealed for about 12 hours in a hot-air oven at about 200° C Beams which have been treated in this manner show a quite remarkable constancy of behaviour, whereas, if the annealing be omitted, small and irregular changes of zero take place within a period of a few days

Theory and Method of Calibration and Weighing

A difficulty which faced the authors with the construction of the first micro-balance of a sensibility exceeding one ten-thousandth of a milligramme was that of constructing and calibrating a set of weights light enough to be used with it. The sensibility of the earlier balances was indeed determined by the use of a rider consisting of a measured length of quartz fibre, cut from a longer length which had been accurately measured and weighed on a delicate assay balance, but the practical difficulties of handling this rider convinced us that a set of such weights would be impracticable in actual size This difficulty, which naturally increased with the attainment of a sensibility of the order of one hundred-thousandth of a milligramme, has been completely overcome by the adoption of what the authors believe to be an entirely new

method of weighing, by means of which weights of the order of one-hundredth of a milligramme can be compared with the standard measures with an accuracy of one five-hundredth of their amount, *i.e* the absolute value of such weights can be determined with certainty to one fifty-thousandth of a milligramme (2×10^{-8} gramme), while changes of weight can be measured of an order as low as one two-hundred and fifty-thousandth of a milligramme

Not only is this new method of weighing remarkable for its accuracy, but also for its convenience and rapidity of working, in which respect it equals, if it does not surpass, the method of compensation, rider-adjustment, and oscillation employed with the ordinary precision balance

The method depends for its successful application on the attainment of perfect homogeneity in the beam, this being the necessary and sufficient condition that the zero position shall be independent of the pressure of the air surrounding it. For if we have a balance of any perfectly homogeneous material, and the length of the arms—which may be considered of negligible weight compared with the loads—be l and l_1, if v and v_1 be the volumes of these loads, the density of the material being ρ, that of the medium in which the beam is immersed σ, then for equilibrium we have the relation $v(\rho-\sigma)l = v_1(\rho-\sigma)l_1$ or $vl = v_1l_1$, that is to say, the equilibrium position is independent of the difference in density between the material of which the beam is composed and that of the medium in which it is immersed. Given such a beam, then, the method consists in employing as counterweight the whole or any fraction of the weight of the air contained in a sealed quartz bulb of accurately determined volume The *rationale* of the method is as follows —If a quartz bulb be filled with air at the same temperature and pressure as the air surrounding it, the effective weight of the contained air will, in accordance with the principle of Archimedes, be zero If, however, the density of the air within the bulb differ from that of the surrounding air, then the inside air will possess a certain effective positive or negative weight. If v be the internal volume of the bulb which was sealed off at P_1 and T_1, then the weight of air within the bulb is $v\sigma_0 T_0/P_0 P_1/T_1$, if σ_0 is the density of air at normal temperature and pressure T_0 and P_0 If now the bulb be immersed in air at P_2 and T_2, its effective weight will be $v\sigma_0 T_0/P_0 (P_1/T_1 - P_2/T_2)$ If, again, the pressure and temperature of the air in which the bulb is immersed are changed to P_3 and T_3, the effective weight will be $v\sigma_0 T_0/P_0 (P_1/T_1 - P_3/T_3)$, and the change in effective weight will be $v\sigma_0 T_0/P_0 (P_2/T_2 - P_3/T_3)$, or, if the temperature be assumed constant, $v\sigma_0 T_0/P_0 (P_2 - P_3/T_2)$ If the balance is adjusted so that the zero position on the scale corresponds to P_2 and T_2, and a small amount of substance to be weighed be placed on the scale pan, the weight of this substance is given by

the effective weight of the bulb Thus in micro-balance A the internal volume of the quartz bulb, determined (as in all cases) by filling with mercury, is 0·0085 c c It was sealed at a temperature of 23° C. and a pressure of 759 mm. The weight of air contained in it is, therefore, 0·01012 milligramme, and by varying the pressure in the vacuum-case any value may be given to its virtual weight between this maximum value (which, of course, would only be exerted in a complete vacuum) and zero

A change of pressure of 1 mm in the case corresponds to a variation of the effective weight of 0 01012/759 = 0·00001333 ($1{\cdot}3 \times 10^{-5}$ milligramme), and an alteration of temperature of 1° C at 20 mm. pressure to a variation of less than 1×10^{-6} milligramme The temperature effect is therefore negligible at all pressures lower than 50 mm * (the variation in volume of the quartz bulb with varying pressure in temperature is also negligible)

As the pressure in the case can easily be read to one-tenth of a millimetre, it is seen at once that an accuracy of determination of $1\,33 \times 10^{-6}$ milligramme can be obtained provided the zero of the instrument remain constant and the beam be homogeneous.

As it is impossible to obtain quartz absolutely free from air-bubbles, the desired homogeneity in the beam was obtained by making all parts of rod drawn from the same sample of the best obtainable rod of large diameter That the requisite degree of homogeneity is actually obtained in this way will be seen from the following results —

Micro-balance, Type A, without attached bulb, sensibility (determined by placing a quartz rider on the beam and observing its position and the deflection produced) of such an order that a deflection of one scale-division corresponds to a change in weight of one one-hundred-thousandth of a milligramme (1×10^{-8} gramme).

Zero under observation from September 23, 1908, to October 8, 1908, during which period thirty-six (36) observations of the zero were made with the following results —

24 readings of 608 and 12 readings of 607 scale-divisions were obtained. (No attempt was made to read closer than the nearest scale-division.) During this period the pressure was varied irregularly from 5 mm to 8 cm

At the conclusion of this series of observations the bulb was attached and sealed up and the beam again adjusted and calibrated.

* The change in effective weight δw for an alteration of pressure δP_1 in the case is given by $\delta w = \frac{w}{T_1} \frac{T}{P} \delta P_1$, or if T can be taken as equal to T_1 by $\delta w = \frac{w \delta P_1}{P}$, and for an alteration in the temperature of the case δT_1 by $\delta w = \frac{w P_1}{P T_1^2} \frac{T . \delta T_1}{}$

The constants of this balance are as follows —

Total weight of beam with attached mirror, bulb, and counterpoise	0·177 gramme.
Length of arm	5 1 cm.
Volume of bulb	0 00865 c.c
Weight of air contained in bulb	$1\,02 \times 10^{-5}$ gramme
Time of complete oscillation	35 seconds.

The extreme sensibility and accuracy of the balance is evidenced by the accompanying table, which gives the values of the pressure inside the balance-case and the corresponding scale-readings The third column gives the calculated values of the scale-reading for each pressure on the assumption that the change in scale-reading is directly proportional to the change in pressure·—

Pressure (in mm of mercury)	Observed reading to nearest mm	Calculated reading
4 3	631	(631)
4 5	632	632
5 6	638	636
5 7	637	636
6 4	639	638
6 6	640	639
7 0	642	642
9 8	650	650
12 2	658	658
15 2	668	668
15 7	670	670
17 4	674	676
22 5	694	693
28 7	715	(715)
35 8	738	736

The greatest difference between the observed and calculated scale-readings in this table amounts to two scale-divisions only, and this occurs in only four out of fifteen readings The average discrepancy is considerably less than one scale-division. As shown above, the change in weight corresponding to a change of pressure of 1 mm is $1\,3 \times 10^{-8}$ gramme, and since a change of pressure of 24 4 mm. produces a shift of 82 divisions in the scale-reading, the change of weight corresponding to one scale-division is equal to

$$\frac{1{\cdot}3 \times 24\,4}{83} = 3\,82 \times 10^{-9} \text{ gramme},$$

i.e less than one two-hundred and fifty-thousandth of a milligramme

It follows that changes of weight exceeding this amount occurring in any

substance attached to the arm of this balance can be observed and measured, even if the changes occur with considerable slowness, since we have shown that the resting point of the balance remains constant over long periods of time

Description of micro-balance B A micro-balance to determine in absolute measure weights not in general exceeding 1 decigramme with an accuracy of "at worst" one ten-thousandth of a milligramme (1×10^{-7} gramme)

This balance, like micro-balance A, is constructed wholly of quartz and consists of a beam almost identical in form and dimensions with that of A and one suspended system only For the central axis it has been found advantageous to replace the small single knife-edge by a pair of knife-edges ground on the ends of two quartz rods about 1 cm. apart

In place of the ordinary double scale-pan system a counterpoise is rigidly attached to one end of this beam, and the weight of the suspended system at the other end is always adjusted to equilibrate this.

Several different methods of attaching the suspended system to the beam have been tried Of these, the ordinary knife-edge and plane of fused quartz have hitherto been found least satisfactory A fine rounded point and plane have given good results up to a sensibility of about one four-thousandth of a milligramme, but proved unsatisfactory at higher sensibilities, while attachment by means of a short and very fine quartz fibre fused to the beam at one end and to a suspended hook at the other has proved much the best mode of suspension *

Attached to the plane which rests on the end point (or to the fibre if such be used) is a hook which carries the suspended system

This consists of a fine quartz rod with a hook at each end, a sealed-up quartz bulb α and quartz scale-pan β, both of which are similarly provided with two hooks, and a quartz counterpoise γ, which can be attached to the bottom of the scale-pan

This type of gravity balance appears to us to offer some important advantages over the customary one In the first place, the difficulties of construction are much diminished by the avoidance of the adjustments for

* The attachment of the hook to the beam by means of the fibre is carried out as follows —

A small T-piece of quartz is attached by one arm of the T to the end of the beam, the other end serving as a holder, to which a small rod of quartz can be fused for adjusting its position; a hook is then put on one end of a piece of fine quartz rod, holding now the beam in one hand and the hook in the other, the leg of the T-piece is fused to a globule in a very small oxy-gas flame, the straight end of the hook is now passed through the flame to touch this globule, and then the two are drawn apart about half a centimetre and at the same time withdrawn from the flame A few trials will result in the production of a satisfactory flexible suspension.

placing the knife-edges in plane and parallel, and for equalising the length of arms.

In the micro-balance with fibre attachment the want of perfect flexibility of the fibre conditions an alteration in the length of arm for different positions of the beam, but, since the counterpoise and substance to be weighed are both attached to the same hook on the fibre, this cannot introduce any error into the weighings, which are in such a case independent of the length of arm

The method of weighing with this balance is as follows:—

(*a*) The quantity of substance to be weighed does not exceed the total weight of air contained in the bulb In this case the pressure inside the balance-case and the resting point having been taken with the scale-pan empty, the substance to be weighed is placed on the pan and the pressure adjusted until the same resting point is obtained If w is the total weight of air contained in the bulb, which was filled at the pressure P, and P′ represents the difference in pressure required to recover the original resting point, then the weight of the substance is wP'/P.

(*b*) The quantity of substance to be weighed exceeds the weight of air contained in the bulb

In this case it is necessary to prepare one or more counterpoises which must be lighter than the original one, and must differ from each other by a known amount not exceeding w It is obvious that such counterpoises can be easily made, and their difference in weight determined with great accuracy by the above described method on the micro-balance itself

With a series of such counterpoises, which take the place of a set of weights, quantities of any substance not exceeding in weight that of the heaviest counterpoise can be weighed with an absolute accuracy equal to that with which the weight of air in the bulb can be ascertained, and with a relative accuracy depending on the capacity of the instrument for recovering its zero, and on the accuracy with which the pressure can be read.

It has been found preferable to take the actual resting point of the balance rather than to determine this from its oscillations. The oscillations of both types of micro-balance have been found to damp rapidly, five to ten minutes being sufficient for the instruments to come to rest. The damping factor is nearly but not quite independent of the pressure, and is greater for the point and plane suspension than for the rigid system, or for the beam with quartz fibre suspended system.

Behaviour of Micro-balance, Type B.

Figures will be quoted showing the behaviour of this balance both with point and plane suspension and with fibre suspension.

Total weight of beam and suspended system .	0·93 gramme
Volume of bulb	0 422 c c
Amount of air contained in bulb	$5\,04 \times 10^{-4}$ gramme
With point and plane attachment, time of swing	13 5 seconds.

The following are typical successive readings of pressure and resting point —

P = 72 2 mm. Resting point = 620.

Four readings after arresting the beam each time gave the same resting point

The scale-pan and counterpoise were removed, after which

P = 72 0 mm Resting point = 620

The zero is therefore not disturbed by manipulations necessary in weighing.

This has been repeatedly shown. The sensibility was such that a change of pressure of 3 3 mm caused a change in the resting point of nine scale-divisions, therefore, since a change in pressure of 1 mm represents a change in effective weight of $\frac{5\,04 \times 10^{-4}}{760}$.

A change of resting point of one scale-division represents a change in weight of $\frac{5\,04 \times 10^{-4} \times 3\,3}{760 \times 9} = 2\,43 \times 10^{-7}$, that is, less than one four-thousandth of a milligramme

The volatility of quartz in the oxy-gas flame can be well shown by an instrument of this sensibility Thus before placing the counterpoise in the flame the readings were

P = 72·0 mm Resting point = 620

After placing in the flame for less than one second,

P = 64 7 mm. Resting point = 620

Difference in pressure = 7 3

Loss in weight = $\frac{5\,04 \times 10^{-4} \times 7\,3}{760} = 4\,83 \times 10^{-6}$ gramme.

The loss in weight is relatively very considerable if the time of heating is slightly increased; thus in a second experiment before heating the counterpoise

P = 64 7 mm Resting point = 620.

After heating for two seconds,

$$P = 41\cdot3 \text{ mm} \qquad \text{Resting point} = 620$$

Therefore loss in weight $= \dfrac{5\cdot04 \times 10^{-4} \times 23\cdot4}{760} = 15\cdot52 \times 10^{-6}$ gramme

The behaviour of the same balance with fibre suspension is shown by the following typical figures —

Time of swing	33·0 seconds
Pressure	13·5 mm
Resting point in five successive readings after arrestment each time	559 or 560
Pressure	6·0 mm
Resting point in five readings after arrestment each time	688

A change in pressure of 7·5 mm causes a change in resting point of 128 scale-divisions, and the sensibility is therefore $\dfrac{5\cdot04 \times 10^{-4} \times 7\cdot5}{760 \times 128} = 3\cdot88 \times 10^{-8}$, or less than one twenty-five thousandth of a milligramme

That no appreciable change in weight is caused by the careful handling of the quartz scale-pan and counterpoise is shown by the recovery of the resting point after these had been removed and replaced, when

$$P = 6\cdot1 \text{ mm} \qquad \text{Resting point} = 689$$
$$\text{Calculated resting point for this pressure} = 690$$

Of the micro-balances described in the foregoing pages, micro-balance A was designed some months ago with the object of measuring the loss in weight which radium salts are supposed to undergo during disintegration

If the generally accepted views as to the nature and magnitude of these changes are correct, the instrument is amply sensitive for the purpose, if greater sensitiveness is desirable it can easily be obtained by still further lightening the beam, or lengthening the time of swing We had hoped to have been able to crucially test this question, but all our efforts to obtain a few milligrammes of radium bromide have been unsuccessful Although we have not yet given up hope of obtaining what we require, we have thought it better to publish a description of this balance as well as that of the less sensitive instrument which we have called Type B, as it seemed to us that many problems might present themselves to other investigators the solution of which might be materially assisted if such an instrument were available

The Effect of a Magnetic Field on the Electrical Conductivity of Flame.

By Prof. H A. Wilson, F R.S., King's College, London

(Received June 14,—Read June 24, 1909.)

The following paper contains an account of some measurements of the change in the conductivity of a Bunsen flame produced by a magnetic field the direction of which was perpendicular to the current through the flame and to the motion of the flame gases. The velocity of the negative ions in the flame has been calculated from the results, and the value of the velocity obtained agrees approximately with that found by other methods

The flame used consisted of a row of 12 small Bunsen flames burning from quartz tubes. The centres of the tubes were 1 cm apart, and each tube had an internal diameter of 0 5 cm Each flame was about 6 cm high, and the adjacent flames touched each other, so that a flame about 14 cm long, 6 cm high, and about 2 cm. thick was obtained.

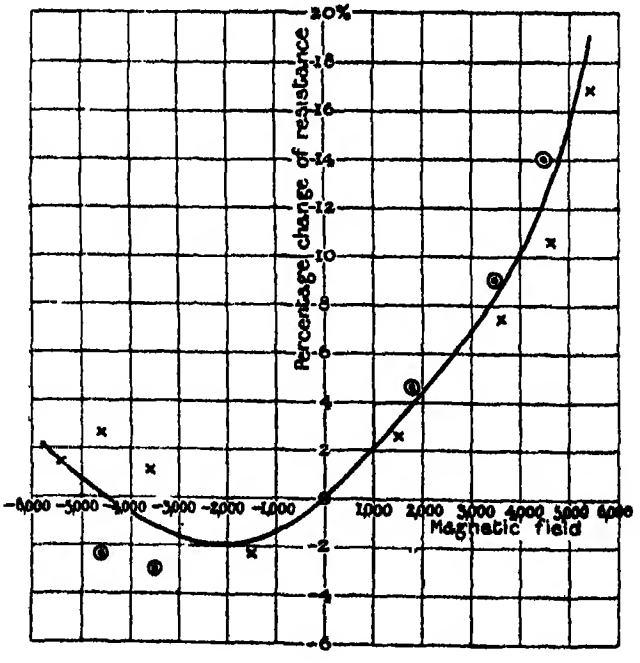
Two platinum disk electrodes were supported in the flame facing each other about 10 cm apart, and were connected through a galvanometer to a battery of secondary cells Some potassium carbonate was put on the negative electrode to increase the current.

The potential gradient along the flame was measured by means of two platinum wires which were supported horizontally in the flame perpendicular to the horizontal line joining the centres of the disk electrodes These wires were connected to an electrostatic multicellular voltmeter The capacity of the voltmeter was increased by connecting it to a $\frac{1}{2}$ microfarad condenser, and large amyl-alcohol resistances were put in the wires leading from the flame to the voltmeter This arrangement was adopted to prevent the small oscillations of the flame making the voltmeter needle unsteady. The flame was placed between the poles of a large Du Bois electromagnet, the conical pole pieces of which had been removed so that a fairly uniform field could be produced in a horizontal direction perpendicular to the line joining the disk electrodes

It was found that passing a current through the magnet produced a gradual change in the conductivity of the flame, which remained when the current was stopped This appeared to be due to the heating of the coils altering the draught of air to the flame In addition to this effect there was a sudden change in the conductivity on turning the magnetic field on or off. It was easy to disentangle the two effects, but only rather rough measurements could be obtained.

The ratio of the potential difference between the two wires to the current was taken as a measure of the resistance of the flame. It has been shown by several observers that this ratio is independent of the current.

It was found that the percentage change in the resistance for a given magnetic field did not vary much with the current Two sets of observations, one with potential differences between the platinum wires of from 200 to 400 volts and the other with from 50 to 150 volts, gave nearly equal results The distance between the platinum wires was 7 cm The results obtained are shown in the figure, the percentage change of resistance being plotted against the strength of the magnetic field. Each point represents the mean of several observations The crosses are the results obtained with the higher potentials and the circles those obtained with the lower It will be seen that with the field in one direction the resistance was increased by an amount increasing more rapidly than the field, whereas with the field in the opposite direction the resistance was diminished with small fields, but slightly increased with fields above about 4000.



Owing to the upward motion of the flame gases there is an induced electric force along the flame which opposes the current when the field is on in one direction and helps it with the field in the other direction If

therefore, we take the mean of the effects for the two directions of the magnetic field, we shall get the value of the effect which would have been obtained with the flame gases at rest

The following table contains the values obtained in this way, using values taken off the curve :—

Magnetic field (H)	Percentage change of resistance (σ)	σ/H^2
1000	0 3	$3 \cdot 0 \times 10^{-7}$
2000	1 3	$3 \cdot 2 \times 10^{-7}$
3000	2 ·6	$2 \cdot 9 \times 10^{-7}$
4000	4 ·8	$3 \cdot 0 \times 10^{-7}$
5000	8 ·0	$3 \cdot 2 \times 10^{-7}$
	Mean	$3 \cdot 1 \times 10^{-7}$

The last column contains the values of σ/H^2, which do not differ much

If it is assumed that the velocity of the positive ions in the flame is small compared with the velocity of the negative ions, then we may apply Sir J J Thomson's* theory of the effect of a magnetic field on the conductivity of metals to the flame According to the theory we have $\sigma/100 = \frac{1}{3} H^2 k^2$ where k denotes the velocity of the negative ions due to one electromagnetic unit of electric force Hence $k = (3\sigma/100\,H^2)^{\frac{1}{2}} = 9 \cdot 6 \times 10^{-5}$ cm./sec For 1 volt per centimetre this gives $k = 9600$ cm./sec

Mr E. Gold† measured the velocity of the negative ions in a Bunsen flame and found $k = 8000$ by one method and 13,000 by another. The result just obtained is nearly equal to the mean of his two results The velocity, of course, must vary to some extent in different Bunsen flames.

Half the difference between the effects with the magnetic field in the two directions gives the effect presumably due to the induced electric force in the flame The following table contains the values of this effect —

Magnetic field (H).	Percentage change of resistance (δ)	δ/H
1000	1 7	$1 \cdot 7 \times 10^{-3}$
2000	3 1	$1 \cdot 6 \times 10^{-3}$
3000	4 3	$1 \cdot 4 \times 10^{-3}$
4000	5 4	$1 \cdot 4 \times 10^{-3}$
5000	7 4	$1 \cdot 5 \times 10^{-3}$

* 'Rapports Congrès International de Physique,' Paris, 1900, vol. 3, p. 144

† 'Roy Soc Proc.,' A, vol. 79, 1907

The last column shows that the effect is approximately proportional to the magnetic field, as was to be expected If v denotes the velocity of the flame gases, we should expect the induced electric force to be equal to Hv, and hence $\delta/100 = vH/X$, where X denotes the strength of the electric force along the flame.

This was not less than 10 volts per centimetre, so that we get $v = X\delta/100H > 10^4$ cm./sec

Now the velocity of the gases in a Bunsen flame is not more than 200 cm /sec, so that it appears that this effect is at least 50 times greater than was to be expected In fact, if it had had the value to be expected it would have been negligible, and the change of resistance would have been independent of the direction of the magnetic field

It appears, therefore, that the effect of the magnetic field on the resistance of the flame can be represented as the sum of two terms, one proportional to the square of the field and the other proportional to the field The first term has the value to be expected, but the other term is much too large

The discrepancy would be removed if the negative ions moved *upwards* with a velocity proportional to the horizontal electric field and equal to 10^4 cm /sec for 10 volts per centimetre, but there does not seem to be any reason for supposing that they do so If the two wires in the flame are connected to a quadrant electrometer the induced electric force in the flame, when no current is flowing, due to the upward motion of the flame gases in the magnetic field, can easily be measured, and it is approximately equal to the product of H and the velocity If $H = 5000$ and $v = 200$ the induced force is 0 01 volt per centimetre, whereas to explain the observed change in the resistance it would have to be 0·5 volt per centimetre for a field along the flame of 10 volts per centimetre

I am not at present able to offer a satisfactory explanation of the magnitude of the part of the effect proportional to the field

In conclusion, I wish to say that my thanks are due to Mr. G. H Martyn, B Sc., for his assistance in carrying out the experiments described.

Surface Flow in Calcite.

By G T. Beilby, F.R.S.

(Received June 17,—Read June 24, 1909)

Part I.—*The Surface Skin produced by Polishing Cleavage Plates of Calcite, with some Measurements of its Thickness*

The researches of Mr C. V. Barker on "The Formation of Regular Growths of Crystals of one Substance upon those of another"* have given a new interest to certain of my earlier observations on the formation and structure of polished surfaces †

Mr. Barker having found that the method of polishing a cleavage face of calcite used by me in my original experiments did not interfere with the parallel growth of sodium nitrate crystals on the polished face, he invited me in February, 1906, to repeat his experiments on this particular point This I did, using my own crystals and methods of polishing These observations fully confirmed those of Mr Barker, for parallel growths were readily obtained, not merely on faces which had been lightly polished by gentle rubbing with clean soft chamois leather, but also on faces which had been polished by the more drastic processes of grinding and polishing with abrasive powders

In a truly polished surface I have never been able to detect any traces of crystalline structure. The flowed layer on the surface is left with a smooth and glassy appearance like that of a varnish or enamel. The greater solubility of this layer, as well as its greater hardness and homogeneity, indicates that an important change in its molecular structure has occurred. I have sought to explain this change by the supposition that the crystalline orientation of the molecules is completely disturbed at the moment of flow and that the subsequent solidification occurs before the molecules have time to arrange themselves in conformity with the orientation of the undisturbed crystal The surface layer is thus left in an amorphous or vitreous condition. If these conclusions are correct, a very interesting question arises as to the nature of the influence of the calcite crystal in determining the parallel growth of sodium nitrate crystals on its surface. If the molecules of the polished surface are not themselves in the regular crystalline arrangement proper to calcite, the parallel growth of the nitrate crystals cannot be due to the influence of the calcium carbonate molecules with which they are in direct

* 'Chem. Soc. Journ.,' 1906, vol. 89, p. 1120

† 'Roy. Soc. Proc.,' vol. 72, p. 221.

contact, and the conclusion appears to be forced upon us that the influence under which the nitrate molecules are oriented proceeds from the crystal as a whole, and *that this influence is sufficiently strong to act across the disturbed layer which has been produced by polishing.*

As a first step towards the solution of this problem, it appeared desirable that a more detailed and searching exploration of the layer developed by polishing should be made Having accumulated a good deal of practical experience in the exploration of surfaces by step-by-step etching or solution, I was satisfied that the method was capable of considerable development, and that it could be used in this case with a fair amount of confidence.

In the method which was adopted, small drops of uniform size and containing known quantities of hydrochloric acid were placed on the polished surface and were allowed to remain there till the acid had dissolved all the calcium carbonate it could take up The drop was then washed off first with water and then with absolute alcohol. The weight of the calcium carbonate removed was calculated on the assumption that the whole of the HCl in the drop was converted into $CaCl_2$. The density of the calcium carbonate was taken to be that of calcite—2·75—though it is probable that the density of the polished layer is considerably greater than this The area of the surface acted on by the drop was measured, and from this the thickness of the layer dissolved by the acid was calculated. Microscopic examination showed that, with all but the most dilute solutions of acid, the amount of carbonate removed was fairly uniform over the whole area over which the drop spread. The strength of the acid used ranged from 0 2 to 0·000125 per cent. HCl; the weaker acids were made directly by dilution from the stronger It was noticed that the more dilute acids lost a part of their solvent power if they were left standing overnight in glass flasks or bottles, the tests were therefore always made with freshly prepared dilutions. It seems probable that the errors involved in this method all tend to cause the calculated effects of the removal of $CaCO_3$ to be in excess of the actual effects, the error being greater as the higher dilutions are reached. This is of special interest in connection with the determination of the minimum depth of disturbance which could be detected by the microscope, as in this case the calculated thickness of the layer removed is practically certain to be a maximum figure. Experiments were made by applying successive drops of acid to the same spot on the polished surface and some interesting observations were made in this way, but in the most complete series of experiments the drops of the different dilutions were placed on fresh parts of the surface. The pits so produced were carefully explored by the microscope, using objectives of various resolving powers and with different means of illumination.

Photomicrographs of the pits were secured, but these are reserved for publication with a more detailed description of these observations.

The etching was made in the first instance with solutions in a descending scale of acidity, the series being pushed as quickly as possible towards the furthest point at which any visible alteration of the polished surface could be detected. When a condensed beam of sunlight was used to give the oblique illumination of the surface, it was possible to detect the effect of a drop of acid which contained only 0·000125 per cent. of HCl The depth of the layer removed by this drop did not exceed 0·62 $\mu\mu$ (1 $\mu\mu = 10^{-6}$ mm). This pit could be seen, though not very distinctly, by illuminating the spot by the critical image of a Nernst filament. Illumination by the nearly critical image of the sun showed the whole surface of the pit brilliantly specular and of a sky-blue colour A photo-micrograph of this was obtained. If it is correct to assume that the solvent effect of the acid was uniformly distributed over the whole surface of the pit, then it follows that a roughening of the surface not more than 2 molecules in depth has been detected.

The results of the step-by-step etching have been summarised in tabular form They show that the mechanical disturbance caused by the polishing agent penetrates to a depth of 500 to 1000 $\mu\mu$. At this depth the disturbance consists mainly of the deeper scratches or furrows which have been ploughed through the thin lamellæ of which the crystal is built up. This ploughing has so completely flowed the crystalline substance that the scratches and furrows have been perfectly healed over and no traces of them are disclosed even when a powerful beam of light is thrown across the surface Yet the action of the solvent acid enables us to follow with the utmost minuteness the lines of disturbance and flow. As the surface is approached, the furrows are finer and more numerous till, at a distance of 100 $\mu\mu$ from the surface, they disappear and the action of the solvent only discloses fine and shallow flow lines similar to those which have been observed in the polishing of metals. In the 50 to 100 $\mu\mu$ layer at the surface there is no trace even of the remains of broken-up lamellæ, and the appearance is absolutely vitreous and homogeneous, like a coating of varnish or enamel.

Tested by the loaded needle, the surface is harder and more tenacious than the original undisturbed face of the crystal. This greater hardness and tenacity became more evident as practical experience in polishing calcite was accumulated. It was found that the resistance of the surface layer to the finer abrasives increased as the polishing proceeded. After a surface skin has been fully developed by polishing, it is able to resist the cutting action of the powders by which the surface had been satisfactorily ground at an earlier stage. It happens not infrequently that the polished surface is

spoilt by accidental scratching just when it has reached the final stage. When this occurs, it is useless to attempt to repolish it by going back only a few stages. In most cases there is no alternative but to clear off the whole skin by grinding and to start again from the beginning. When clearing off this skin, one is made to realise how thoroughly its properties differ from those of the natural crystal surface

HCl per cent	Depth of layer in $\mu\mu$	No of molecules in thickness	Illumination by an intense incident beam at an angle of 30° with the surface	Illumination by a slightly oblique beam of transmitted light	Scratches across the field
0·000125	0·62	2	Distinct specular appearance on a deep sky-blue ground	No visible structure even with an objective of N A 0·95 and magnification × 700	A few accidental scratches
0·00025	1 25	4	The same	The same	,,
0·0005	2 50	8	Brilliant specular appearance	The same	,,
0 001	5·00	16	The same, but more distinctly granular	The same	,,
0·002	10 00	32	The same, oblong granular across the flow lines	Flow lines faintly visible	,,
0·005	25 00	80	The same; granules larger	Flow lines more distinct	,,
0·01	50 00	160	The same	The same, with granular texture	Regular scratches appear
0 02	100 00	320	The same, specular layer disappearing	Scratches becoming uncovered	Over 200
0·05	250·00	800	Specular appearance only on edge of scratches	Disturbance confined to scratches	Over 100
0·10	500 00	1600	The same	The same, but fewer scratches	About 30
0 2	1000 00	3200	The same	A few scratches in the undisturbed lamellæ Their sides partly etched into rounded forms and partly broken and splintered	About 5

The acidity of the solvent is given in fractions of 1 per cent. of HCl
The depth of the layer is given in $\mu\mu$. 1 $\mu\mu = 10^{-6}$ millimetre
The diameter of the $CaCO_3$ molecule is taken as approximately 0·3 $\mu\mu$.

The increased hardness and tenacity of the surface skin is believed to be due to the same cause as the corresponding change, which my earlier observations have shown to occur in metals,* namely, to the sudden breaking down of the crystalline units into a mobile or liquid phase which immediately congeals into a vitreous or amorphous solid in which the capacity for further

* 'Phil Mag.,' August, 1904

flow is enormously reduced. In this new and harder state molecular movement of the surface still occurs on polishing, but it is confined to a layer only a few molecules in depth, and the flow is so limited that surface scratches and irregularities are no longer obliterated by it The difficulty in finishing off a calcite surface with a really fine polish is very great, and is largely due to this cause, for the longer the surface is polished the less capable it becomes of flowing into a true liquid-like skin The most perfect surface microscopically is obtained when a perfectly fresh cleavage face is lightly polished by the finger covered with soft chamois leather

Part II.—*The Parallel Growth of Crystals of Sodium Nitrate on Polished Plates of Calcite.*

The observations in Part I show that polishing develops over the crystal surface a true skin, the substance of which is in various respects profoundly different from the crystal substance from which it has been produced Whatever hypothesis may be put forward as to the intimate nature of this change of structure, there can be no doubt as to the reality of the change. There appears also to be no room for doubt that the liquid-like surface of the outermost layer of the skin, on which the perfection of the polish depends, is due to the fact that this layer has passed through a liquid phase and has solidified under the influence of surface tension. There is abundant evidence to show that the aggregation of molecules at a surface can be so completely controlled by cohesion that crystalline orientation cannot occur On this profoundly altered skin, with its smooth liquid-like surface, it has been found that the crystalline influence of the calcite is still sufficient to cause crystals of sodium nitrate to deposit in parallel order, that is with their molecules similarly oriented to those of the calcite.

It naturally occurred to me to try whether the orienting influence could make itself felt through layers of other indifferent substances. Films of gold and of platinum were deposited on cleavage plates of calcite, and on these parallel growths of sodium nitrate were still obtained But there was always the suspicion that these extremely thin metallic films were not really impervious, and that they might have become soaked with the nitrate solution, thus forming a physical connection between the crystal surface and the crystallising salt.

More success attended the production of deposited layers of calcium carbonate and oxalate. When a drop of lime water is placed on a calcite surface, in addition to the pellicle of carbonate which forms over its surface, a fine and fairly adherent layer of carbonate is deposited on the calcite. This layer is translucent, and under high magnification it is minutely.

granular; with care it can be polished till it becomes almost as transparent as the calcite itself. Films of various thicknesses were deposited and polished, and were then tested with sodium nitrate for parallel growths. The results showed that films could be obtained of sufficient thickness absolutely to stop the parallel growths, *but thinner and apparently equally perfect films were obtained which did not interfere with these growths.* I hope to be able to determine the critical thickness of the deposited layer which is just sufficient to cut off the orienting influence of the calcite.

Conclusions

1. The polished surface of a cleavage plate of calcite, even when illuminated by an intense oblique beam, shows no traces of the grave disturbance to which it has been subjected by the operation of polishing.

2. By the step-by-step removal of the surface layers by a solvent the nature and extent of the disturbance of these layers is disclosed, and it is found that ploughing and breaking of the lamellæ has occurred to a depth of from 500 to 1000 $\mu\mu$. The complete obliteration of the traces of this disturbance referred to above (1) shows that the disturbed substance as it passed through the mobile to the solidified state must have extended through the whole depth of the disturbed layers, otherwise the healing of the deeper furrows would have been imperfect, and their traces would have been shown up by the oblique beam.

3. The laying bare of the disturbed layers by the action of the solvent shows that this action is selective, the substance which has been flowed and solidified being first attacked. The molecular arrangement of the flowed substance must, therefore, be different from that of the original crystal.

4 The surface layer formed by polishing is harder than the unaltered crystal surface, and it is equally hard in all directions, which the natural surface is not. The new surface forms a genuine protecting skin over the crystal face.

5. The presence of this protective skin does not interfere with the parallel growth of crystals of sodium nitrate on the polished surface. Among a large number of experiments with polished surfaces produced in a variety of ways no case occurred in which this parallel growth was prevented.

6. By the deposition and subsequent polishing of layers of non-crystalline calcium carbonate and oxalate on the already polished skin, a further skin was interposed between the true surface of the calcite crystal and the deposited crystals of sodium nitrate. Up to a certain thickness this did not interfere with the parallel deposition of the sodium nitrate crystals, but

beyond this thickness the deposition was quite irregular and had no relation to the orientation of the crystal beneath.

Most of these observations have been before me for fully three years, during which period they have from time to time been repeated with any variations which appeared likely to put the questions involved to a more crucial test The most important of these questions is, can the directive influence of a crystal make itself felt through a layer of unoriented, or of differently oriented molecules? So far as these observations go, the answer appears to be in the affirmative, and the sole reason for hesitating to accept this answer without reserve is that it will, if accepted, fundamentally affect our views of crystalline aggregation and the growth of crystals. The greatest caution ought, therefore, to be exercised in drawing a final conclusion in this sense.

I hope to continue these observations and to extend them in other directions I should like here to acknowledge the kindness and courtesy of Prof. Miers and Mr C. V. Barker in making their original communications to me three years ago

On the Mechanism of the Absorption Spectra of Solutions.

By Robert A. Houstoun, M.A., Ph.D, D.Sc., Lecturer on Physical Optics in the University of Glasgow.

(Communicated by Prof. A. Gray, F.R.S. Received June 25, 1909.)

On the theory of electrons the colour of bodies is explained in three ways: (1) The body is homogeneous and the electrons in it execute vibrations about their mean positions in the molecules. (2) The body is homogeneous and the electrons in it are free to move about from molecule to molecule as is the case with metals. (3) The body may consist of small spheres or grains of colouring matter embedded in a homogeneous transparent medium. We get an absorption spectrum with bands in it from (1) and (3), but not from (2). We can distinguish (3) from (1) by the polarised light scattered by the grains, the quantity of light scattered is, however, small in comparison with the quantity absorbed in the grains. The granular structure is also revealed by ultramicroscopic examination. This investigation is concerned only with (1).

An absorption band, then, corresponds to a principal oscillation of the molecule or whatever system the absorption spectrum is characteristic of. Let the principal co-ordinates of this system be $q_1, q_2, \ldots q_n$. Then, when a light wave is being transmitted across the medium, if the motion is not damped, the typical equation of motion assumes the form

$$a_r\ddot{q}_r + c_r q_r = Q', \tag{1}$$

Q' being the resultant electric intensity at the point. Q' is related to Q, the electric intensity of the light wave, by the relation

$$Q' = Q + gP, \tag{2}$$

P being the polarisation and g a constant. To take account of absorption introduce the dissipation function,

$$2F = b_{11}\dot{q}_1^2 + b_{22}\dot{q}_2^2 + \ldots + 2b_{12}\dot{q}_1\dot{q}_2 + \ldots.$$

The typical equation thus becomes

$$a_r\ddot{q}_r + b_{r1}\dot{q}_1 + b_{r2}\dot{q}_2 + \ldots + b_{rr}\dot{q}_r \ldots + b_{rn}\dot{q}_n + c_r q_r = Q'. \tag{3}$$

Let the transmitted radiation be of the type $e^{i\theta t}$. If, as is usual, we assume that the friction terms do not appreciably affect the periods, we may solve the other $(n-1)$ equations on the assumption that the motion is undamped and substitute the results above. We obtain

$$q_1(c_1 - a_1\theta^2) = Q' \ldots q_n(c_n - a_n\theta^2) = Q',$$

and on substituting

$$(c_r+b_{r,r}i\theta-a_r\theta^2)\,q_r+\frac{b_{r1}Q'i\theta}{c_1-a_1\theta^2}\ldots+\frac{b_{rn}Q'i\theta}{c_n-a_n\theta^2}=Q'.$$

Thus
$$q_r=\frac{Q'\left[1-i\theta\left(\frac{b_{r1}}{c_1-a_1\theta^2}\ldots+\frac{b_{rn}}{c_n-a_n\theta^2}\right)\right]}{c_r-a_r\theta^2+b_{r,r}i\theta} \tag{4}$$

Let K be the specific inductive capacity of the medium. Then

$$K=1+4\pi P/Q. \tag{5}$$

The polarisation must be a linear function of the co-ordinates, therefore

$$P=f_1q_1+f_2q_2\ldots+f_nq_n. \tag{6}$$

Using (2), (5), and (6) and writing $(\nu-i\kappa)^2$ for K, we obtain

$$\frac{(\nu-i\kappa)^2-1}{(\nu-i\kappa)^2\frac{g}{4\pi}+\left(1-\frac{g}{4\pi}\right)}=4\pi\Sigma\frac{f_rq_r}{Q'} \tag{7}$$

We shall now assume that we are dealing with a well-defined absorption band, one on each side of which there is a more or less transparent region We shall also assume that we are dealing with solutions and coloured glasses which transmit an appreciable amount of light, when a millimetre or two thick, even at the darkest point of the visible spectrum We can then neglect κ^2 in comparison with ν^2 and can also neglect the variation in ν produced by the absorption band As the index of refraction of the solvents and glasses varies only 1 or 2 per cent throughout the visible spectrum, ν may be considered constant throughout the absorption band

The imaginary part of the left side of (7) is

$$-2\nu\kappa\Big/\left\{\nu^2\frac{g}{4\pi}+\left(1-\frac{g}{4\pi}\right)\right\}^2.$$

If q_r refer to the band under consideration, the only term on the right side of (7) which has an appreciable imaginary part is q_r/Q'. Rewriting (4),

$$\frac{q_r}{Q'}=\frac{1-i\mathrm{A}}{(c_r-a_r\theta^2)+ib_{r,r}\theta}=\frac{(c_r-a_r\theta^2)-\mathrm{A}b_{r,r}\theta-ib_{r,r}\theta-i\mathrm{A}(c_r-a_r\theta^2)}{(c_r-a_r\theta^2)^2+b_{r,r}{}^2\theta^2},$$

where A is a function of θ. Hence

$$2\nu\kappa=\frac{\mathrm{C}(b_{r,r}\theta+\mathrm{A}\{c_r-a_r\theta^2\})}{(c_r-a_r\theta^2)^2+b_{r,r}{}^2\theta^2}, \tag{8}$$

where C is a constant given by

$$4\pi f_r\left\{\frac{\nu^2g}{4\pi}+\left(1-\frac{g}{4\pi}\right)\right\}^2.$$

So far the discussion has been as general as possible. Assume, now, that

the band is due to a class of electrons vibrating about their mean positions; assume, also, as Drude has it in his Optics, that this class of electrons is uninfluenced by any others in the field. Then $A = 0$ and $C = 4\pi Ne$, where N is the number of electrons of this class per unit volume. Let m be the mass of one of them and e its charge in electrostatic units. Then $a_r = m/e$.

We may thus rewrite (8) in the simpler form

$$\kappa = 2\pi Neb_{rr}\theta \Big/ \nu \left\{ \left(c_r - \frac{m}{e}\theta^2 \right)^2 + b_{rr}^2\theta^2 \right\}. \qquad (9)$$

An expression of this type has been used to represent absorption bands by Ketteler, Pfluger, the author, and others, the constants being chosen to fit the experimental results, and it has been found that the only term in it varying appreciably with θ is the bracket in the denominator. We may therefore rewrite (9)

$$\kappa = 2\pi Neb_{rr}\theta_0 \Big/ \nu \left\{ \left(c_r - \frac{m}{e}\theta^2 \right)^2 + b_{rr}^2\theta_0^2 \right\}, \qquad (10)$$

where θ_0 is the value of θ at the maximum, θ_0 being given by $\theta_0^2 = c_r e/m$.

The maximum value of κ, $\bar{\kappa}$, is given by $\bar{\kappa} = 2\pi Ne/(\nu b_{rr}\theta_0)$. Let θ_1 be the value of θ for which κ has half its maximum value, then

$$\left(c_r - \frac{m}{e}\theta_1^2 \right) = \pm b_{rr}\theta_0.$$

Substituting for c_r,

$$\frac{m}{e}(\theta_0^2 - \theta^2) = \pm b_{rr}\theta_0$$

Hence

$$\bar{\kappa} = \frac{2\pi Ne^2}{\nu m(\theta_0^2 - \theta_1^2)}.$$

Let us now substitute for θ_0 and θ_1, $2\pi v/\lambda_0$ and $2\pi v/\lambda_1$, where v is the velocity and λ_0, λ_1 the wave-length of light *in vacuo*. Then

$$\bar{\kappa} = \frac{Ne^2\lambda_0^2\lambda_1^2}{2\pi\nu m v^2(\lambda_1^2 - \lambda_0^2)}.$$

Express e in electromagnetic units and v^2 will disappear from the denominator. Also, since λ_1 does not differ much from λ_0, we may write

$$\frac{\lambda_0^2\lambda_1^2}{\lambda_1^2 - \lambda_0^2} = \frac{\lambda_0^3}{2(\lambda_1 - \lambda_0)}.$$

Hence

$$\bar{\kappa} = \frac{Ne^2\lambda_0^3}{4\pi\nu m(\lambda_1 - \lambda_0)}. \qquad (11)$$

Let p be the number of electrons per molecule of colouring matter belonging to the absorption band under consideration. Then N/p is the

number of molecules per unit volume. Let c be the concentration of the solution in gramme-molecules per litre and let m_H be the mass of an atom of hydrogen. Then $c/(1000\, m_H)$ is also equal to the number of molecules of dissolved colouring matter per unit volume. Therefore

$$\frac{N}{p} = \frac{c}{1000\, m_H}.$$

Substituting for N from (11), we obtain

$$\frac{pe}{m} = 4000\, \pi \frac{\nu k}{c} \frac{m_H}{e} \frac{(\lambda_1 - \lambda_0)}{\lambda_0^3}. \qquad (12)$$

If we substitute 9660 for e/m_H, this gives

$$\frac{pe}{m} = 1\,300\, \frac{\nu k}{c} \frac{(\lambda_1 - \lambda_0)}{\lambda_0^3}. \qquad (13)$$

Formula (13) holds only on the very special assumptions made by Drude. Absorption bands in general have not the simple shape demanded by Drude's theory. I have, nevertheless, applied the formula to all the well-defined absorption bands on which I could get measurements. In view of our ignorance of the subject, any method that may throw light on the mechanism of absorption deserves to be followed up.

There is a great diversity of ways for specifying the absorption of a solution, but this does not make much difference in applying the formula. If the tables give the fraction of the incident light transmitted by a power of 10 or e, λ_1 is the wave-length for which the index has half its maximum value. On the other hand, if the fraction of the incident light transmitted is given directly, and I be its maximum value, λ_1 is the wave-length for which the fraction transmitted is $\sqrt{I}$ In most cases we get a somewhat different result according as we take λ_1 on the red or violet side of the band, I have, as a rule, taken it on the red side, as that is generally better defined. Once λ_1 was chosen it was never altered, no matter what value of pe/m it gave. The following table (p. 610) gives some results.

The measurements of Katz and Kalandek agree as well as can be expected in the case of eosin in water, but differ in the case of fuchsin in alcohol. We cannot, however, be certain that the substance used was the same in each case. Some of Katz's results were not utilised as they promised nothing new. Zsigmondy's results are given in the form of curves and the numbers were taken from the curves. The colouring matter exists in these glasses as an oxide or silicate, and apparently Garnett's theory does not apply.* Pulfrich does not state the strength of solution employed, and I

* Prof. Zsigmondy informs me that he has not examined them ultramicroscopically, but that it is to be assumed that they are optically homogeneous.

had to standardise his results by making a rough determination of the index of extinction for one wave-length. There are no measurements available on the didymium salts, but pe/m for their bands would probably have a low value, owing to their narrowness and the strength of the solution necessary to make them visible. It is not possible to utilise the work that has been done on the ultra-violet absorption of organic substances as it is not quantitative.

Substance	λ_0.	λ_1	pe/m.	Source of data.
Fuchsin in alcohol	550	585	$1\cdot8\times10^{7}$	Stanislaw Kalandek, 'Phys. Zeit.,' 9 Jahrg., pp. 128—135
„ aniline	565	595	$1\cdot7\times10^{7}$	
Phloxin in water	515	560	$1\cdot4\times10^{7}$	
Crystal violet in alcohol	575	611	$4\cdot9\times10^{7}$	
„ aniline	605	635	$4\cdot7\times10^{7}$	
Corallin in alcohol	465	515	$1\cdot6\times10^{6}$	
„ aniline	455	505	$1\cdot7\times10^{6}$	
Methylene blue in water	665	690	$5\cdot4\times10^{6}$	
„ aniline	675	660	$7\cdot7\times10^{6}$	
Water blue in water	575	646	$8\cdot1\times10^{6}$	
Eosin in water	515	533	$6\cdot9\times10^{6}$	
Eosin in water	516	525	$9\cdot2\times10^{6}$	Georg J. Katz, 'Inaug.-diss. Erlangen,' 1898.
„ alcohol	527	542	$2\cdot3\times10^{7}$	
„ turpentine	539	550	$6\cdot5\times10^{5}$	
Cyanine in alcohol	587	609	$5\cdot8\times10^{6}$	
„ ether	597	628	$6\cdot9\times10^{4}$	
„ chloroform	597	615	$7\cdot8\times10^{6}$	
„ benzol	608	627	$1\cdot5\times10^{4}$	
Hexamethylpararosaniline in water	587	597	$3\cdot4\times10^{6}$	
„ „ alcohol	608	624	$1\cdot3\times10^{7}$	
„ „ chloroform	618	630	$2\cdot4\times10^{6}$	
Fuchsin in water	550	578	$1\cdot7\times10^{6}$	
„ chloroform	550	570	$2\cdot4\times10^{6}$	
„ alcohol	558	578	$1\cdot6\times10^{6}$	
Pentamethylmonophenylpararosaniline in chloroform	582	605	$4\cdot3\times10^{7}$	
Pentamethylmonophenylpararosaniline in water	592	606	$3\cdot6\times10^{6}$	
Pentamethylmonophenylpararosaniline in alcohol	582	608	$1\cdot7\times10^{7}$	
Pentamethylmonophenylpararosaniline in ether	582	598	$1\cdot0\times10^{5}$	
Pentamethylmonophenylpararosaniline in benzol	603	620	$8\cdot4\times10^{5}$	
Cobalt chloride in water	504	540	$2\cdot5\times10^{3}$	Houstoun and Russell, 'Roy. Soc. Edin. Proc.,' vol. 22, Part II, p. 66
Uranyl nitrate in water	486	491	84	
	473	478	75 ?	
Three glasses coloured with CoO	644	668	$5\cdot0\times10^{4}$	R. Zsigmondy, 'Ann. d. Phys.,' (4), vol. 4, 1901, p. 60
	681	660	$3\cdot0\times10^{4}$	
	640	700	$3\cdot0\times10^{4}$	
Two glasses coloured with Cr_2O_3 „	620	690	$3\cdot2\times10^{4}$	
	655	725	$1\cdot2\times10^{4}$	
Three glasses coloured with NiO	640	670	$1\cdot3\times10^{4}$	
	640	672	$1\cdot0\times10^{4}$	
	610	662	$1\cdot1\times10^{4}$	
Potassium permanganate in water	547	570	$5\cdot1\times10^{5}$	C. Pulfrich, 'Wied. Ann.,' vol. 14, 1881, p. 177

The values of pe/m obtained for the anilin colouring matters, when dissolved in water, alcohol, or chloroform, are very striking, since the ratio of the charge to the mass of an electron is about $1{\cdot}8 \times 10^7$. *We are evidently dealing here with electrons, and Drude's assumptions are not far wrong* When the solvent is benzol or ether, the values of pe/m are considerably lower This raises the question whether the colouring matter is not in colloid solution in these cases. We have no observations to go upon, but in this connection a paper by Dr Sheppard* is interesting It may be noticed that a calculation made by Drude from the dispersion of solid cyanine gave $pe/m = 8{\cdot}5 \times 10^6$ for its absorption band.

The values of pe/m obtained for the inorganic salts suggest ions, though here potassium permanganate is difficult to place Probably, though, we shall have to fall back on the more general theory and possibly also take account of the scattering of light by granules.

In conclusion, I should like to acknowledge my indebtedness to Prof. Larmor for the benefit of his criticism

* S. E Sheppard, "On the Influence of their State in Solution on the Absorption Spectra of Dissolved Dyes," 'Roy Soc Proc.,' May, 1909

On the Absorption of Homogeneous β-Rays by Matter, and on the Variation of the Absorption of the Rays with Velocity.

By W. Wilson, M.Sc., Hatfield Scholar of the University of Manchester.

(Communicated by Prof E Rutherford, F R S. Received June 19,—Read June 24, 1909.)

The present work was undertaken with a view to establishing, if possible, the connection between the absorption and velocity of the β-rays. So far, no actual experiments have been performed on this subject, but Schmidt* has determined the velocity of the rays emitted by radium E and uranium X. These have absorption constants of 40 cm.$^{-1}$ and 14 cm.$^{-1}$, and their velocities, assuming the Lorentz formula for the variation of e/m with velocity of an electron, are 2 31 and $2\,76 \times 10^{10}$ cm. per sec. respectively. The matter has been attacked theoretically by Sir J J. Thomson.† He deduces a formula connecting the "diffusion" coefficient of absorption with the velocity, and finds that the variation takes place inversely as the fourth power of the velocity.

Seitz‡ has made a series of experiments on the number of cathode particles passing through thin sheets of matter, and finds the coefficient of absorption to vary inversely as $av^6 - bv^4$, where a and b are constants and v the velocity of the electrons.

It has been generally assumed that a beam of homogeneous rays is absorbed according to an exponential law, and the fact that this law holds for the rays from uranium X, actinium, and radium E has been taken as a criterion of their homogeneity §

This assumption is open to many objections, for the exponential law may be due to rays of different types being mixed in certain proportions. If the distribution of the rays and their velocity do not change in passing through matter, and if the absorption of the particles is proportional to the number present, we should expect an exponential law of absorption, but if their speed diminishes, the absorption should be greater the greater the thickness of matter traversed

In the present experiments radium, which has been shown by Kaufmann‖

* Schmidt, 'Phys Zeit.,' 1907, p. 361

† Sir J J. Thomson, 'Conduction of Electricity through Gases,' 2nd edition, p. 376.

‡ Seitz, 'Annalen der Physik,' vol 12, p 860, 1903.

§ Hahn and Meitner, 'Phys. Zeit.,' 1908, p. 321.

‖ Kaufmann, 'Gött. Nachr,' 1903, p 90.

to emit β-rays with velocities ranging between very wide limits, was used as a source of radiation, and by means of a magnetic field β-particles of different velocity were separated out and their absorption measured. Without entering at present into further details, it can be stated that the ionisation did not vary exponentially with the thickness of matter traversed, but, except for a small portion at the end of the curve, followed approximately a linear law.

The following experiments deal with this question and with the variation of the absorption of the β-rays with velocity.

Experimental Arrangement.

The general arrangement of the apparatus finally used is shown in figs. 1 and 2.

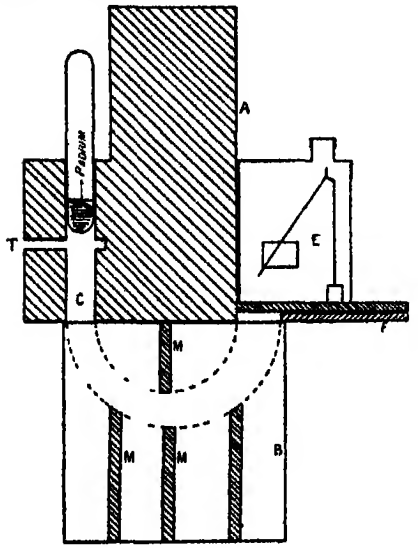


Fig 1

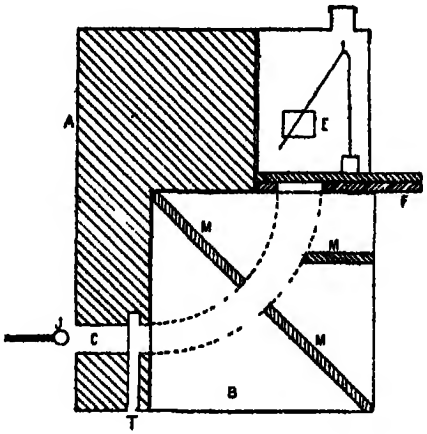


Fig 2.

A lead block A, containing a hole C, 0·8 cm in diameter, rested on the pole-pieces B of an electromagnet. In C was placed a glass tube containing about 30 milligrammes radium bromide. The preparation of radium employed was of about 7 per cent. purity, and it was found that most of the β-particles were absorbed before getting out of the active matter itself, so in the later experiments a corresponding quantity of radium emanation in equilibrium with its products, contained in a thin-walled bulb blown on the end of a capillary tube, was used as a source of radiation.

The chief experimental difficulty was the presence of a large γ-ray effect which, when the radium was used, often amounted to as much as

60 per cent. of the whole ionisation in the electroscope. To get the ionisation due to the β-rays alone this had to be subtracted from each reading of the ionisation, and the results consequently suffered in accuracy, especially when a few layers of absorbing material had been added, and the whole effect had become only slightly greater than that due to the γ-rays alone.

By the use of the emanation this effect was much reduced, and the accuracy of the results thereby greatly increased. With the arrangement shown in fig. 2 the γ-ray effect was not more than 20 per cent. of the whole, even for rays of very high or very low velocity.

The apparatus was arranged so that the rays could pass in a fairly well-defined beam into the magnetic field, which was perpendicular to the plane of the diagram. The field was found to be practically uniform, and the rays described circular paths and entered the small electroscope E through a hole 1·2 cm. diameter in a lead plate F. The distance from this hole to the hole from which the β-rays emerge is 3·9 cm., and from these data the curvature of the path of the rays could be determined. The radii of curvature for the arrangements shown in figs. 1 and 2 were 2·1 and 4·0 cm. respectively.

The electroscope was raised a little from the lead plate F, so that sheets of metal could be inserted in the path of the rays directly under the electroscope, and the variation of the ionisation with increasing thickness of matter determined. The field was measured by means of a Grassot fluxmeter, and also by comparing the induction through a small coil, whose necessary dimensions were known, when placed in the field, with a known induction. The two results were identical.

By pushing lead plates into the field at different distances from the base of the electroscope it was found that the rays were not scattered much in passing through the air, and that their paths were circles determined from the geometrical shape of the apparatus.

In order to reduce as much as possible the amount of scattered β-radiation which enters the electroscope, and also to ensure a greater purity of the rays used, lead screens were placed as shown at MMM. These, it will be seen, will from their positions very effectively cut out rays reflected from the pole-pieces, etc.

In addition to these reflected rays there is a large amount of secondary radiation set up by the γ-rays. The effects due to this cause were eliminated as follows. After every reading of the ionisation in the electroscope a lead plate of sufficient thickness to absorb all the β-rays which fell on it was pushed into the slot T and prevented any β-rays from entering the

magnetic field. At the same time it was so thin that the γ-ray effect on the pole-pieces, etc., was practically unaltered. In the second type of apparatus care had to be taken to place the slot in such a position that the lead plate did not come in the direct path between the radium and the electroscope. The value of the ionisation now found in the electroscope gives the amount to be deducted for both direct and secondary γ-radiation. A certain amount of secondary β-radiation is set up in the screen itself by the γ-rays, but since only that portion of this reaches the electroscope which has exactly the same properties as the rays we are considering, it will not affect the results.

The procedure in making an experiment was as follows. The field was adjusted to the right strength, and the ionisation in the electroscope measured. A reading was then taken with the lead plate in position, and the ionisation thus found subtracted from the previous reading. This was repeated for every layer of absorbing material which was placed under the electroscope.

Form of Absorption Curves.

As stated before, the ionisation in the electroscope was found not to fall off, according to an exponential law, with the thickness of matter traversed, but more rapidly the greater the distance penetrated. For the rays of higher velocity the relation was linear, except when the radiation had been cut down to a large extent, when it fell off more slowly with increasing thicknesses of matter.

Curves showing the absorption of the rays for some different velocities are shown in fig. 3 and in fig. 4. In the latter, the logarithm of the ionisation is plotted against the thickness of matter traversed. This brings out the difference between the law of absorption found and the exponential law more strongly. For the very easily absorbed rays the linear law was not so strongly marked, and the reason for this will be discussed later.

Reliability of the Experiments.

Experiments were then performed to determine whether the effect observed is really a property of the rays or due to the experimental conditions.

Three causes which might possibly influence the absorption curves are —

1. Lack of saturation of the ionisation current.
2. The shape and size of the electroscope and of the opening by which the rays enter.
3. The proximity of the magnetic field to the electroscope, which might cause irregularities by bending the rays inside it.

The first of these was tested for by comparing the time taken for the gold leaf to go over two widely different portions of the scale for various values of ionisation. If the ratio remains constant it is a proof of saturation.

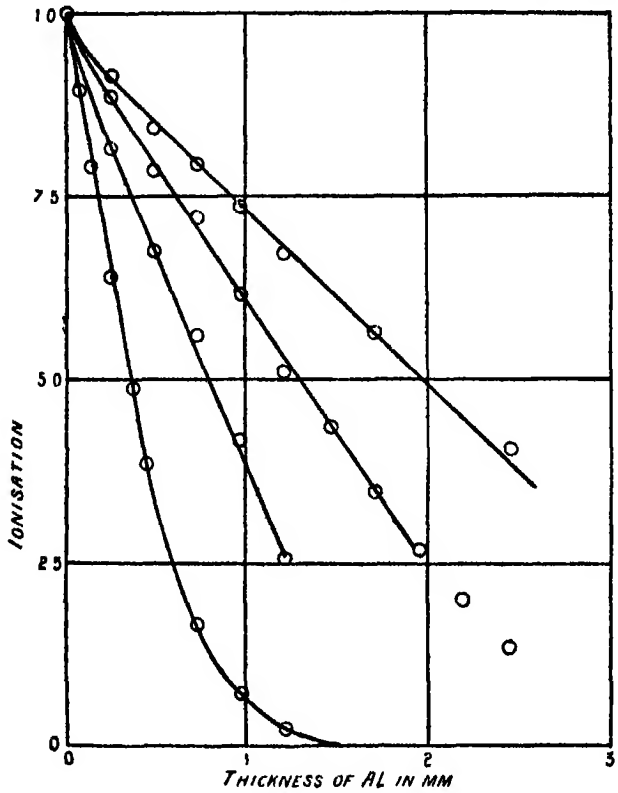


Fig 3

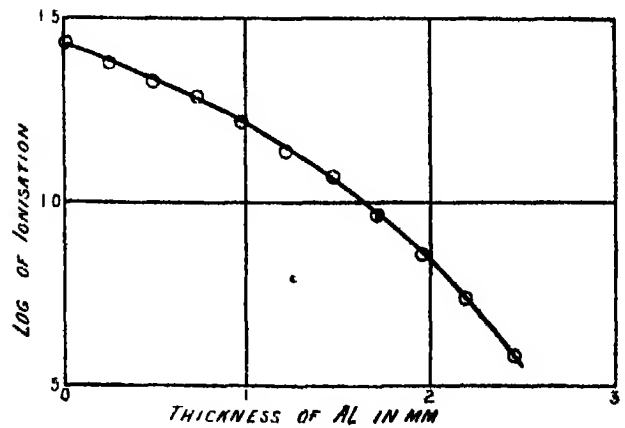


Fig. 4

Saturation was found to be complete A further test which fully confirms the other was made by measuring the absorption curves for the same type of rays, the strength of the sources being in the ratio of about one to eight The two curves were identical

The second point was tested for by placing a tray containing a preparation of actinium under the electroscope and determining the absorption of the β-rays emitted by it It was found to be an exponential law with the coefficient of absorption 30 cm $^{-1}$, which is the value obtained when measured in the ordinary way

A magnetic field was then applied to test the third point, and the same law of absorption still held good A small amount of radium emanation in equilibrium with its products was also placed under the electroscope and no difference in the absorption curves for the β-rays emitted by it resulted when a magnetic field was applied

Further, the absorption curves obtained, using the two types of apparatus shown in figs 1 and 2, are practically the same, although the magnetic field required to deflect the same type of rays into the electroscope is much greater in the first place than in the second

It also seemed possible that the effect might be due to the rays striking the absorbing screen at right angles and becoming more and more scattered as they penetrate the matter This would cause their path to become more tortuous and the rate of absorption would be thereby increased. That this is not the case was seen by allowing a practically parallel beam of actinium rays to fall normally on a screen placed under a larger electroscope, when the absorption still took place according to the general exponential law found for these rays.

Variation of Absorption with Velocity

We have seen that the absorption of homogeneous β-rays, when measured by the ionisation, takes place according to a linear law The relation between ionisation and thickness of matter traversed is given by $I = k(a-x)$, where a is the thickness of matter for which the ionisation would become zero if the law were rigorously true, and ka the initial ionisation a determines the rate of absorption of the rays and is a constant for rays of a given velocity. Absorption curves were determined for rays of different velocities, using aluminium, copper, and tin as absorbing substances The values obtained for the variation of a for aluminium with velocity are given in Table I. The velocity was calculated from the value of magnetic field × radius of curvature of the rays by means of the formula $HR = mv/e$, where H is the strength of the field and R, the radius of curvature of the rays.

e/m is determined by means of the Lorentz formula, $\frac{e}{m} = \frac{e}{m_0}\left(1-\frac{v}{c}\right)^{\frac{1}{2}}$, where c is the velocity of light and e/m the ratio of the charge to the mass of the electron for low velocities The value of e/m_0 is $1\cdot74 \times 10^7$ electromagnetic units.

If we define the absorption coefficient as λ, where $dI = -\lambda I dx$, I being the ionisation and x the thickness of matter traversed, we obtain from our previous equation

$$k = \lambda k (a-x)$$

The initial value of λ is therefore a^{-1} With this law of absorption λ is not constant, but increases as the thickness of matter traversed by the rays is increased The values of the initial value of λ are given in the fourth column of Table I. The values of a are plotted against HR and velocity in figs. 5 and 6 respectively, and in fig 7 the initial values of λ are plotted against HR

On account of the very complicated nature of the effects influencing the determinations of the absorption coefficients, it is not probable that any simple expression can be obtained connecting the values of a or λ with the velocity Between velocities 2·1 and $2\cdot9 \times 10^{10}$ cm per sec the following relation gives a very good agreement—

$$a = 5\cdot5\left(\frac{mv}{e}-1500\right)10^{-4}$$

for aluminium as absorbing substance, where a is expressed in millimetres.

For copper the equation is

$$a = 1\cdot2\left(\frac{mv}{e}-1000\right)10^{-4}$$

Table I

R	HR	V	a	λ
cm	Gauss cm.	10^{10} cm.	mm	cm $^{-1}$
4·0	1310	1·79	0·14	71·5
	1860	2·20	0·26	38·5
	2760	2·545	0·70	14·3
	3860	2·74	1·32	7·6
	4450	2·795	1·67	6·0
	5390	2·855	2·08	4·81
	6350	2·899	2·64	3·79
	8000	2·920	3·29	3·04
	8580	2·930	4·09	2·44
	8960	2·937	4·69	2·13
2·1	980	1·46	0·095	105·0
	1820	2·19	0·235	42·5
	2160	2·35	0·35	28·6
	2640	2·515	0·535	18·7
	3300	2·665	0·96	10·4
	4300	2·782	1·28	8·35
	5190	2·844	2·00	5·00
	5490	2·859	2·12	4·17

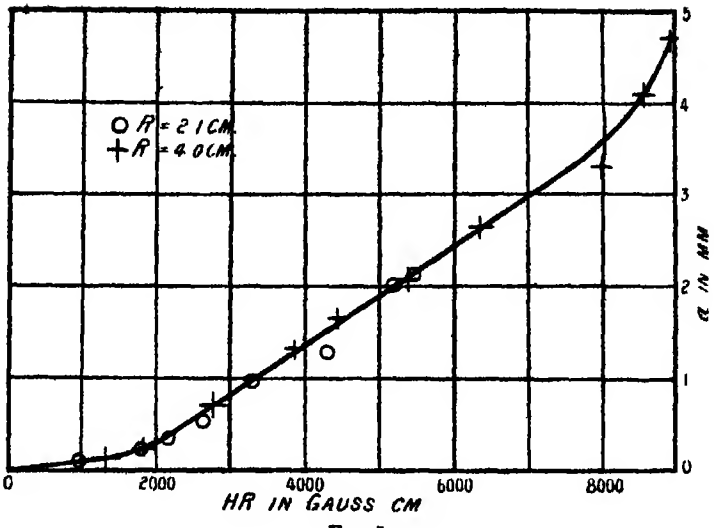


FIG. 5

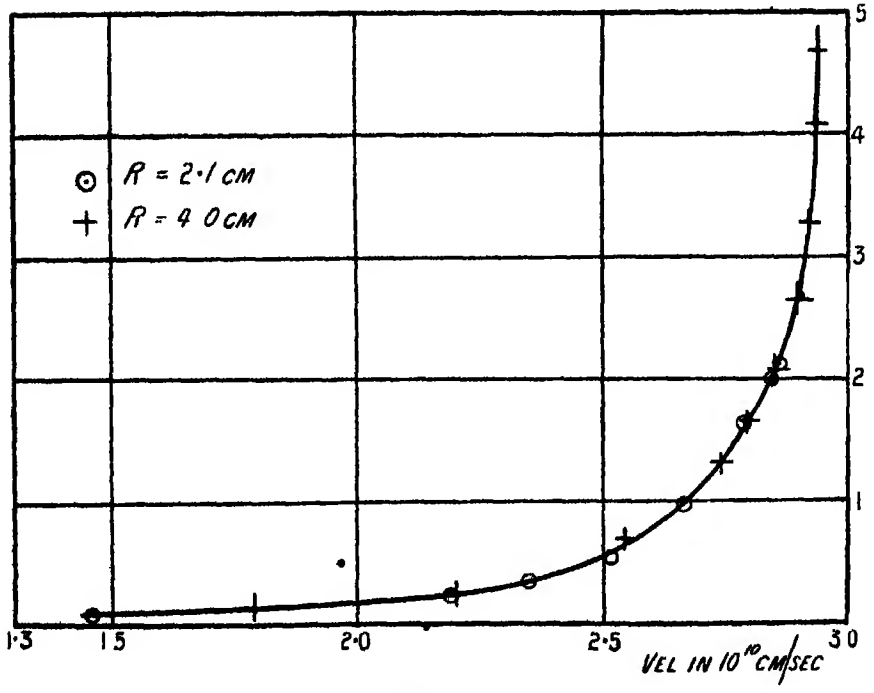


FIG. 6

In Table II the values of a for different velocities of rays when copper and tin were used as the absorbing media are given. The linear law of absorption

does not appear to hold so well for these substances as for aluminium, but the departure from the exponential law is still very great. For copper, fairly definite values of a could be obtained, but with tin the values are only

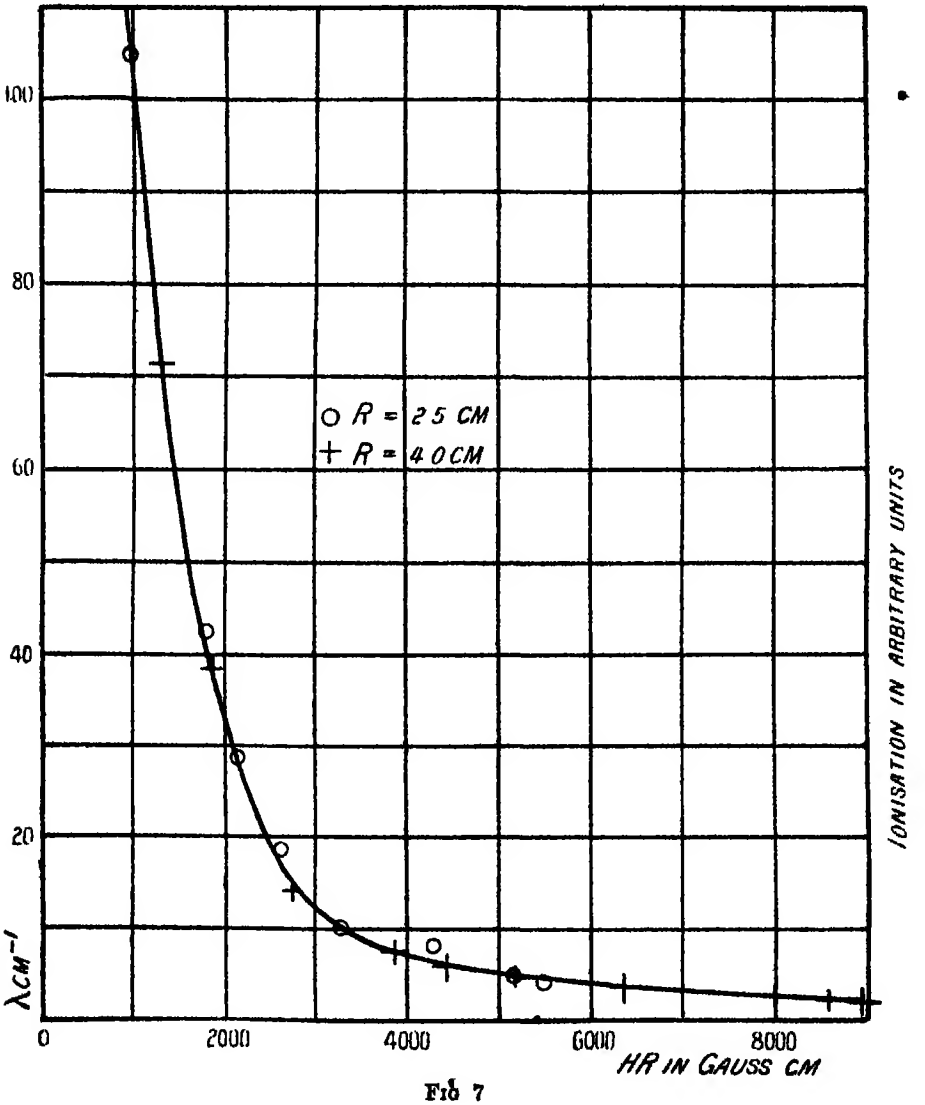


Fig 7

approximate. The indeterminate character of the curves obtained for tin is very likely connected with the secondary radiation from it, as peculiarities in this connection have been noted by all other observers who have worked on this type of experiment

It will be interesting to consider the effect of the density of the absorbing substance on the absorption curves, but more experiments will have to be performed before anything definite can be said on this point

Table II

HR	a in mm.	
	Tin	Copper
1860	0·130	0·118
3130	0·370	0·275
4400	0·626	0·430
6350	0·775	0·700
8570	0·940	0·985

The values of a given for copper in Table II are plotted in fig 8 against the values of HR.

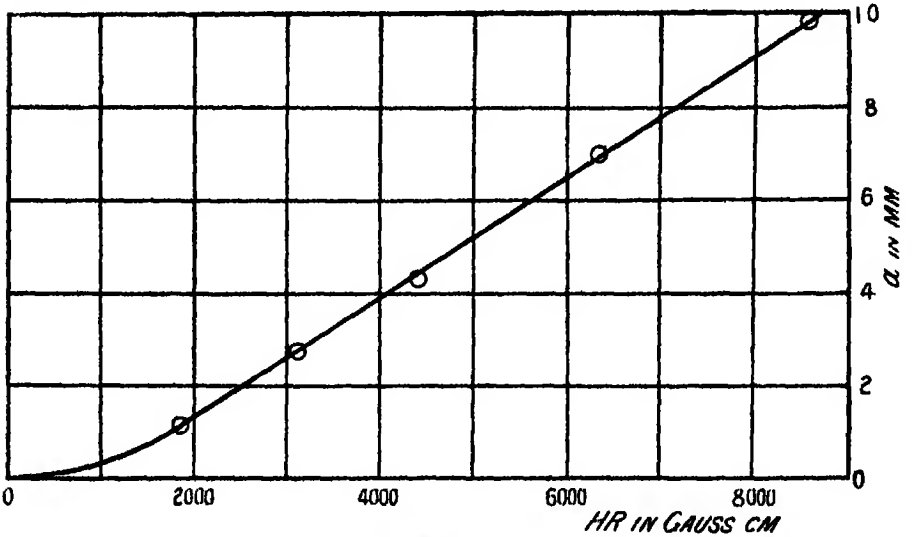


Fig. 8

Explanation of the Exponential Law found by various Observers for the Absorption of Rays from Radio-active Substances

Before entering into a discussion as to the meaning of the absorption curves obtained, it is preferable to try to explain why various observers have found that the rays from uranium X, radium E, and actinium are absorbed according to an exponential law with the thickness of matter traversed.

The fact that homogeneous rays are not absorbed according to an exponential law suggests that the rays from these substances are heterogeneous Now, Schmidt* gives a curve *which to some extent* shows the manner in which the rays from uranium are distributed about their mean velocity Using a similar arrangement to the author, he obtains a curve, shown in fig 9 (*a*), in which the ionisation in the electroscope is given for different field strengths

The area of this curve, then, represents the ionisation we would get in the electroscope if all the rays of all velocities from the uranium were allowed to enter together, instead of being deflected into the electroscope separately by the magnetic field.

From the absorption curves we have found we can determine the amount by which the rays corresponding to each separate portion of the curve are absorbed, and so can build up a similar curve for the rays after passing through any thickness of matter. Curve *b*, fig. 9, is a specimen curve obtained in this manner, showing the distribution of ionisation with magnetic

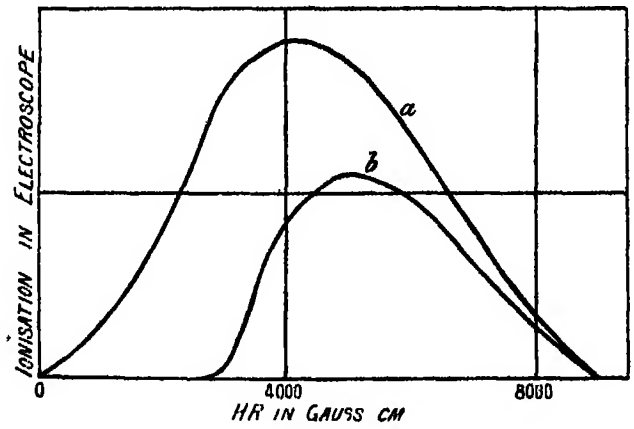


Fig 9

field after the rays have passed through 0 6 mm of aluminium The more slowly moving rays are completely absorbed, while the ionisation due to the more rapid particles has only decreased by a small amount The areas of such curves give us the ionisation we would get in the electroscope if all the β-rays from the uranium were allowed to enter it together after passing through the corresponding thickness of aluminium

In Table III are given the areas found in the above manner for curves drawn for many different thicknesses of aluminium

* Schmidt, 'Phys. Zeit.,' January 1, 1909

Table III.

Thickness of aluminium in mm. traversed	Area in arbitrary units	Log of area to base 10
0	746	2·873
1	658	2·818
2	556	2·745
4	437	2·640
6	341	2·533
8	262	2·418
10	204	2·310
12	158	2·199
14	121	2·082
16	90	1·954
20	55	1·740

The values given in this table are plotted in fig 10, and it will be seen that the curve is an exact exponential whose absorption coefficient is

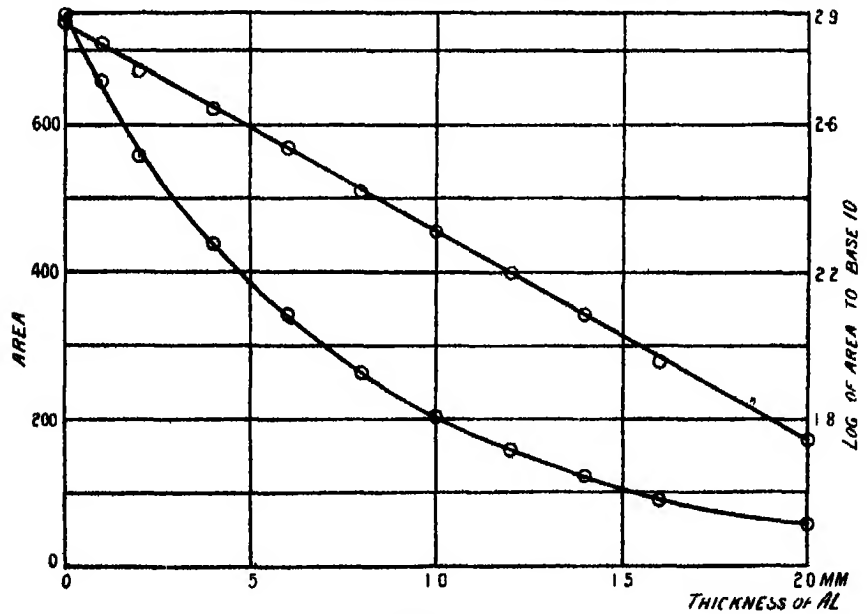


Fig 10

13·1 cm.$^{-1}$, a value not very different from that observed for the β-rays of uranium by Rutherford, 13·7, and Schmidt, 14. The curve extends over quite a wide range, the final area being about one-thirteenth of the initial one. It is thus clear that the exponential curve for absorption of rays is not, as has been widely assumed, a test of their homogeneity, but that in order that the exponential law of absorption should hold, we require a mixture

of rays of different types This view of the subject is strongly confirmed by some recent experiments of Soddy,* who finds that the rays from uranium are heterogeneous, some having velocities which correspond to HR = 6500. The mean velocity according to Schmidt† is that corresponding to HR = 4110 Gauss cm

We have seen that for any special velocity of rays the ionisation due to them after passing through a thickness of matter x is given by $i = k(a - x)$, where a determines the rate of absorption, and is fixed by the velocity of the rays, while ka is determined by the initial value of the ionisation

If we now assume that the numbers of the particles in a heterogeneous beam are distributed with regard to a in such a manner that the ionisation due to the rays of each particular velocity is $af(a)$, the ionisation due to them after passing through a thickness of matter x is given by

$$i = f(a)(a - x)$$

The ionisation due to the whole number of particles entering the electroscope together would be

$$I = \int_x^\infty f(a)(a - x)\,da$$

Now if I decreases with x according to an exponential law, we have

$$I = I_0 e^{-\lambda x} = \int_x^\infty f(a)(a - x)\,da$$

From this equation we obtain $f(a) = \lambda a I_0 e^{-\lambda a}$, or $af(a)$, which gives the initial ionisation for each different velocity of rays, is $\lambda a I_0 e^{-\lambda a}$

This is somewhat of the same type as the distribution of velocities used in problems on the kinetic theory of gases, but differs from it in the power to which e is raised It has a maximum when $a = \lambda^{-1}$

We thus see that it is possible to obtain a heterogeneous beam of particles, of which the different types of rays are absorbed according to a linear law, but the absorption of the whole beam takes place according to an exact exponential law.

Mechanism of the Absorption of the β-Rays.

There are two ways in which the absorption of a beam of particles can take place In one the particles lose energy as they pass through matter and finally cease to be effective as ionising agents This has been shown to be the manner in which the absorption of the α-rays takes place In the other the particles are stopped in mid career while their velocity is still high, and

* Soddy, 'Le Radium,' February, 1909

† *Loc cit*

Schmidt* and McClelland and Hackett† have with considerable success worked out a theory in which particles are assumed to pass through matter with a constant velocity, a sudden stopping of a certain proportion of the rays taking place in each thin layer of the matter. This view has received considerable support from some experiments of Makower,‡ who, by measuring the actual number of the β-particles from radium getting through various thicknesses of matter, has shown that this number varies in exactly the same manner as the ionisation

Sir J J Thomson has considered the question from a theoretical standpoint and finds that if the absorption is due to scattering and stopping alone, the law it follows should be exponential with the thickness of matter traversed

Crowther§ finds that scattering is "complete" after the rays have passed through very small thicknesses of matter, and Hahn and L Meitner have shown that when a parallel beam of the β-rays from actinium strikes the absorbing screen perpendicularly there is only a slight deviation from the exponential law of absorption

These experiments are against the view that the increase of the absorption coefficient for greater thicknesses of matter which we have found is due to the rays striking the absorbing screen normally and becoming more and more scattered as they pass through the matter

Change of the Velocity of the Rays in passing through Matter

Schmidt‖ has also attacked this problem, using as a source of radiation the rays from radium E He deflected the rays in a similar manner to that described above into an electroscope by means of a magnetic field and obtained a curve connecting ionisation in the electroscope and strength of field From the strength of field for which the ionisation was a maximum he deduced the velocity of the rays, and found that the position of the maximum point did not alter if he allowed the rays to pass through different thicknesses of aluminium before entering the magnetic field From this he concluded that the velocity of the rays does not change appreciably in passing through matter The absorption curves obtained in my experiments, however, suggest that the velocity of the rays decreases with thickness of

* Schmidt, 'Jahr. der Rad.,' 1908, and 'Ann. der Physik,' 1907, vol 23, p 671
† McClelland and Hackett, 'Roy Soc. Dubl. Trans.,' vol. 9, No 4, 1907
‡ Makower, 'Phil Mag.,' January, 1909.
§ Crowther, 'Roy Soc. Proc.,' A, p 308, 1908
‖ Schmidt, 'Phys Zeit.,' June, 1907

matter traversed and further experiments on this point were made as follows —

In fig 11, curve *a*, is shown the connection between ionisation and the electroscope and strength of field when a certain preparation of radium is used as a source of radiation As in the case of the uranium rays (fig 9), we

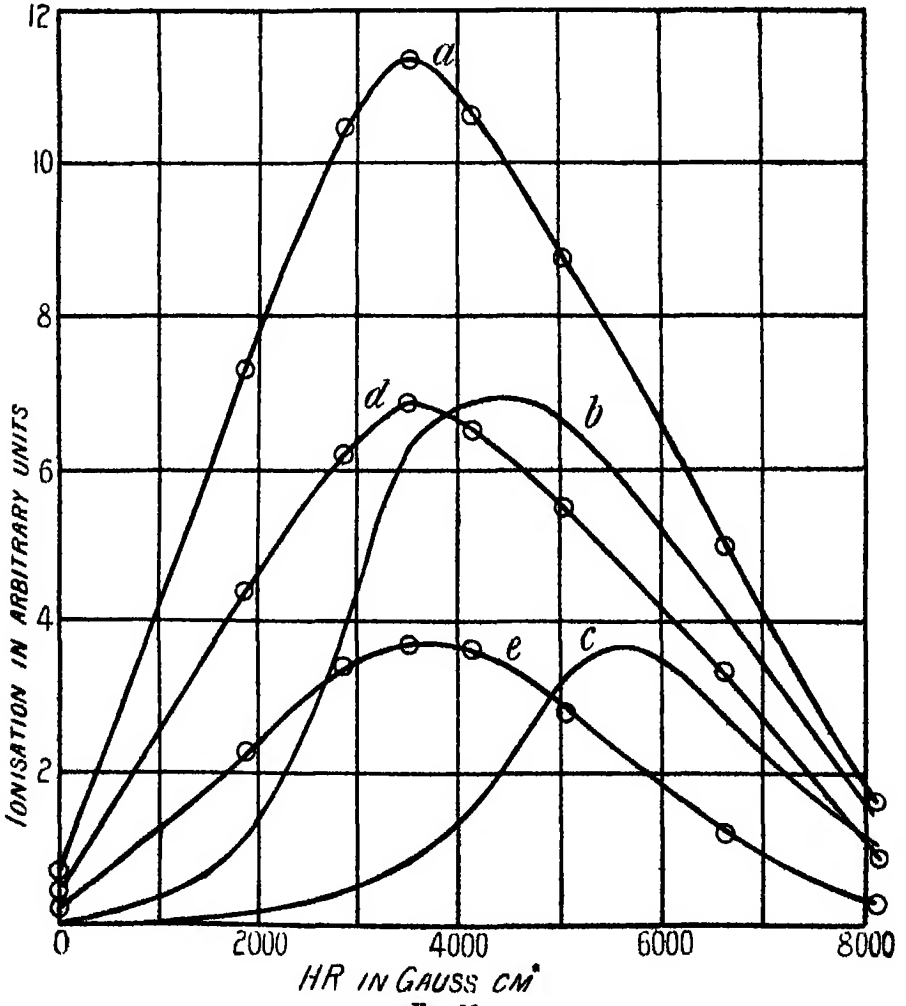


Fig 11.

can determine the shape of this curve after the rays have passed through various thicknesses of matter Curves shown at *b* and *c* for the rays after passing through 0 489 and 1 219 mm of aluminium were obtained experimentally by varying the field while sheets of aluminium of a given thickness were placed *under the electroscope* The rays from the radium were

then allowed to pass through screens of these thicknesses placed at *J* (fig 2) *before entering the magnetic field.*

If the particles do not decrease in velocity in passing through the matter, the curves obtained in this case connecting ionisation with strength of magnetic field should fall on *b* and *c* If the velocity decreases, however, they should fall to the left of these This was found to be the case, the curves being shown in the figure at *d* and *e*, and the particles therefore decrease in velocity as they pass through the matter

The velocity is found from the position of the maxima to fall from $2\cdot78 \times 10^{10}$ cm per sec to $2\cdot69 \times 10^{10}$ cm per sec while the rays passed through 0·489 mm. of aluminium, and from $2\cdot865 \times 10^{10}$ cm per sec to $2\cdot69 \times 10^{10}$ cm per sec. while they pass through 1·219 mm.

This experiment also explains why the experiments of Schmidt apparently show no change in the velocity of the rays. According to the views expressed in this paper he was dealing with heterogeneous rays and the position of the maximum should therefore move to the higher fields if the velocity of the rays does not change The actual decrease in velocity, however, brings the maximum point back to practically the same position as before

Deviation from the Linear Law

The bending away of the final portions of the absorption curves from the straight line may be due to three causes —

1 The size of the hole from which the β-rays emerge, and by which they enter the electroscope, makes the beam of rays used in these experiments not quite homogeneous This would cause the more rapidly moving rays to become relatively more and more important as the rays traverse the matter, and a departure from the straight line should therefore be observed Experiments made without the screens MMM (figs 1 and 2) give curves which depart earlier from the straight line The rays in this case are more heterogeneous, and this is what we should expect

2 The γ-ray effect, which has to be subtracted from each reading of the ionisation, also increases relatively in importance as the rays penetrate matter, and its exact value is very hard to determine If its effect were taken slightly too small it would produce a deviation from the linear law, in the direction observed

3. As Sir J J Thomson points out in "Conduction of Electricity through Gases," p 378, the mechanism of the absorption is not the same for the slow rays which cannot penetrate the atom and rapid ones which can do so The departure from the law may be due to this cause, and also to changes in the ionisation produced by the rays on account of their decrease in velocity

4. It is very probable that if we start with quite homogeneous rays, after they have passed through matter they will become heterogeneous owing to the velocity of each separate particle not being altered by the same amount.

The fact that the absorption curves for low velocities do not show the linear law so strongly marked as those of high velocities is very likely in part due to the greater heterogeneity of the rays, but there is also a great likelihood that this law would not even hold for pure slow rays The law depends on a variety of conditions which require for their full consideration data not yet acquired, and it is more than likely that the conditions change very much with the velocity of the rays, and the linear law propounded can only be regarded as approximate

Conclusions

The results obtained in this paper can be summed up as follows —

1 The absorption by matter of homogeneous β-rays does not take place according to an exponential law, when measured by the ionisation, but according to a law which is practically linear

2 The rays emitted by such substances as uranium X, radium E, and actinium are heterogeneous, and groups of rays can be built up which represent their properties with respect to absorption

3 The β-rays decrease in velocity in passing through matter.

4 The variation of the absorption of the β-rays with velocity has been determined experimentally, but does not appear to follow any simple law

I wish to thank Prof Rutherford for suggesting the subject of this research, and for his great help and encouragement during its progress

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The Properties of Colloidal Systems I.—*The Osmotic Pressure of Congo-red and of some other Dyes.*

By W M. Bayliss, F R.S

(Received April 19,—Read May 6, 1909)

[This paper is printed in Series B (No. 548), vol. 81]

OBITUARY NOTICES

OF

FELLOWS DECEASED.

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JOHN KERR, 1824—1907

DR. JOHN KERR was born on December 17, 1824, at Ardrossan, in Ayrshire. In the Album of the University of Glasgow for 1841, the year of his matriculation, he is described as "*Joannes Kerr filius natu secundus Thomæ piscarii in oppido Ardrossan*" His father, Thomas Kerr, seems to have removed to Skye when his son was yet very young, for the boy received part of his early education at a village school in that island

His attendance at University classes extended over the years from 1841 to 1849 He obtained prizes in Natural Philosophy in 1845–6, when the class was taught, during the illness of Prof Meikleham, by David Thomson, who afterwards became Professor of Natural Philosophy in Aberdeen. In the following year—the first of William Thomson's professorship—Kerr appears to have devoted himself specially, no doubt under the influence of the new professor, to the study of the mathematical theories of magnetism and electricity In the four years from 1846 to 1849 he received special prizes for examinations and essays in these subjects, and the "Earl of Eglinton's Prize of Twenty Sovereigns" as the most distinguished student in mathematics and natural philosophy for the degree of M A. In 1849 he graduated with "Highest Distinction in Physical Science," and his University career came to a conclusion

In 1848 and 1849 Prof Thomson was converting the old "professorial wine-cellar" into a physical laboratory, the first in Great Britain, probably in the world, and Kerr was one of the earliest members of the experimental corps who did pioneer work in the murky seclusion of what was known among the students of the time as the "coal-hole"! He was a divinity student, and, like some others of his class, did excellent scientific work, indeed, as Thomson declared in his Bangor Address in 1885, among the divinity students of the time were some of the best researchers, and they became all the better clergymen for having seen something of scientific methods and handled scientific instruments.

After some time spent in teaching, for he does not seem to have assumed officially the clerical duties for which he qualified, Kerr was appointed in 1857 to the post of Lecturer in Mathematics in the Glasgow Free Church Training College for Teachers In that institution he set up a small laboratory, modestly provided with apparatus, to a great extent at his own expense, and there he spent such time as he could spare from his class work in carrying on physical researches, aided, like Thomson himself, by a few devoted students He retained his post for forty-four years, and retired in 1901 to spend the remaining six years of his life among his instruments, working out special points connected with his optical researches.

His old students on that occasion showed their gratitude to him for his

teaching, and their respect for his achievements and the fine qualities of mind and heart which had endeared him to them as a friend as well as a master by presenting him with some silver plate.

Kerr wrote one book only, but within its scope and purpose it was a very good one. It was entitled 'An Elementary Treatise on Rational Mechanics,' and no doubt represented to a considerable extent such a view of the subject as a mature student, who had passed through the classes of Thomson, would naturally take Proofs attributed to Thomson appear in one or two places and the book contains, like Thomson and Tait's 'Natural Philosophy,' which appeared a year later, a chapter on "Cinematics" It adheres, however, to the old division of the subject into statics and dynamics, and therefore opens with a discussion of statics, in which the parallelogram of forces is proved by the old and little satisfying statical considerations. The dynamical (kinetic) part is based on Newton's Laws of Motion, a return to which was a feature of the 'Natural Philosophy', but it does not include the extension of the third law to the modern theory of energy, which, as Thomson and Tait first pointed out, is contained in the *Principia*. It would appear probable, therefore, that this view of the third law had not formed part of Thomson's teaching at the period when Kerr was a student On the whole the 'Rational Mechanics,' if revised and extended in some places and abridged in others, would form a sound introduction to the study of dynamics for the students of the present day

Kerr's name will, however, be always associated with his two great discoveries the birefringence developed in glass and other insulators when placed in an intense electric field, and the change produced in polarised light by its reflection from the polished pole of a magnet. Led by the view already experimentally verified in various ways, that the Faraday lines of induction in an insulating medium correspond to some marked physical change in the medium, he set to work to discover whether this change had any optical effect Taking a plate of glass about two inches thick, he bored holes from its ends until their extremities were within about a quarter of an inch of meeting. In these he inserted the terminals of the secondary circuit of an induction coil When the coil was in action, the intervening glass was subjected to intense strain. A beam of light, plane polarised by a Nicol's prism, was sent across the glass, at right angles to the line joining the terminals, and received on the other side by an analysing Nicol, arranged so as to give extinction of the beam when the glass was free from strain. Soon after the coil was excited, the light was seen to pass through the analysing Nicol, and to have become elliptically polarised. The maximum effect was produced when the wave-front of the incident light was arranged, as just described, to be parallel to the Faraday lines in the glass, and to have its plane of polarisation inclined to them at an angle of 45° Midway between these two positions of the plane of polarisation, that is when that plane was parallel to the lines of force, or at right angles to them, no effect was produced.

Kerr measured the effect by means of a compensator of strained glass, and

found that the glass appeared to have received, by the action of the electric field, a crystalline structure such as would have been produced by compressing it along the lines of electric force The effect was not instantaneously produced, it appeared about two seconds after the coil was started, and increased for twenty or thirty seconds afterwards In a piece of amber the opposite effect to that in glass was found to be produced, that is the material behaved as if it were extended along the lines of force He devised glass cells, which he seems to have constructed to a great extent himself with great labour and much mechanical skill, for the extension of his experiments to liquids He found that the effect was well marked in carbon disulphide and in paraffin oil, and was similar to that in glass He obtained results for a large number of organic liquids, of which the mere enumeration would take considerable space

His papers on this subject are contained in the 'Philosophical Magazine,' beginning with vol 50 (1875), in which his first great paper appeared, and continued at intervals for many years.

The view was expressed by Quincke that, for the production of birefringence by electric strain, non-uniformity of the electric field was essential, but Kerr showed that when a uniform field was produced in a stratum of carbon disulphide by means of parallel plate electrodes of considerable area, the effect was still found to exist He proved the same thing also later for solid substances, and his results were confirmed by Quincke

Kerr's second discovery was communicated first to Section A of the British Association, at the Glasgow Meeting in 1876, and the writer of this notice well remembers the excitement which the exhibition of the phenomenon caused among the distinguished physicists there assembled It was described in a paper published in the 'Philosophical Magazine' in the following year (vol 4, 1877)

The soft iron pole-piece of one core of a horseshoe electro-magnet was carefully polished on one face, and a beam of plane polarised light was thrown upon it and reflected into an analysing prism Various effects were produced according to the position of the reflecting surface with reference to the direction of magnetisation and to the plane of incidence of the light

When the light falls normally on a pole of an electro-magnet—that is when the polished surface is perpendicular to the direction of magnetisation, and the light is along that direction—the plane of polarisation appears to be turned through a small angle. In reality, however, the light has become elliptically polarised in the act of reflection, but each ellipse has a long axis and a very short one, and a plane parallel to the long axis may be taken as the new plane of polarisation of the light regarded as still plane polarised This new plane makes only a small angle with the former position of the plane of polarisation, and such that the direction of the apparent turning is opposite to that in which the exciting currents are circulating round the pole Thus the apparent direction of turning depends on whether the reflecting surface is a north or a south pole.

When the light is incident obliquely on the polished surface (still at right angles to the direction of magnetisation), and has its plane of polarisation either in or at right angles to the plane of incidence—so that, when the surface is unmagnetised, there is no effect of reflection at the metallic surface on the position of the plane of polarisation—the effect of exciting the magnet is to produce again a small apparent rotation of the plane of polarisation In reality the light is elliptically polarised, as in the former case. The direction of the apparent rotation is again opposite to that of the currents which produce the magnetisation.

In other experiments, the reflecting surface was magnetised tangentially Kerr showed that when, in this case, the plane of incidence is perpendicular to the lines of magnetisation, and also when the incidence is normal, no effect is produced On the other hand, when the incidence is oblique, elliptic polarisation, similar to that already described, results. When the light is polarised in the plane of incidence, the direction of the apparent turning is opposite to that of the circulation of the exciting currents, on the other hand, when the light is polarised at right angles to the plane of incidence, the apparent rotation is in the same direction as these currents for angles of incidence between 0 and 75°, and changes sign for larger angles of incidence

These results have been verified and extended by various experimenters, especially Righi and Kundt, and mathematical theories of the effects have been worked out, first by FitzGerald, and more recently by Larmor in his general discussion of magneto-optic phenomena. Kerr's papers are to be found in the 'Phil Mag.,' *loc cit supra*, and vol 5 (1878)

In a paper published in the 'Phil. Mag.,' vol 26 (1888), Kerr gave an account of experiments on the double refraction of light in strained glass, and his last paper ('Roy Soc Proc,' 1894) dealt again with the subject of electro-optics

Before Kerr had made his scientific discoveries his University had recognised his educational work, and the merit of his book on Rational Mechanics, by creating him an honorary Doctor of Laws In 1890, he was elected to the Royal Society, and he received, in 1898, a Royal Medal for his scientific researches He died on August 18, 1907, at his residence in Glasgow.

Most of the apparatus and appliances—induction coil, glass cells, electro-magnet, etc—with which Kerr made his discoveries have been acquired by the University of Glasgow, and are preserved there in the new Institute of Natural Philosophy

Dr. Kerr was a shining example of a man engaged in engrossing routine work, and hampered by narrow resources, yet devoting himself with splendid success to scientific research of the highest order. His career shows what can be achieved by patient, slow, unremitting work from day to day, when guided by a true physical instinct and a well trained and well balanced mind. The almost feverish haste of many may well feel rebuked when it is remembered that Kerr was 51 years of age ere he ventured to publish his first paper, and that every detail of his work, when it was made known, stood

the test of the scrutiny of the best scientific investigators of the time. As has been truly said by another, the name of this quiet and unostentatious teacher and experimentalist will be linked for all time with that of Faraday He would not himself have desired any better immortality

A. Gr.

Dr. JAMES BELL, C B, 1825—1908.

Dr. James Bell, C B., formerly Principal of the Chemical Laboratory at Somerset House, was a native of County Armagh, and was born in 1825. For his chemical education Mr. Bell was mainly indebted to the late Professor Williamson. Entering the Inland Revenue Laboratory as an assistant to Mr George Phillips, he became Deputy Principal on the death of Mr. Kay, and on the retirement of Mr Phillips in 1874, Principal, holding that position until his resignation in 1894, when the laboratory was reorganised and placed upon its present footing

Dr Bell's succession to the Principalship was nearly coincident with the parliamentary inquiry into the working of the first Food and Drugs Act of 1872. One result of this inquiry was to make the Somerset House Laboratory a sort of Court of Appeal in cases of disputed analyses, the amended Act of 1875 enabling either party to a suit to refer the incriminated article of food, drink, or drug to the Commissioners of Inland Revenue for the opinion of their chemical advisers Dr Bell thus became identified with the subject of food-analysis and practically all his published work has reference to it For his services in this connection he was made a Fellow of the Society in 1884, and, two years later, an honorary Doctor of Science of the Royal University of Ireland. He had previously obtained the Degree of Doctor of Philosophy of Erlangen. He was made a Companion of the Bath in 1889

Dr. Bell was one of the original Members of the Institute of Chemistry and was President of that body in 1888

He died at Hove on March 31, 1908, and was buried at Ewell, Surrey, where he had resided when Principal of the Inland Revenue Laboratory

T E T.

R. L J ELLERY, 1827—1908

Robert Lewis John Ellery, C M G, Government Astronomer of Victoria from the early fifties until his retirement in 1895, died at his residence, Observatory House, Domain, Melbourne, on the morning of January 14, 1908, at the age of eighty-one Twelve months before he had been stricken with paralysis, but had recovered, and, though his illness had left him feeble, he was in very fair health until the end of December, 1907, when his diminished strength was overtaxed by a spell of exceptionally hot weather

The review of Mr Ellery's long life of scientific activity takes us back to the beginnings of official astronomy and meteorology in Australia Foundations for the work had been laid by private enterprise, for, so early as 1821, Sir Thomas Brisbane had, on being appointed Governor of New South Wales, erected the Paramatta Observatory at his own expense, and initiated a fine series of observations, made by himself and his assistants, Dunlop and Rümker But this noble private work came to an end about 1848, and it was not until some years later that the Governments of three Australian Colonies nearly simultaneously decided to establish observatories. Work was commenced at Williamstown, near Melbourne, by Ellery in 1853, by Todd at Adelaide in 1852, and by Scott at Sydney in 1856 The early history of Ellery's work at Williamstown has fortunately been preserved for us in a letter to the Astronomer Royal, then President of the Royal Astronomical Society ('Mon Not., R.A.S.,' xv, p 154), and a few extracts may be given here. The letter is headed with the curious title, "Account of Operations connected with the Advancement of Commercial Astronomy in Australia," which is more probably due to the recipient than to the sender

In June, 1853, a time-ball signal—visible to the shipping in Hobson's Bay—was erected on the mast or flagstaff, Gellibrand's Point Soon after its erection I was appointed to take the necessary observations for the regulation of this signal At that time two chronometers (neither of them first-class instruments) and two indifferent sextants, with a small artificial horizon, composed the staff of instruments. . . . In August a small transit instrument was offered for sale, and bought by Government, with the works of a new regulator clock by Evans, of Birmingham Authority was given me to get a room built for the instrument and clock, which was done, attached to my quarters.

With these instruments the time-signal has been given daily, Sundays excepted, up to February, 1854, when a few instruments which had been ordered by the Colonial Government arrived, consisting of 30-inch transit by Potter (late Bates), regulator-clock mercurial pendulum by Frodsham, sextant by Potter. . . . Up to this present time no other addition has been made with the exception of one or two plain meteorological instruments . . The time-keeping is, of course, the main object at present but, in addition, I am tabulating a regular list of zenith and circumpolar stars (of course of right ascension only, as the instrument has only a setting circle reading to minutes). I have, I think, determined the longitude of the observatory pretty closely, from the mean

of 35 sets of moon-culminations . The exact instant of the drop of the Williamstown time-ball is marked by my only assistant, my wife ; and if any error occurs, from wind or otherwise, it is thus accurately accounted for, and published in the daily papers. It is proposed to erect time-balls on the electric telegraph stations, which are now in course of erection at Geelong and Port Phillip Heads, so as to make every use of the observatory possible in giving mean time to different parts of the colony .

The sum of £2500 was voted by the Legislative Council for the building a stone observatory at Williamstown this has not been done yet, but the Governor has placed in the estimates for next year the sum of £660 for the purchase of astronomical and meteorological instruments, according to a list of what would absolutely be required that I was instructed to send in (Transit Circle and Equatorial by Troughton and Simms, etc.) . I had omitted to say that since December last I have kept a regular journal of three-hourly meteorological observations—at least so far as my limited staff of instruments allowed me, having only a simple but very good pediment barometer, Zambra's thermometer, and aneroid barometer Having neither rain-gauge, anemometer, or hygrometer, of course my observations are very limited , but, such as they are, they represent some interesting facts with regard to Australian meteorology

Mr Ellery was the son of an English surgeon, Mr John Ellery, and was born at Cranleigh, in Surrey, on July 14, 1827 He was educated for the medical profession, but went out to Australia in 1851, "tempted by the discovery of gold" It is stated that he practised as a surgeon in Williamstown, but it seems clear that this cannot have been for long, and that his duties as Government Astronomer must have claimed his time and attention It may be added here that he was twice married, his first wife—the "only assistant" of the above extract—whom he married in 1853, died in 1856, and in 1858 he married her sister, who survives him The ladies were daughters of the late Dr John Shields, of Launceston.

Returning to the proposal for the observatory, it was apparently made in a letter to the public press by Mr Ellery himself, in the interests of the shipping The suggestion was adopted, and he was asked to carry it into practice It is recorded that "the work at first was slight, and Mr Ellery, besides being Government Astronomer, was storekeeper of the marine depôt" But developments soon followed In 1856 the Government determined upon a geodetic survey of Victoria, and Mr. Ellery was made director of this work, which lasted from 1856 to 1874, in addition to his other duties Indeed, his attention was claimed in many directions We have seen above how he began to make meteorological observations of value, and this work developed rapidly in Australia, so that in September, 1888, he presided at a Meteorological Conference of the Directors of Australasian Observatories, held at the Melbourne Observatory, with the view of improving intercolonial meteorology His name is given as Colonel R L J Ellery, F R S , F.R A S , which reminds us of his career as a citizen soldier In 1873 he organised the Victorian Torpedo Corps, which subsequently became the Submarine Mining Engineers He commanded the corps until 1889, when he retired with the rank of lieutenant-colonel. He was one of the founders of the Royal Society of Victoria, and was its President from 1856 to 1884. Indeed, "he was one of the originators," writes his successor, Mr Baracchi, "of every scientific

movement in Australia during half a century" The close of his career was fittingly marked by his Presidency over the Australasian Association for the Advancement of Science at the Melbourne meeting of 1900.

The main work of a State observatory, connected closely with a geodetic survey, naturally consisted in meridian observations, and several valuable catalogues of stars were published during Mr Ellery's directorate The first contains the observations of 546 stars made in the years 1853—1863 at Williamstown, where Mr Ellery began work But in 1863 the observatory was transferred to the Government Domain in Melbourne, and the first Melbourne Catalogue, published in 1874, contains the observations of 1227 stars from 1863 to 1870 The second (1211 stars, epoch 1880), published in 1890, contains the observations from 1871 to 1884, while the third (epoch 1890) is still awaiting publication for lack of funds. The main work of preparing these catalogues was undertaken by the first assistant, Mr E J White, the story of whose appointment in 1858 is thus related in the Melbourne 'Argus' of January 15th last —

In that year Donati's great comet appeared. The University professors—Professor Wilson and others—wrote to 'The Argus' complaining that there were in Victoria no instruments capable of being used for proper observation However, among the motley population of the diggings an astronomer was working This was Mr E J White, now F.R.A.S, who in the intervals of sinking holes and felling trees used to make astronomical observations from his log hut in California Gully, Bendigo He wrote an account of his observations of the comet, and sent it to 'The Argus.' Within a few days two troopers rode up to his claim to announce to him that the Governor contemplated making him a Government astronomer He hesitated about leaving the free-and-easy digger's life, but eventually joined Mr Ellery in the observatory work

The appointment of this able assistant was not the only result of the appearance of Donati's comet The interest aroused by the event led ultimately to the transference of the observatory to Melbourne, and also to the erection of the great Melbourne reflector Professor Wilson, who was Secretary of the Board of Visitors of the Observatory, and was no doubt assisted by Mr Ellery's advice, made representations to the Government that Lord Rosse had discovered changes in some of the northern nebulæ, and that it was important to examine the nebulæ of the Southern Hemisphere in continuation of this work, and that, in consequence, the erection of a large telescope in Melbourne would "materially promote the advancement of science." The Government were urged to consult scientific men in England on the matter. Enquiries were accordingly made, and reached England in October, 1862 The project for a large telescope in the Southern Hemisphere had been discussed in 1852—1853 at some length by Mr Nasmyth, Lord Rosse, Colonel Sabine, Sir D. Brewster, Mr Piazzi Smyth and his father, Mr. Lassell, Sir J W Lubbock, Sir John Herschel, Dr Robinson, Colonel Cooper, Lord Wrottesley, the Astronomer Royal, and others, but the application to Government had come to nothing owing to the Crimean War. The correspondence was revived, and the general opinion was in favour of a large reflector, either similar to Mr Lassell's four-foot equatorial, which had

been successfully at work in Malta for a year, or after a different design by Mr. Grubb While the choice was under consideration, Mr Lassell, in 1864, offered to present his telescope to the Colonial Government, if it were thought suitable for the work. But the generous offer was ultimately declined in favour of a new instrument, which was ordered from Mr Grubb in 1865 The construction of it took three years, and it was carefully tested at Dublin in 1868, to the great satisfaction of the Committee of the Royal Society appointed for the purpose The instrument was accordingly sent out to Melbourne and erected, but Mr Ellery at once reported its performance disappointing, and there followed a period of grave anxiety and doubt. From the correspondence, which was printed for private circulation, it appears that the fault lay chiefly in one of the two speculum mirrors—the other one gave much better results The maker attributed the trouble to the manner in which a thin coating of varnish, with which the mirrors had been protected for transit, had been removed by Mr Le Sueur, the assistant instructed in England and sent out with the telescope, and Mr Le Sueur ultimately resigned his appointment But Mr Ellery's reports indicate other possible causes for the poor definition. One thing clearly emerges from the printed account—that Mr Ellery showed great skill in dealing with a very trying situation In spite of the unexpected difficulties he got the big instrument to the work for which it was designed, and many drawings of nebulæ were made. A few of these were published in 1885 in a volume labelled Part I, and in the Preface we read "it is now intended to issue the drawings, descriptions, observer's notes, etc, in parts" But no other numbers have since been issued The examples given in Part I show that much information of value was obtained, and though in some ways such work has been superseded by photography, the epoch at which they were made gives a special value to these observations It seems very desirable that they should see the light of publication.

Meanwhile, in order to get the best possible results with the telescope, Mr Ellery at once set about learning to refigure and polish the mirrors, and he ultimately refigured them with success in 1889—90 He was able to report that the "performance of the great telescope is now certainly better than it ever has been previously" ('Mon. Not, R.A.S' li, p 231) It was one of the factors in Mr Ellery's success that he had considerable mechanical skill, and took pleasure in doing things of a mechanical kind to the closing years of his life Such skill is always valuable to an astronomer, especially when at a distance from instrument makers. For, instance, when it was found that the *réseaux* for the work of the Astrographic Chart could not be transmitted to Australia without the appearance of numerous pinholes in the silver film, which rendered them useless, Mr. Ellery promptly devised a plan of making *réseaux* for himself, by ruling a film of asphalt varnish in the observatory ruling machine He applied a parabolic governor to the chronograph, and invented in 1868 a chronograph-pen, which anticipated the modern fountain-pen, he devised a dark field micrometer and

made a clock, and doubtless many other instances of his skill are left unrecorded.

The self-reliance, which necessarily comes from working at a great distance from other workers, appears not only in his mechanical inventiveness, but in his statement of scientific results We may take as an example the following extract from his report in February, 1892 He was just undertaking, at the age of sixty-five, a totally new piece of work, viz, the Melbourne share of the Astrographic Chart The telescope had arrived in December, 1890, and had been set up in January, 1891 Observers had to be trained, and, before the end of 1891, when the report was written, the supply of plates had failed. Nevertheless, as the result of this early few months' work, Mr. Ellery announces that "to gain one magnitude (ratio 2 512), the time of exposure must be increased 3 16 times, or, to gain two magnitudes, it must be increased ten times" ('Mon Not,' lii, p 265) This is a definite statement on a matter which has been much debated in the years since elapsed The writer of this notice, in 1905, analysed the result of extensive investigations made at Greenwich, and arrived at identically the same result as Ellery had obtained and announced fifteen years earlier (see 'Mon Not,' lxv, p. 763, line 29), and other workers have independently reached a similar conclusion

Ellery takes high rank as a pioneer He met and overcame nearly all the difficulties associated with the starting of scientific work in a new land. One alone proved somewhat intractable—the difficulty of getting published the observations made with so much courage and toil The observations made with the great telescope and the Third Melbourne Catalogue are referred to above, but there are also the zone observations The project for a great zone-survey of the Southern Hemisphere has now been almost forgotten, and yet a great deal of work was done under this scheme The Melbourne zones were from 150° to 160° N P D, and were reported nearly complete in 1878, but nothing has been published, and now that one photographic Durchmusterung has been carried through at the Cape, and another of greater accuracy is under way, it seems possible that these earlier measures never will be published But we must not forget that two Melbourne catalogues were published in good time, and a third, with possibly also the lithographs of the nebulæ, may still be saved. In any case it is earnestly to be hoped that the work on the Astrographic Catalogue, initiated so ably by Ellery in the closing years of his long career, will not suffer in the same way, and that, profiting by the experience of the past, the printing of it may be undertaken in the near future

H. H. T.

SPENCER COMPTON CAVENDISH, DUKE OF DEVONSHIRE, 1833—1908

SPENCER COMPTON CAVENDISH, eighth Duke of Devonshire, eldest son of the Earl of Burlington, who subsequently became seventh Duke of Devonshire, was born July 23, 1833. He was educated privately under his father's eye, who, it is said—and in all probability truly said—wished the defects of his own education to be corrected in that which he gave his sons The elder Duke had taken very high honours at Cambridge, both in mathematics and classics, at a time when these were the only branches of knowledge in which students had any opportunity of distinguishing themselves at the old Universities of England, but concentration on these subjects had not well qualified him to take the prominent part in politics and in the Senate to which his social position called him, and he felt that a general education in a wider field was more important for his sons than academic distinction.

To Trinity College, Cambridge, Lord Cavendish, as he was then called, went in due course, and though he did not emulate his father in the triposes, nevertheless took a good place in the second class of mathematical honours in 1854, and received the degree of M A, to which, as a nobleman, he was entitled under the statutes of the University then in force, after residing two years and passing the examination Very soon afterwards he entered on a political career, for he was attached to Earl Granville's mission to Russia at the close of the Crimean War He had not the natural gifts of vivacity and readiness in conversation which make a man shine in social gatherings, and the coldness of Lord Granville's reception at the Court of the Czar could hardly have tended to make his position more agreeable, so his first diplomatic mission was also his last He was in a far better fitting place when, in 1857, he entered, as Liberal Member for North Lancashire, the House of Commons, where the character he had inherited from his father, of thoughtfulness, sincerity, and devotion to duty, was sure to be appreciated when he came to be known. He made few mistakes, was slow in making up his mind, but stuck to his opinions when he had formed them After the general election in 1859 he passed suddenly, and to the public unexpectedly, into the front rank of the Liberal Party The occasion was the Address at the opening of the new Parliament, when he moved the amendment expressing want of confidence in the Ministers, which was carried in a full House, and turned Lord Derby out of office. By this time his father had succeeded to the Dukedom of Devonshire, and he had taken the title of Marquis of Hartington, and was doubtless selected from the independent members of his party to move the amendment because he was the representative of a great Whig

family This was, however, merely the occasion of his coming to the front; it was the confidence inspired when his personal character came to be known that kept him there. For he was not an eloquent speaker, nor great in debate, though very painstaking in making sure of his ground, and his convictions were always well defined in his own mind, so that he never failed to give them expression in a way which made them clear to his hearers. Of course he took office in the new Ministry, and soon became a Cabinet Minister The variety of offices which he subsequently held is remarkable. Beginning at the Admiralty, he was at different times Secretary for War, for Ireland and for India, was Postmaster-General, and President of the Council, including at that time the Committees of Council on Education, both Elementary and in Science and Art He seemed equally well suited for all these posts, because he was capable of taking the trouble to make himself acquainted with the business he had to do, whatever it might be, and the sense of duty which impelled him to try and do it as thoroughly well as lay in his power He was a steady supporter of the Irish Church Disestablishment, and lost his seat in North Lancashire in consequence, a consequence, perhaps, in some degree due to the racial antipathy of the Lancashire folk to the Irish, who competed with them for work while content with a lower scale of living As Irish Secretary he found the management of Irish affairs no easy task, and had to promote the passing of the Peace Preservation Act, in spite of Mr. Gladstone's unwilling attitude and his own intimate personal friendship with his chief Other troubles beside the agrarian one confronted him in Ireland, which he met with consistent firmness, but it all led him to form that judgment of the Irish national character which made him subsequently a firm opponent of Mr Gladstone's Home Rule policy for Ireland, and an outspoken adversary of any course which would endanger the Union. His independent judgment did not prevent Mr Gladstone from urging him to undertake, in 1875, the Liberal leadership of the House of Commons, nor him from maintaining his respect and friendship for Mr Gladstone. For five years he led the Opposition, a most difficult and onerous task, discharged in a way most honourable to himself with complete consistency to his principles He had an innate distrust of measures prompted by sentiment or by optimistic views of human nature not warranted by experience, and firmly maintained that organised crime and intimidation could only be met by exceptional legislation

After the defeat of the Conservative Party at the General Election in 1880, Lord Hartington was sent for by the Queen to form the new Ministry, but he pointed out that Mr Gladstone's powerful personality made it well nigh impossible for a Liberal Ministry to succeed under any chief but him. Lord Granville took the same view, and both helped to persuade Mr. Gladstone to return to office Lord Hartington at first took post under him at the India Office, and later went to his old post at the War Office. He worked loyally and manfully under Mr Gladstone as long as he could honestly do so. How great was the strain under which this was done will be understood

when it is borne in mind that in this period the Egyptian Soudan was evacuated, and he had to defend in the House of Commons the delay of the relief of Khartoum, a delay entirely due to his colleagues and contrary to his own advice. In this period, too, occurred the assassination of his brother, Lord Frederic Cavendish, which debarred him from taking a part in Irish affairs, to which he could be supposed to have been biassed by personal feelings As a member of the Government he gave a general support to Mr. Gladstone's Franchise Bill, but took no prominent part in the debates in the House of Commons on the question whether the franchise should be enlarged without a simultaneous redistribution of seats. Outside, however, his speeches materially assisted the sort of compromise by which both the Franchise and the Redistribution Bills ultimately passed Very soon after this the Government was defeated on the Budget and the Ministry resigned, and Lord Hartington did not resume office until the coalition of Unionists ten years later In the short interval that followed with Lord Salisbury in office, when Mr. Gladstone indicated his leaning to Home Rule, Lord Hartington made it quite clear that he would make no further concessions in that direction, and he voted against the motion which turned Lord Salisbury out On the same occasion a large section of Liberals, including so staunch a member of the party as Mr John Bright, showed their distrust of Mr Gladstone by not voting at all. This breach with his old friend and leader must have been very painful to both of them, but Lord Hartington was firm in the position he had taken, and, when Mr Gladstone's Home Rule Bill came on, spoke against it, both in and out of the House, putting his arguments, without rhetorical effort, in a clear, logical form, which was extremely effective, and brought about the defeat of the Bill at the second reading

Lord Salisbury, who was then asked by the Queen to form the new Ministry, offered to make way for Lord Hartington, but he declined the offer, considering that his following was not yet strong enough or sufficiently consolidated However, in the Parliament thereupon elected he and the Liberal Unionists supported Lord Salisbury, while they were nevertheless ready to rejoin the rest of the Liberal party if the Home Rule policy were dropped The hope of such a result gradually faded, nevertheless Lord Hartington's efforts to keep all the Unionists in line were so successful that when, in 1892, Mr Gladstone returned to power, he had not a sufficient majority to carry a measure of Home Rule Lord Hartington had become Duke of Devonshire in 1891, and it was he who moved, in the House of Lords, the rejection of the second Home Rule Bill, when he was supported by many of the Liberal Peers. The triumph of his exertions appeared at the general election in 1895, when the Unionists came in with a majority hitherto almost unexampled, which invited the formation of a Ministry including the leading men of both wings. The Duke, along with others of his following, notably Mr. Joseph Chamberlain, took office under Lord Salisbury. His office was that of President of the Council, and he held it

until the reconstruction of Mr Balfour's Ministry in 1903 That office did not involve much administrative work, though it made him responsible for the control of the Education Department. If his influence there was not much in evidence, it was not because he lacked interest in the education of the country—how important that was in his eyes was abundantly shown elsewhere—but because the action of the education office was so hampered by the jealousies of different denominations and parties in religion that the question of the best education for the masses had to be subordinate to the question of what would work at all smoothly The two education bills of which he had charge and passed were, in fact, compromises to which he assented, because better seemed at the time impracticable He had at the same time other work to do. Some years earlier he had been Chairman of a Royal Commission on the defence of the Empire, and the report of that Commission had met with general approval, but had led to no action One of its most important recommendations had been the appointment of a permanent Council of Imperial Defence, independent of parties. This proposal was now revived, and the Duke became the first President of this Council.

In 1903, when Mr. Chamberlain raised the question of fiscal reform, the Duke did not object to an enquiry as to the effect of our fiscal policy on our relations with the Colonies, and on industrial competition with other countries, but the question seems to have presented itself differently to him when he found that enquiry meant, in the minds of many of his colleagues in the Ministry, the justification of a foregone conclusion which was opposed to his firm convictions The result was his withdrawal from the Ministry and a split of the Liberal-Unionist party He insisted that any tax on food must weaken us by pressing on our working population, and that duties imposed by way of retaliation were neither justifiable nor likely to give a firm standing in competition, nevertheless, he never wavered in his belief, that the Union was good policy He was now seventy years of age, and though he continued to take part in the debates in Parliament, he was more independent of party, and of course to the outer world appeared a less prominent figure than heretofore

Apart from politics, he found time and energy to take an active part in many other matters of public concern. In 1891 he was chosen Chancellor of the University of Cambridge in succession to his father. He filled the office until his death, and showed the same high sense of duty and the same willingness, at the expense of much time and personal trouble, to promote the interests of the University and to enlarge its influence and usefulness, as his father had shown before him. The awakening of the University from the lethargy engendered by having been, for two and a-half centuries, hardly stirred from without by any but religious and political controversies, while its Chancellors were chosen for the influence they could exert in maintaining its privileges and independence, had begun from within under the inspiration of the Prince Consort, who was Chancellor from 1847

to 1861. Originally a chartered trade union of independent teachers under no control but that of the Chancellor, elected by themselves, it had remained almost wholly unendowed. Most of the professors and all the other officers of the University were paid by fees alone, until nearly the end of the Prince Consort's tenure of the Chancellorship The Colleges, quite independent corporations, mostly well endowed, monopolised the care of the young students and the direction of their studies Want of funds, more than anything else, has hindered the University in enlarging its curriculum, and to remedy this the late Duke devoted a great amount of patient and self-denying labour, continued quite up to the end of his life His father had early seen the difficulty in which the University was involved, and had himself built and equipped the Cavendish Laboratory for Physical Investigation, which under Clerk Maxwell, Lord Rayleigh, and J J Thomson has become world famous, and so much frequented that it has had twice to be enlarged Other laboratories have been built by the University, partly with money borrowed on the security of the annual contributions from the Colleges, assisted by liberal donations from individuals interested in the sciences with which they were connected The demand rapidly outran the means of meeting it, and at an influential gathering of members of the University at Devonshire House it was decided to make an appeal for contributions. The subscription list was opened with £10,000 from the Chancellor and a like sum from Lord Rothschild The appeal took the form of a circular letter written by the Chancellor, at first to the members of the University and ultimately to the public But the Chancellor exerted himself by writing in person to the wealthy people of his acquaintance who were likely to be interested in higher education, and presided at the meetings which were called at intervals to keep up an interest in the movement The donations have exceeded £100,000, but the Duke was greatly disappointed that they did not amount to much more, when half a million had been asked for to meet urgent needs, and he took much pains to ascertain why the interest in higher education of the University type was so much less in England than in the United States and Canada He found that there was a wide-spread belief amongst the men who had amassed large fortunes that the education given at Cambridge was not the best preparation for the practical business of life, and especially that time was wasted in the study of the classical languages without, in most cases, any adequate result, and he did not fail to press this on the attention of the leading members of the University When the Liberal Party came into power in 1905 he sympathised with the proposal, which arose within the University, to endeavour to get an Act of Parliament to modify its constitution so as to give those actually doing the work of the University fuller control of the courses of study, and was disappointed that it went forward so slowly.

Apart from his desire for a general widening of the range of study, he took a particular interest in the development at Cambridge of a school of scientific agriculture. Before he became Chancellor he was a member of a syndicate, appointed by the University in 1890, to consider the promotion of education

in that subject, and later it was through his influence that the Drapers' Company provided a liberal endowment for the Professorship of Agriculture. When the school had taken root, he again came to the front, in the endeavour to obtain subscriptions for suitable buildings and further equipment for it. He was again disappointed in the result. The amount subscribed, though a substantial help to the University, was inadequate, and he remarked. "We must hope that the fruit of our labour will come in legacies." It cannot be said that his labours as Chancellor were a failure The expansion of the University in regard to teachers and taught, and in the opportunities for learning, has at no time been greater than under his presidency, and he will always be remembered there as a benefactor In the matter of higher education Cambridge had no monopoly of his attention In 1877 he was chosen Lord Rector of the University of Glasgow, and quite recently he was elected Chancellor of the Victoria University at Manchester. He was some time President of the National Association for the Promotion of Technical and Secondary Education, and the success of the local grammar schools in the parts of the country with which he was specially connected always had his sympathetic regard Indeed, he was always ready to do his part in promoting the welfare of the districts in which his property lay. For fifty years he was Provincial Grand Master of the Freemasons for Derbyshire

Rather late in life, in 1892, he married Louise, widow of the seventh Duke of Manchester, and left no family.

He had never been physically a very strong man His weak organs were his lungs In 1904, when the King went to Cambridge to open the Sedgwick Museum, he was laid up with a bronchial attack, and unable to receive him On June 12, 1907, he went to Cambridge to confer honorary degrees on the Prime Minister and a group of distinguished persons He performed the duties of his office, spoke well at the dinner in the Hall of Trinity College, and to the public appeared to be in his usual health, but to those near him who knew him it was quite plain that he was far from well, and that it was only his strength of will to do his duty which carried him through. It was but a few days later that he was prostrated by a severe attack while on a visit to the King at Windsor Castle. This caused the greatest alarm to his friends, but he very slowly recovered, and in November wrote that he was quite convalescent, both fit and inclined to attend to business, and asked for information of what was going on in the University, particularly about the movement to obtain a reform of the constitution of that body He spent the winter in Egypt, and in the spring was to return to England, and contemplated making, after Easter, his first public appearance at Manchester as Chancellor of the Victoria University. In descending the Nile he caught a chill, which brought on another severe attack, from which he rallied but never wholly recovered, and later, on his way home, at Cannes, he had a relapse which proved fatal on March 24, 1908.

The Duke will certainly live in the history of his country, but it is too

soon to estimate the position which he will occupy in that history, and were it otherwise, the writer of this notice is not qualified to foretell it. He would assuredly have been Prime Minister if no personality so towering as Gladstone's had stood in the way He was not the originator of any great line of political action, nor ambitious of such a distinction. But his character was essentially English, patriotic, liberal, and fair-minded. He never suffered sentiment to obscure his vision, and was never carried away by the zeal of other people. Always practical, he trusted to experience against theory, and his policy was to deal with men as their actions showed them to be, not as he wished them to be Careful in forming his judgments, he had little occasion to review them, and did not flinch in carrying them out

G D L

COLONEL ANDREW WILSON BAIRD, 1842—1908.*

Andrew Wilson Baird, the eldest son of Thomas Baird of Cults, Aberdeen, was born on April 26, 1842. He received his earlier education at Marischal College, Aberdeen, and in 1859 he entered the Military College of the East India Company at Addiscombe. When that college was closed in 1860 he was transferred with the other cadets to the Military Academy at Woolwich.

He obtained his commission as Lieutenant in the Royal Engineers in December, 1861, and after the usual course of instruction in military engineering at Chatham, he went to India in February, 1864 †

In India he served under the Bombay Government as Special Assistant in the Harbour Defences, and up to the end of 1865 had charge of the construction of certain harbour batteries He was then made Special Assistant Engineer in the reclamations of the foreshore at Bombay.

From January until July, 1868 he was taken from his peaceful work to serve as Assistant Field Engineer in the Abyssinian Expedition under Sir Robert Napier, afterwards Lord Napier of Magdala. He was mentioned in despatches for his conduct of the embarkation of troops and baggage, and received the war medal for that expedition.

Shortly after his return to India Lieutenant Baird was appointed to the Trigonometrical Survey of India and was attached to the Triangulation

* This Notice is founded on information supplied by Mrs Baird, and on the private knowledge of the author, and on his correspondence with Mr. Roberts and others.

† The dates of his several commissions are as follows :—lieutenant, 1861 ; captain, 1874 ; major, 1881 ; lieutenant-colonel, 1888 , colonel, 1893.

Parties in Kathiawar and Guzerat He suffered much from the extreme heat in this and part of the country, and was compelled to take leave of absence in England in the spring of 1870.

His zeal and accuracy in scientific work had become known to General (then Colonel) Walker, the Surveyor-General, and it was whilst he was in England that he was selected to organise the tidal work in India. The admirable manner in which he carried out his task naturally led ultimately in 1885 to his election to the Royal Society

In the first instance, tidal observations were only undertaken by the Survey of India with the object of determining the mean sea-level as a datum for the Trigonometrical Survey The first observations were of a very imperfect character, and it was not until 1855 that a self-registering tide-gauge was used, and even then the observations were restricted to a month in duration At a later date Colonel Baird himself gave the following account* of the origin of the tidal survey —

"Subsequently to 1855 it was desired to investigate the relations between the levels of land and sea on the coasts of the Gulf of Cutch, which were believed by geologists to be gradually changing This necessitated a more exact determination of the mean sea-level than had hitherto sufficed for the operations of the Survey and thus, in 1872, General Walker, who was then Superintendent of the Survey, decided on having the observations at the stations in the Gulf of Cutch carried on in accordance with the recommendations of the Tidal Committee of the British Association, by self-registering gauges set up for at least a year to determine the existing relations of land and sea, and again for another year when a sufficient interval had elapsed for a sensible change to have taken place †

"I was then (1872) in England, and was deputed by the Secretary of State for India to study the details of tidal registrations and the reduction of observations by harmonic analysis, in accordance with the method suggested by Sir William Thomson I also made the necessary arrangements to secure self-registering barometers and anemometers, to indicate the atmospheric conditions prevailing at each tidal station concurrently with the changes of sea-level I then proceeded (in December, 1872) to India, selected sites for three stations in the Gulf of Cutch, tested all the instruments to be employed at Bombay before sending them to their destinations (where no artificers would be available to modify them), fixed them in position, and supervised the registrations which were carried on for a year at each station

"In 1877 the Government of India issued instructions to the effect that systematic tidal observations should be undertaken at all the principal Indian ports, and at other ports on the coast-lines where results might be acquired which would be of general scientific interest apart from their practical usefulness in affording data for the calculations of the rise and fall of the

* Preface to Baird's 'Manual of Tidal Observations.'

† These stations have recently been reopened and the previous mean levels have been found to be virtually unchanged.—G H D.

tides and the construction of tide-tables for the purposes of navigation. I was entrusted with the carrying out of the operations, under the direction of General Walker."

As indicated in the foregoing quotation it was determined by the Government of India that the tidal survey should be as thorough as possible, and the method of harmonic analysis, then recently inaugurated by Sir William Thomson, was to be adopted

Mr. Edward Roberts had already prepared the requisite computation forms for a Committee of the British Association, and the tides at a few ports had already been reduced under his supervision.

The construction of an instrument for the mechanical prediction of tides from harmonic constants was already in contemplation, although it was not until 1879 that the first tidal predictions, those for the year 1880, were actually produced.

After the surveys in the Gulf of Cutch to which Baird refers, he was sent to England in 1876 with the object of studying the details of the methods to be employed Whilst he was there he was in constant communication with General Walker (himself on leave), Lord Kelvin, and Mr Roberts. Mr Roberts reduced the observations at the three ports of the Gulf of Cutch, and Baird had good opportunities of studying the methods.

When Baird was placed in charge of this new department he had to rely almost entirely on his own initiative His knowledge of mathematics had naturally become somewhat rusty from disuse, and moreover he could have had no knowledge of the particular mathematical developments required in the method of harmonic analysis He also knew but little of tide-gauges, of the methods of installation, and of the various mechanical difficulties involved in the successful use of that instrument

His work was of a very varied character, for he had to determine all the details of the installation of tide-gauges, to arrange for their supervision, and for the accuracy of the readings He had further to organise an office for computers at Poona, in which he was assisted by Mr Connor

When all this had been done it remained to work the machine he had created The tide-curves had to be collected from the several stations, corrected for clock errors, and measured The resulting numbers had to be submitted to the laborious processes of reduction, and the resulting constants transmitted to Mr Roberts for the production of the requisite tide-tables by means of the tide-predicting instrument which had been constructed for the Indian Government It speaks volumes for Baird's scientific power and for his administrative ability that the organisation which he devised has remained unchanged from the time of its initiation up to now

Such undertakings as the creation of a new department naturally do not attain to their full development at once, and we have forestalled matters by giving this general sketch of the work Thus for 1880 tidal predictions were only published for two ports, for 1881 for eight ports, and for 1882 for

15 ports. The increase was gradual, and the tables for 1908 contain predictions for 40 eastern ports.

I first met Baird at Lord Kelvin's house in December, 1882, when we discussed a revision of the mathematical development of the harmonic method. The result was the report to the British Association in 1883, in which certain changes of procedure were recommended From that time until the end of Baird's tenure of his position in the Survey of India I was in constant communication with him

Perhaps the most convincing proof of the care and foresight with which the tidal department was founded is afforded by the fact that there is nothing to record as to its subsequent career, but it is obvious that the work demanded, and still demands, constant attention to scientific and administrative details

Towards the end of Baird's tenure of his post he wrote a useful handbook, entitled 'A Manual of Tidal Observations' It was published in 1886,* and a passage from the preface has been quoted above as to the early history of the department The work contains a valuable account of his practical experience, together with full instructions for the application of the harmonic method of reduction.

In 1881 Baird was sent as Commissioner for India to the International Congress of Geography at Venice It is remarked in the report of the meeting that his communication on levelling and on tidal work shows the great progress made in this branch of science A medal of the first class was awarded to him for his exhibit of the drawings of a tide-gauge showing the improvements introduced by him †

On his return to India in the spring of 1883 he resumed his position in charge of the tidal and levelling operations, until in July 1885, he was appointed to officiate for Colonel Riddell, R E, on leave, as Mint Master at Calcutta He subsequently acted for Colonel Riddell and General White as Mint Master on several occasions both at Calcutta and Bombay, and in the intervals he held the appointment of Assistant Surveyor-General Finally, in 1889, he was permanently appointed Master of the Mint at Calcutta, which position he held until his retirement in 1897 During his tenure of the Mastership he carried out an almost complete reorganisation of the manufacturing department of the Mint, whereby economy and efficiency were largely promoted.

In 1895—96 the Government of India saw reason to fear that the coinage of India was becoming seriously worn, and it was suspected that some of the older issues ought to be withdrawn Baird was charged with an enquiry into the state of the coinage His report‡ contains an elaborate investigation of

* By Taylor and Francis

† See Report of the 'Terzo Congresso Geografico Internazionale tenuto a Venezia,' 1881, pp. 237—8; also 'Proceedings of the Royal Geographical Society,' December, 1881

‡ No. 278, dated 17th March, 1896 "From Colonel A. W. Baird, R E, F R.S., etc, Master of the Mint, Calcutta, to the Secretary to the Government of India, Finance and Commerce Department."

the subject. Although the paper was not published, I have been allowed to see a copy. It is naturally intensely technical, but even to a reader wholly ignorant of coinage as I am, it possesses great interest. Such points as the average annual wear of the rupee and of the smaller coins, the mean annual accretion of dirt on the old coins, are studied with scientific accuracy, and it is curious to note that the weight of the accretion is almost proportional to the age of the coin Plans for the withdrawal of the old coins from circulation are further discussed with great ability.

The Government adopted almost all his suggestions, and it was in consequence of his work that the coinage was re-established on a thoroughly satisfactory footing

It is on such men as Baird and on such unwearying services as those sketched above that the administration of the Indian Empire depends Official recognition of this is recorded by the Companionship of the Star of India, which he received in 1897 In science he has left a permanent mark as the successful organiser of the first extensive operations in tidal observations by the new methods. The treatment of tidal observations is now made by harmonic analysis in every part of the world, and this extensive international development is largely due to the ability with which he carried out this pioneer work in India

On his retirement in 1897 he went to live near Elgin, and amused himself with the improvement of the house and garden on the little property which he had bought there, and at first he enjoyed some sport His health, however, gradually became very delicate, and he frequently passed the winter abroad in warmer climates. His death occurred suddenly in London on April 2, 1908, in consequence of failure of the heart.

Whilst in England in 1872 he married Margaret Elizabeth, only daughter of Mr. Charles Davidson, of Forrester Hill, Aberdeen, who survives him. He leaves also two sons, both in military service, and five daughters.

G H. D.

WILLIAM ASHWELL SHENSTONE, 1850—1908

The life of a schoolmaster is usually laborious, and when he devotes the greater part of his holidays to study and experimental research, little time is left for incident outside the daily routine Shenstone's career was uneventful, but it brought him into frequent contact with the scientific world, in which he made many lasting friendships

Born at Colchester on December 1, 1850, he came of a family collateral with that of the poet, as it appears that his grandfather, Joseph Shenstone, born at Halesowen between 1790 and 1800, was a son of a cousin of that writer W A Shenstone's father, James Burt Byron Shenstone, was established as a pharmacist at Colchester, and after the usual course of education at the Colchester Grammar School the son was received into the business At the age of twenty he obtained, as the result of competitive examination, one of the Bell Scholarships, given annually by the Pharmaceutical Society After a course of study in the School of that Society he won the Perera Medal and other prizes During the time he was occupied in his father's business a portion of the premises was burnt down, and Shenstone was exposed to cold and damp, with the result that a serious illness followed, from the effects of which he never completely recovered. The injury which resulted was, in fact, the primary cause of his last illness nearly forty years later

After two years spent in the laboratory of the Pharmaceutical Society, Shenstone accepted an engagement with Dr W. A. Tilden, at that time chief science master at Clifton College Here he displayed those qualities of patience, diligence, accuracy, and enthusiastic devotion to experimental work, which distinguished him throughout life In 1874 work was begun on the terpenes, at that time an unexplored department of organic chemistry, and Shenstone joined in the investigation with his usual ardour The result was a paper, under the joint names of Tilden and Shenstone, published in the 'Transactions of the Chemical Society,' in which it was shown that the numerous hydrocarbons of this family then known were reducible to three classes according to the character of their nitrosoderivatives

From Clifton, in 1875, Shenstone passed to the Science Mastership at Taunton School, then under the Head Mastership of the Rev William Tuckwell After about two years he moved to a similar position at Exeter School. Here he built and fitted up a school laboratory (described in 'Nature,' July 26, 1878) which attracted some attention, as it served to show that the practical teaching of physics and chemistry was within the reach of all properly conducted schools, and need not be avoided on the ground of expense, as had been up to that time so frequently alleged.

Shenstone was always an attractive lecturer, and interested himself much in educational problems, especially in connection with the improvement of methods for teaching physical science. When, in 1880, he succeeded Dr Tilden as Science Master at Clifton, he found a field in which he could bring into operation some of his ideas. Here he was one of the first science masters to associate practical work with all theoretical teaching, and here he made use of a modified "heuristic" system, in which an attempt is made to stimulate the disposition to enquiry which is inherent in most young minds. The extent to which this system in the hands of a competent teacher can achieve success is to some extent indicated by the very wide circulation of Shenstone's two books ('Practical Introduction to Chemistry'), and there can be no doubt that this pioneering work has materially influenced the teaching of natural science throughout the kingdom.

The greater part of his leisure was devoted to original research. During the earlier part of his time at Clifton he managed to carry out a difficult piece of work on ozone, and on the properties of certain highly purified substances, from which he drew important conclusions. The results of these experiments led him to the conclusion that chlorine, bromine, and iodine, when in the highest known condition of purity, and dried by contact under special conditions with phosphoric oxide for six months or more, do at once combine with mercury when brought into contact with that metal similarly purified and dried. The remarkable indifference displayed by some substances, when completely deprived of moisture, to the ordinary conditions of chemical change, have led some chemists to believe that it is impossible for any two substances to interact chemically except in the presence of a third substance, usually water, though only in minute quantity. The question must be regarded as still open, but it will be difficult for any future experimenter to improve on Shenstone's method or his manipulative skill.

Shenstone also joined Professor Tilden in a research on the "Solubility of Salts in Water at High Temperatures," the results of which form the subject of a paper in the 'Philosophical Transactions' for 1884. In most of this work a good deal of complicated glass apparatus was required, and the whole of that which was needed in his work on ozone, on the purification of chlorine, etc., was constructed with his own hands. Shenstone was an accomplished glass blower and in 1886 he published a small book, the first of its kind, giving instructions suitable for laboratory workers, which was translated into German and is to be found in most chemical laboratories. Shenstone also succeeded in the far more difficult art of manipulating vitrified quartz, the properties of which he demonstrated in a lecture at the Royal Institution in 1901. Apparatus constructed of this material is of the utmost value in certain chemical and physical operations owing to the fact that it is unaffected by nearly all chemical agents except hydrofluoric acid, and that its coefficient of dilatation by heat is practically *nil*. Hence, unlike glass, it may be suddenly quenched in cold water from a red heat

without injury. He devoted much time during the last two or three years of his life to the practical question of producing this material on a manufacturing scale, so as to bring it within the reach of scientific workers at a moderate cost, and to these efforts the physicists and chemists of to-day owe the beautiful clear variety of silica vessels now commonly in use.

Shenstone was elected into the Royal Society in 1898. In his later years he devoted some time to literary work and produced a 'Life of Liebig' (Cassell and Co.), and the year before he died he gathered into one volume, under the title 'The New Physics and Chemistry,' a series of essays contributed to the 'Cornhill Magazine,' in which he presented a remarkable and, on the whole successful, attempt to explain in popular language the modern views of physicists and chemists on subjects such as matter, motion, the ether, the nature of atoms and molecules, radio-activity, etc.

He married, in 1883, Mildred, daughter of the late Rev. Reginald Durrant, and he leaves a widow with a son and a daughter.

Shenstone possessed a fortunate combination of enthusiasm with peculiar skill in handling apparatus, and under more favourable conditions would certainly have done a much larger amount of scientific work. But what he did accomplish was a marvel to those who knew him well. In spite of the lameness, the cause of which has already been referred to, and which sometimes led to general disturbance of health, the brave, bright spirit, which endeared him to his friends, helped him over mountains of routine work and kept him ever eager for the more attractive fields of science. He died at Mullion, Cornwall, on February 3, 1908, and is buried in the village churchyard.

W. A. T.

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HARRISON AND SONS, Printers in Ordinary to His Majesty, St Martin's Lane.

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